Dissertation

Next-Leading Order Corrections for Proton Decay in Supersymmetric Unification

(超対称大統一理論における陽子崩壊の高次補正)

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ABSTRACT

The discovery of a Higgs boson with a mass of 125 GeV at the Large Hadron Collider (LHC) experiments opened the door to investigating the new physics beyond the standard model. While the standard model (SM) of particle physics is quite consistent with various experimental data, there remain the problems that the SM does not explain. Supersymmetric grand unified theories (SUSY GUTs) are one of the promising extensions of the SM. The framework of SUSY GUTs gives a big-picture of our world below the Planck scale at which quantum gravitational effects become important. Proton decay is a significant phenomenon to verify the SUSY GUTs.

The SUSY GUTs also have several problems. The familiar one is a kind of fine-tuning problem called the Doublet-Triplet Splitting problem. The missing partner model which solves this problem requires higher-dimensional representation Higgs multiplets. Various exotic models containing higher-dimensional multiplets have been proposed in order to solve problems in the SUSY GUTs.

In the context of the SUSY GUTs, supersymmetric standard models (SUSY SMs) are the low-energy effective theories of them. Lately, the low-energy theories have been constrained because of no signature of new particles at the LHC. After the Higgs discovery, some modifications of the minimal SUSY SM have been proposed in order to explain the observed Higgs mass and no signature of the SUSY particles. In this thesis, we consider the models with extra vector-like multiplets and ones with heavy scalar SUSY particles (sfermions). These models have been focused attention on because they predict diverse phenomenology.

The low-energy observables associated with hadron physics are divided into two parts: the Wilson coefficients and the hadron matrix elements. The former includes the effects of UV physics, such as SUSY GUTs, as radiative corrections. Next-Leading order (NLO) corrections to the Wilson coefficients have already been performed partially. The latter requires non-perturbative calculations due to the strong coupling at the hadronic scale. The lattice simulation of the hadron matrix elements has also been progressing.

In this thesis, we have derived the threshold corrections for proton lifetime, which are comparable with the other NLO correction, but have not been estimated up to present. Furthermore, we have estimated the size of threshold corrections at the GUT scale and the SUSY scale in various models numerically.

For the threshold corrections at the SUSY mass scale, the effect is found to be about a few percent in decoupled sfermion scenarios. The threshold effect at the GUT scale is strongly model dependent. The effect is less prominent in the minimal SUSY $SU(5)$ model with extra vector-like matters in spite of the large unified coupling. We, however, have found that proton lifetime gets longer by about 60% in the missing partner model.

Contents

Chapter 1

Introduction

1.1 Overview and Organization

The standard model (SM) of particle physics is a phenomenological model concerning fundamental interactions of matters and force carriers. This model was proposed in the late 1960s and was developed from the 1960s to the 1970s [1–7]. Many experiments and observations have verified it so far. The ATLAS and CMS collaborations reported that a scalar boson was discovered at the LHC experiment in July 2012 and it was totally consistent with the SM Higgs boson [8, 9]. The collaborations have also reported there is no remarkable deviation from predictions of the SM in spite of their efforts.

However, there is strong evidence that a model beyond the SM (BSM) is needed. Indeed, several experiments and observations have already shown the evidence of the BSMs. First, the mass of neutrinos should not be zero for explaining the neutrino oscillation experiments [10–23] while neutrinos are massless in the SM. The standard cosmology also gives good explanation for the thermal history of the Universe. The latest observation by the Planck satellite requires sufficient densities of non-baryonic matters and hypothetical energy, which are called "dark matter" (DM) and "dark energy", respectively [24]. In particular, there is no DM candidate in the SM, and hence a new particle (stable and electrically neutral particle) and/or a massive object (such as primordial black hole) is required. The SM does not also explain the baryon-antibaryon asymmetry in the Universe, that is Sakharov's condition [25] is not satisfied within the SM.

From theoretical aspects, we have also expected the new physics beyond the SM to explain some problems. One of them is known as the hierarchy problem in the Higgs boson mass parameter, which requires the precise cancellation between the bare parameter and radiative corrections if the SM is valid up to the Planck scale. The charge quantization of matters is also one of the unexplained problems. In fact, the arbitrary real number is generically allowed as a $U(1)$ charge while all electric charges for matters are discretized in the SM.

As we will discuss in Section 1.3, supersymmetry (SUSY) is a promising candidate of the extensions of the SM. New particles, which have opposite spin-statistics of the SM particles, are naturally introduced by extending the space-time to the one with Grassmann coordinates. These new particles are called "sparticles" and play important roles to solve the problems if the mass scale of sparticles is around a few TeV. The scalar partner of top quark ensures the cancellation of the Higgs mass parameter by virtue of the non-renormalization theorem in SUSY field theories. The lightest fermionic partner of the SM bosons become a candidate of the DM.

The LHC measurements of properties of the SM particles, especially the Higgs boson, open the door to exploring the BSM. The observed mass of the Higgs boson is 125 GeV, which is the combined result from the ATLAS and CMS collaborations [26]. In the minimal setup of the SUSY SM (MSSM), the mass of the Higgs boson should be lighter than the Z boson at tree-level. This fact gives further information about the SUSY SM: large quantum corrections to the Higgs boson mass are required in the MSSM. Alternatively, the Higgs mass is lifted up by extra fields coupling with the Higgs multiplets in the extended MSSM. For instance, familiar extensions are the following: extra vector-like matter scenarios [27, 28], specific mass spectrum [29], and high-scale/split SUSY scenarios [30, 31].

The SUSY SM also implies the existence of a kind of underlying theories. Actually, by using the low-energy values for the SM gauge couplings and the renormalization group (RG) running, the strength of the couplings converges at very high energy ($\sim 10^{16}$ GeV). This implies that the SM gauge interactions unify into a single gauge interaction. This framework is called "Grand Unified Theory" (GUT).

In this framework, the SM gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ originates from the spontaneous breaking of the unified gauge group such as $SU(5)$ [32], $SO(10)$ [33,34], E_6 [35], and so on. Although this framework gives solutions to some problems in the SM, it also causes new problems; Doublet-Triplet Splitting Problem that is a kind of fine-tuning problem, Yukawa Unification, and so on. In order to resolve these problems, several extended models have been proposed so far, which will be discussed in Chapter 2. For instance, the missing-partner model is known as the model solving the doublet-triplet splitting problem. Such extended models often require higher-dimensional representation fields, and they may lead to the large quantum corrections to the observables.

The GUTs cannot be testable by producing new particles directly since the new particles obtain their masses through the GUT-breaking vacuum expectation values and their mass scale lies around 10^{16} GeV. They, on the other hand, are testable by indirect measurements such as *Nucleon Decay*. Indeed, many unified models predict the presence of the baryon-number violating processes owing to the unified description of matters. At the Kamioka observatory, some unification scenarios have been excluded; for instance, the non-supersymmetric $SU(5)$ grand unified model with the low unification scale 10^{14} GeV and the minimal SUSY $SU(5)$ GUT with TeV-scale SUSY models (for instance, see review [36]).

The important decay modes in the SUSY $SU(5)$ GUTs are the following. One is that a proton decays into a neutral pion and a positron, another is that a proton decays into a charged Kaon and an anti-neutrino. The former arises from an extra gauge boson mediation, on the other hand, the latter arises from an extra fermion mediation. The latter decay mode strongly constrains the SUSY GUT models since proton lifetime relates to the masses of sparticles. In fact, the minimal SUSY $SU(5)$ GUT with TeV-scale sparticles is excluded by the Super-Kamiokande experiment [37,38]. Several ways to evade from the constraint have been proposed: imposing global continuum or discrete symmetries, embedding into an extra-dimension, PeV-scale SUSY, and so on.

The Super-Kamiokande experiment has recently reported the lower limits on the decay modes [39, 40]. The partial lifetime for $p \to e^+ + \pi^0$ should be longer than 1.6×10^{34} years and the current lower limit on the partial lifetime for $p \to K^+ + \overline{\nu}$ mode is 6.6×10^{33} years. The sensitivity of the next generation detector at the Kamioka mine, the Hyper-Kamiokande, has been studied; in particular, it may achieve the sensitivity to the proton lifetime which is about ten times as long as the current bounds [36]. The fact implies that there is the potential to discover proton decay signature in the future experiment.

There have been theoretical developments in evaluating the proton decay rate. One of them is getting to control uncertainty of the hadron matrix elements by lattice simulations [41]. It is expected that errors in the lattice simulations will be reduced in the future. There is a large hierarchy between the hadronic energy scale and the unification scale. The energy hierarchy leads to significant corrections, which are referred to as renormalization group (RG) evolution, to observables. About three decades ago, leading-order (LO) corrections to the effective Hamiltonian were estimated for certain decay modes at the various energy scales: SM corrections [42], supersymmetric SM corrections [43], and quantum chromo-dynamics (QCD) corrections [44]. The next-leading-order (NLO) calculations have been partially carried out in several literature: SM corrections [45], QCD corrections [46], and supersymmetric corrections [47]. In the NLO calculation, not just the RG evolutions but also the finite corrections, which are also called "Threshold Corrections", will not be negligible. There has been no estimation of the threshold corrections to the baryon-number violating processes yet.

In this thesis, we derive the one-loop threshold correction to the baryon-number violating operators, analytically. Since, however, the finite corrections strongly depend on the mass spectrum of integrated degrees of freedom, we focus on specific models: the minimal SUSY $SU(5)$ GUT model with and without extra matters and the missing-partner SUSY $SU(5)$ model. In particular, we focus on the corrections to a specific decay mode, $p \to e^+ + \pi^0$ mode, which has less dependence on the GUT models. Further, operators which give rise to $p \to K^+ + \overline{\nu}$ decay are protected from quantum corrections by the non-renormalization theorem [48] in the supersymmetric field theories.

Quantum corrections to observables are often enhanced in some extended models. Indeed, in extra vector-like matter scenarios [27, 28], the gauge couplings become larger at the GUT scale. It is naïvely expected that radiative corrections via the gauge couplings are sufficient to affect observables. The other is the GUT models including large-dimensional representation Higgs fields. For instance, in order to realize the observed Yukawa couplings, large representation fields are used: a **45**-dimensional field in $SU(5)$ models [49], **120**, **126** and **210**-dimensional fields in $SO(10)$ models (See Refs. [50–52]). Due to the large degrees of freedom, the effects of quantum corrections are increased.

In this thesis, we focus on the nucleon decay operators in the specific SUSY $SU(5)$ GUT models where the radiative corrections are enhanced: the minimal SUSY $SU(5)$ GUT model with extra vector-like matters and the missing-partner SUSY $SU(5)$ GUT model. In the former model, as we mentioned, it is expected the large unified coupling at the GUT scale affects to proton lifetime. In the latter model, an $SU(5)$ gauge group is broken by non-minimal representation, **75**-dimensional Higgs. In this model, it is also supposed many fields supply the large radiative corrections to the prediction of proton lifetime.

In our study, we adopt a supergraph technique [53–56]. "Supergraph" is an extension of the Feynman-graph calculation on superspace which is the extended spacetime with Grassmann coordinates. Since SUSY requires partner particles, one should calculate more diagrams in the SUSY GUTs at even one-loop level. In the supergraph technique, we treat multiplets related to each other by SUSY as a single field, called "superfield". Therefore, it is suitable to use the supergraph technique in order to calculate corrections to the observables in the SUSY GUTs.

Organization of this thesis

This thesis is organized as follows. In this chapter, we briefly introduce the standard model of particle physics and its supersymmetric extension. The brief review of supersymmetric grand unification models is given in Chapter 2. In this chapter, we also discuss proton decay in the SUSY GUTs. We devote Chapter 3 to our derivation of the threshold corrections to the baryon-number violating operators at the scales where the SUSY particles and GUT particles are decoupled. Numerical studies of the threshold effects on proton lifetime are also shown in the last section of this chapter. Finally, we conclude this thesis in Chapter 4.

1.2 Standard Model

We know that our world consists of elementary particles; quarks, leptons, gauge bosons, and Higgs particle. The interactions and dynamics among the particles are described by the quantum field theory on the four-dimensional space-time. The fundamental theory around the Fermi scale ($\sim 10^2$ GeV) is called the standard model (SM). The Lagrangian of the SM is based on the gauge principle and the renormalizability. The SM, in particular, is a gauge theory with a product group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. The SM Lagrangian is decomposing into four parts; the matter kinetic terms $\mathcal{L}_{\text{matter}}$, the gauge kinetic terms $\mathcal{L}_{\text{gauge}}$, the scalar kinetic terms and the scalar potential $\mathcal{L}_{\text{Higgs}}$, and Yukawa interactions $\mathcal{L}_{\text{Yukawa}}$.

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{matter}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}} , \tag{1.1}$$

Table 1.1: Representation of Fields in SM

	Symbols	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
Quarks	$q_i = \begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix}$	**3**	**2**	$\frac{1}{6}$
	u_{Ri}	**3**	**1**	$\frac{2}{3}$
	d_{Ri}	**3**	**1**	$-\frac{1}{3}$
Leptons	$l_i = \begin{pmatrix} \nu_{Li} \\ e_{Li} \end{pmatrix}$	**1**	**2**	$-\frac{1}{2}$
	e_{Ri}	**1**	**1**	-1
Higgs	H	**1**	**2**	$\frac{1}{2}$

with

$$\begin{aligned} \mathcal{L}_{\text{matter}} &= \sum_{\psi=q',u'_R,d'_R,l',e'_R} i\overline{\psi}_i \not{D}\psi_i \,, \\ \mathcal{L}_{\text{gauge}} &= -\frac{1}{4}G^A_{\mu\nu}G^{A\,\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\,\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \,, \\ \mathcal{L}_{\text{Higgs}} &= (D_\mu H)^\dagger D^\mu H - V(H) \\ \mathcal{L}_{\text{Yukawa}} &= (Y_u)_{ij}\overline{u}'_{Ri}Hq'_j - (Y_d)_{ij}\overline{d}'_{Ri}H^C q'_j - (Y_e)_{ij}\overline{e}'_{Ri}H^C l'_j + \text{h.c.} \,. \end{aligned} \tag{1.2}$$

Here, D_μ is a covariant derivative and $\not{D} = \gamma^\mu D_\mu$ with a γ-matrix γ_μ. The field strength is defined by $F^a_{\mu\nu} \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^a_{\ bc} A^b_\mu A^c_\nu$ with a gauge field A^a_μ, a gauge coupling constant g, and a gauge structure constant $f^a_{\ bc}$. $G^A_{\mu\nu}$, $W^a_{\mu\nu}$, and $B_{\mu\nu}$ are respectively the field strength tensors for $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$. The field content for the other fields is given by Table 1.1. The prime for matter fermions represents gauge eigenstates and the subscripts of matter fields $i, j = 1, 2, 3$ denote generations. The conjugated Higgs field in Eq. (1.2) is defined as $H^C = i\sigma_2 H^*$.

Below the Fermi scale, the electroweak (EW) symmetry ($SU(2)_L \times U(1)_Y$) breaks into the electromagnetic (EM) symmetry $U(1)_{\text{EM}}$ and weak gauge bosons get non-zero masses. In fact, in the SM, the electroweak symmetry breaking (EWSB) is triggered by the tachyonic Higgs and the Higgs potential is given by

$$V(H) = -\mu^2 H^\dagger H + \frac{\lambda}{2}(H^\dagger H)^2 \,. \tag{1.3}$$

with positive μ^2. At the minimum of this potential, the Higgs field H obtains the non-zero vacuum expectation value (VEV). The Higgs field is expanded around its VEV as follows.

$$H = \begin{pmatrix} i\xi^+ \\ \frac{1}{\sqrt{2}}(v + h + \xi^0) \end{pmatrix}, \quad v = \sqrt{\frac{2\mu^2}{\lambda}}. \tag{1.4}$$

The observed value for the Higgs VEV $v = 246.22$ GeV [57]. $\xi^{0,+}$ is absorbed by the weak gauge bosons as the Nambu-Goldstone boson. The physical degree of freedom is the neutral scalar field h with a mass of $m_h = \sqrt{\lambda}v = 125.09$ GeV [26].

After the EWSB, the electroweak gauge bosons get their masses (and $W^\pm$ and Z denote their mass eigenstates). The masses of the EW gauge bosons are given by

$$\begin{aligned} m_W &\equiv \frac{1}{2} g v = 80.385 \pm 0.015 \text{ GeV}, \\ m_Z &\equiv \frac{1}{2}\sqrt{g^2 + g_Y^2}\, v = 91.1876 \pm 0.0021 \text{ GeV}, \end{aligned} \tag{1.5}$$

with gauge couplings g and g_Y for $SU(2)_L$ and $U(1)_Y$, respectively (the observed values for these gauge bosons are given in [57]). The EM coupling e relates to g and g_Y as

$$e \equiv \frac{g g_Y}{\sqrt{g^2 + g_Y^2}}. \tag{1.6}$$

The gauge bosons corresponding to the Cartan generators of $SU(2)_L \times U(1)_Y$ mix with each other, and their mass eigenstates are different from the gauge eigenstates. The gauge and mass eigenstates are mutually associated together via the weak mixing angle θ_W defined by

$$\sin^2\theta_W \equiv \frac{g_Y^2}{g^2 + g_Y^2}. \tag{1.7}$$

We use the observed values for the EM coupling and the weak angle as the input values of numerical calculation. The detail for input values is given in Appendix B.

Fermion masses and flavor violating interactions arise from the Yukawa interactions. When the Higgs boson obtains the VEV, Yukawa terms become the Dirac mass terms for quarks and leptons after redefining the flavor basis in order to diagonalize the Yukawa matrices.

$$\begin{aligned} \mathcal{L}_{\text{mass}} &= \frac{v}{\sqrt{2}}(Y_u)_{ij}\overline{u}'_{Ri}u'_{Lj} + \frac{v}{\sqrt{2}}(Y_d)_{ij}\overline{d}'_{Ri}d'_{Lj} + \frac{v}{\sqrt{2}}(Y_e)_{ij}\overline{e}'_{Ri}e'_{Lj} \\ &= \sum_i \left(m_{u_i}\overline{u}_{Ri}u_{Li} + m_{d_i}\overline{d}_{Ri}d_{Li} + m_{e_i}\overline{e}_{Ri}e_{Li} \right). \end{aligned} \tag{1.8}$$

In general, the complex matrix M is diagonalized by bi-unitary transformation $M_{\text{diag.}} = U^\dagger M V$ with unitary matrices V and U. The gauge and flavor (mass) bases for fermions relate with each other as follows.

$$\begin{aligned} u'_L &= V_u u_L\,, \quad d'_L = V_d d_L\,, \quad e'_L = V_e e_L\,, \\ u'_R &= U_u u_R\,, \quad d'_R = U_d d_R\,, \quad e'_R = U_e e_R\,. \end{aligned} \tag{1.9}$$

We diagonalize the mass matrices for Y_u and Y_e simultaneously by the redefinition of fields for up-type quarks and leptons, but cannot take the down-type quarks diagonalized. We parameterize the unitary transformation of the left-handed down-type quarks as

$$\begin{pmatrix} d'_L \\ s'_L \\ b'_L \end{pmatrix}_{\text{weak}} = U_{\text{CKM}} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}_{\text{mass}}, \tag{1.10}$$

where U_{CKM} is called the Cabibbo-Kobayashi-Maskawa (CKM) matrix [58,59]. We also show the numerical entries for the CKM matrix in Appendix B. Since the gauge basis for quarks is different from the flavor basis, the flavor-changing interactions are given by the CKM matrix in the SM. Furthermore, three generations are assumed in the SM, and thus there is a complex phase in the CKM matrix and the phase gives rise to the CP-violation.

Problems in Standard Model

Though the SM explains various particle phenomena well, the SM is expected to be extended for the following reasons.

- No candidate of dark matter
- Origin of neutrino masses
- Baryon asymmetry of the Universe
- Why hypercharge is quantized
- and so on.

Our universe comprises three components: the baryonic matters ($\sim$ 4.9%), the cold dark matter (DM) ($\sim$ 26.5%), and the dark energy ($\sim$ 68.3%) [24]. The SM does not explain these components except the baryonic one. In particular, the DM closely connects with models beyond the SM if it behaves as a particle. Indeed, the mass scale of the DM is close to the electroweak scale if the DM is initially in the thermal equilibrium by interacting with the SM particles and explains the correct density.

While many neutrino oscillation experiments have shown that neutrinos have the non-zero masses, there is no gauge invariant mass term for neutrinos in the SM. This fact requires new particles in order to give neutrino masses. The familiar one is the see-saw mechanism [60,61], which requires the SM singlet fermions called right-handed neutrinos.

Moreover, we do not explain the baryon-antibaryon asymmetry of the universe in the SM. The SM does not satisfy the condition, which is known as Sakharov's condition [25], since the CP-violation in the SM is small and there is no departure from the thermal equilibrium.

Also, there is no reason for the $U(1)_Y$ charge being quantized though arbitrary real numbers are allowed for Abelian charges. If not, the hydrogen which consists of a electron and a proton (= three quarks) is not electrically neutral. However the experimental upper bound for the electric charge of a hydrogen is $|Q_e + Q_p| < 1 \times 10^{-21}\,e$ [57].

Not just the experimental (or observational) sides, but there are some mysterious features in the SM; the hierarchy problems in the Higgs mass [62] and the cosmological constant [63], the origin of the mass hierarchy of fermions, the origin of the gauge group and matter contents, and so on.

The gauge hierarchy problem is the naturalness problem in the Higgs mass parameter. A correction to the squared mass of a scalar field is proportional to the UV cutoff scale since the bare parameter for the scalar field is not protected by any symmetry. The physical mass of the scalar field is obtained by the cancellation between the bara mass and the quadratic divergence. In the case of the SM, the observed Higgs mass is

$$m_{h,\text{Bare}}^2 + \Delta m_h^2 = m_h^2 = (125.09\ \text{GeV})^2\,, \tag{1.11}$$

with

$$\Delta m_h^2 = -\frac{3y_t^2}{4\pi^2}\Lambda^2 + \cdots\,. \tag{1.12}$$

Here, we consider only a dominant correction. Λ is the UV cutoff scale in which new physics appears. It is certain that new physics appears at $\Lambda = M_{\text{Pl}} = 2.4 \times 10^{18}$ GeV where quantum gravity effects are not ignored. If the cutoff scale lies around 10^{18} GeV, large cancellation between $m_{h,\text{Bare}}^2$ and Δm_h^2 is required in order to explain the observed mass of the Higgs boson.

Several solutions to the gauge hierarchy problem have been proposed. The extended models proposed as the solution often require new particles whose masses lie around $10^2 \sim 10^3$ GeV. * In particular, the partner particle which has $SU(3)_C$ charge ensures the cancellation: *e.g.* top-partner scalar in the supersymmetric models (such as the minimal supersymmetric standard model: MSSM) and top-partner fermion in the pseudo-Nambu-Goldstone boson (PNGB) Higgs models (such as the Little Higgs model [69–72]). †

Anyway, we give brief introduction of the MSSM and its extensions based on the discovered Higgs boson in the next section.

*Recently, the models which do not necessarily require new particles with a mass of a few TeV have been proposed: Cosmological relaxation [64,65], $\mathcal{N}$naturalness [66], and Clockwork models [67,68].

†The LHC experiments run1-2 have severely constrained the masses of $SU(3)_C$ charged particles. After the Higgs discovery, new ideas have been proposed, which do not necessarily require the $SU(3)_C$-charged

1.3 Minimal Supersymmetric Standard Model

Supersymmetry (SUSY) is one of the promising extensions of the SM. In the supersymmetric theories, we extend the space-time coordinates x^μ into the ones with the Grassmann coordinates $(\theta_\alpha, \theta^{\dagger\dot{\alpha}})$, which is called "superspace". Lagrangian on the superspace is constructed of the constrained superfields $S(x^\mu, \theta_\alpha, \theta^{\dagger\dot{\alpha}})$: chiral (anti-chiral) superfields Φ ($\Phi^\dagger$) and vector superfields V.

The chiral (anti-chiral) superfield is constrained as

$$\overline{\mathcal{D}}_{\dot{\alpha}}\Phi = 0\,, \quad (\mathcal{D}_\alpha \Phi^\dagger = 0)\,, \tag{1.13}$$

with the superspace covariant derivative $\mathcal{D}$ ($\overline{\mathcal{D}}$). The chiral (anti-chiral) covariant derivative defined as:

$$\begin{aligned} \mathcal{D}_\alpha &\equiv \frac{\partial}{\partial\theta^\alpha} - i(\sigma^\mu\theta^\dagger)_\alpha\partial_\mu\,, \qquad \mathcal{D}^\alpha \equiv \frac{\partial}{\partial\theta_\alpha} + i(\theta^\dagger\overline{\sigma}^\mu)^\alpha\partial_\mu, \\ \overline{\mathcal{D}}^{\dot{\alpha}} &\equiv \frac{\partial}{\partial\theta^\dagger_{\dot{\alpha}}} - i(\overline{\sigma}^\mu\theta)^{\dot{\alpha}}\partial_\mu\,, \qquad \overline{\mathcal{D}}_{\dot{\alpha}} \equiv \frac{\partial}{\partial\theta^{\dagger\dot{\alpha}}} + i(\theta\sigma^\mu)_{\dot{\alpha}}\partial_\mu\,. \end{aligned} \tag{1.14}$$

Here, σ^μ and $\overline{\sigma}^\mu$ are covariant Pauli matrices. The chiral superfield contains a complex scalar $\phi(x)$, a two-component Weyl spinor $\psi(x)$, and an auxiliary field $F(x)$ as component fields. In particular, the chiral superfield is expanded as

$$\begin{aligned} \Phi(x,\theta,\theta^\dagger) =& \phi(x) - i\theta^\dagger\bar{\sigma}^\mu\theta\partial_\mu\phi(x) - \frac{1}{4}\theta^2\theta^{\dagger 2}\partial^\mu\partial_\mu\phi(x) \\ &+ \theta\psi(x) + \frac{i}{\sqrt{2}}\theta^2\theta^\dagger\bar{\sigma}^\mu\partial_\mu\psi(x) + \theta^2 F(x)\,. \end{aligned} \tag{1.15}$$

In the supersymmetric extension of the SM, the SM quarks, leptons, and Higgs field are embedded in chiral superfields. In supersymmetric Yang-Mills (SYM) theories, the supergauge transformation for chiral superfield (Φ^a) with a gauge index a is given as follows;

$$\Phi^a \to \Phi'^a = (e^{i\Lambda}\Phi)^a\,, \qquad \Phi^\dagger_a \to \Phi'^\dagger_a = (\Phi^\dagger e^{-i\Lambda^\dagger})_a\,, \tag{1.16}$$

where $\Lambda = \Lambda^A T^A$ is a gauge parameter with a generator T^A and is a chiral superfield.

A vector superfield V, which includes a gauge field, satisfies the real condition $V = V^\dagger$. There remain unphysical components in V only by imposing the above condition. In fact, since the gauge field $A_\mu(x)$ is a component field of V, we remove the unphysical degrees

particles in order to cancel the quadratic divergence. The ideas are called "Neutral naturalness". New particles are charged under a different $SU(3)$ and the cancellation is ensured by discrete $\mathbb{Z}_2$ symmetry. For instance, the fermionic top-partner models called "Twin Higgs" [73–76], the bosonic top-partner models called "Folded SUSY" [77,78].

of freedom by the supergauge transformation. The supergauge transformation of the vector superfield V is given by

$$e^V \to e^{V'} = e^{i\Lambda^\dagger} e^V e^{-i\Lambda} \,, \tag{1.17}$$

under the transformation for (anti-chiral) chiral superfields in Eq. (1.16). The vector superfield including only physical fields is expanded as

$$V_{\mathrm{WZ}}(x,\theta,\overline{\theta}) = \theta^\dagger \overline{\sigma}^\mu \theta A_\mu(x) + \theta^{\dagger 2}\theta\lambda(x) + \theta^2\theta^\dagger\lambda^\dagger(x) + \frac{1}{2}\theta^2\theta^{\dagger 2}D(x) \,. \tag{1.18}$$

Here, we fix the supergauge to be the Wess-Zumino gauge, in which there remains an ordinary gauge degrees of freedom. $\lambda(x)$ and $D(x)$ are respectively a two-component Weyl spinor and an auxiliary field associated with the gauge multiplet. For a non-Abelian vector superfield, the following chiral superfield is covariant under the supergauge transformation.

$$\mathcal{W}_\alpha = -\frac{1}{4}\overline{\mathcal{D}}^2 (e^{-V}\mathcal{D}_\alpha e^V) \,, \tag{1.19}$$

with $V = 2gV^A T^A$, a gauge coupling g, and a generator T^A.

The gauge invariant Lagrangian in terms of superfields is given by

$$\mathcal{L} = \int d^4\theta \, \mathcal{K}(\widetilde{\Phi},\Phi) + \left[\int d^2\theta \, W(\Phi) + \text{h.c.}\right] + \left[\int d^2\theta \, \frac{1}{4} f_{ab}(\Phi)\mathcal{W}^{a\alpha}\mathcal{W}^b_\alpha + \text{h.c.}\right] \,. \tag{1.20}$$

Here, we define the superfield with tilde as $\widetilde{\Phi} \equiv \Phi^\dagger e^V$. $\mathcal{K}(\widetilde{\Phi},\Phi)$ and $W(\Phi)$ are referred to as Kähler potential and superpotential, respectively. The superpotential $W(\Phi)$ is a holomorphic function of chiral superfields. $f(\Phi)$ is also a holomorphic function of chiral superfields called "gauge kinetic function". In a renormalizable theory, the forms of these functions are determined as follows:

$$\begin{aligned} \mathcal{K}(\widetilde{\Phi},\Phi) &= \widetilde{\Phi}^i\Phi_i \,, \\ W(\Phi) &= \frac{1}{2}M^{ij}\Phi_i\Phi_j + \frac{1}{3!}y^{ijk}\Phi_i\Phi_j\Phi_k \,, \\ f_{ab}(\Phi) &= \frac{1}{4g^2}\delta_{ab} \,, \end{aligned} \tag{1.21}$$

where $i, j, \ldots$ are labels of chiral (anti-chiral) superfields.

MSSM

The supersymmetric extension of the standard model [79–82] is the promising model beyond the standard model (BSM). Now, we briefly explain the minimal supersymmetric standard model (MSSM). Superfields in the MSSM are given in Table 1.2. Superpotential defined

Table 1.2: Superfields in MSSM

	spin-0	spin-1/2	spin-1	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	Z_{2R}
Q	$\widetilde{q}$	q	-	$\mathbf{3}$	$\mathbf{2}$	$1/6$	-1
L	$\widetilde{l}$	l	-	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	-1
$\overline{U}$	$\widetilde{\overline{u}}$	$\overline{u}$	-	$\overline{\mathbf{3}}$	$\mathbf{1}$	$-2/3$	-1
$\overline{D}$	$\widetilde{\overline{d}}$	$\overline{d}$	-	$\overline{\mathbf{3}}$	$\mathbf{1}$	$1/3$	-1
$\overline{E}$	$\widetilde{\overline{e}}$	$\overline{e}$	-	$\mathbf{1}$	$\mathbf{1}$	1	-1
H_u	H_u	h_u	-	$\mathbf{1}$	$\mathbf{2}$	$1/2$	1
H_d	H_d	h_d	-	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	1
G	-	$\widetilde{g}$	G_μ	$\mathbf{8}$	$\mathbf{1}$	0	1
W	-	$\widetilde{W}$	W_μ	$\mathbf{1}$	$\mathbf{3}$	0	1
B	-	$\widetilde{B}$	B_μ	$\mathbf{1}$	$\mathbf{1}$	0	1

as a holomorphic function of chiral superfields, and thus, two Higgs doublets are required to be introduced in supersymmetric theories. One of two couples to up-type quark superfields, and another couples to down-type and electron-type superfields. The superpotential corresponding to the Yukawa interaction is

$$W_{\text{Yukawa}} = (Y_U)_{ij} H_u \overline{U}_i Q_{Lj} - (Y_D)_{ij} H_d \overline{D}_i Q_{Lj} - (Y_E)_{ij} H_d \overline{E}_i L_{Lj} \,, \tag{1.22}$$

where $i, j, \ldots$ denote generations. In this notation, the indices of $SU(3)_C$ and $SU(2)_L$ are not written down explicitly. In particular the indices of $SU(2)_L$ are contracted by the totally anti-symmetric tensor.

We also assume a Z_2-parity called "R-parity" invariance which assigns SM particles and these superpartners to parity-even and parity-odd, respectively. The definition of the R-parity is

$$P_R = (-1)^{3(B-L)+2s} \,, \tag{1.23}$$

with the baryon number B, the lepton number L, and spin of component fields s. In order to be consistent with the above definition, we assign the Z_2-parity for the MSSM chiral superfields as Table 1.2. This parity ensures the stability of the lightest supersymmetric particle (LSP), and the LSP is a dark mater candidate. Furthermore, the baryon number violating and the lepton number violating operators as

$$\begin{aligned} W_{\Delta L=1} &= \frac{1}{2}\lambda^{ijk} L_i L_j \overline{E}_k + \lambda'^{ijk} L_i Q_j \overline{D}_k + \mu^i L_i H_u \,, \\ W_{\Delta B=1} &= \lambda_B^{ijk} \overline{U}_i \overline{D}_j \overline{D}_k \,, \end{aligned} \tag{1.24}$$

are forbidden by this parity. Although such operators generate the most terrible nucleon decay interactions induced by exchanging sfermions at tree level, they are prohibited by the R-parity.

From the gauge invariance, only the following supersymmetric mass term for the Higgs superfields is allowed.

$$W_{\rm mass} = \mu H_u H_d \,. \tag{1.25}$$

The full superpotential for the MSSM is constructed as the summation of above two superpotentials.

$$W_{\rm MSSM} = W_{\rm mass} + W_{\rm Yukawa} \,. \tag{1.26}$$

This superpotential generates a lot of new interactions in addition to the Yukawa interactions in the SM Lagrangian.

So far any sparticles have not been discovered at the collider experiments, and hence the SUSY should be broken at least at the EW scale. In order not to spoil advantages in the SUSY SM, SUSY should be broken softly, which the breaking terms are called "soft supersymmetry breaking terms". The soft SUSY breaking Lagrangian is given by

$$\begin{aligned} -\mathcal{L}_{\rm soft} = & \frac{1}{2}M_3\widetilde{g}\widetilde{g} + \frac{1}{2}M_2\widetilde{W}\widetilde{W} + \frac{1}{2}M_1\widetilde{B}\widetilde{B} + \text{c.c.} \\ & + (A_u)_{ij}\widetilde{\overline{u}}_i\widetilde{q}_j H_u - (A_d)_{ij}\widetilde{\overline{d}}_i\widetilde{q}_j H_d - (A_e)_{ij}\widetilde{\overline{e}}_i\widetilde{l}_j H_d + \text{c.c.} \\ & + \sum_{\phi=\widetilde{q},\widetilde{l},\widetilde{\overline{u}},\widetilde{\overline{d}},\widetilde{\overline{e}}} (m^2_\phi)_{ij}\phi_i^\dagger\phi_j + m^2_{H_u}H_u^\dagger H_u + m^2_{H_d}H_d^\dagger H_d + (bH_uH_d + \text{c.c.}) \,. \end{aligned} \tag{1.27}$$

Here, the gaugino masses $M_{1,2,3}$, the scalar-trilinear coupling matrices $A_{u,d,e}$, the holomorphic Higgs soft mass b, the non-holomorphic soft masses for Higgs doublets $m^2_{H_u}$ and $m^2_{H_d}$, and the non-holomorphic soft masses for sfermions m^2_ϕ ($\phi = \widetilde{q}, \widetilde{l}, \widetilde{\overline{u}}, \widetilde{\overline{d}}, \widetilde{\overline{e}}$).

In this minimal extension of the standard model, the scalar potential of the Higgs bosons arises from supersymmetric interactions and soft breaking terms as the following form:

$$\begin{aligned} V =& (|\mu|^2 + m^2_{H_u})|H_u^0|^2 + (|\mu|^2 + m^2_{H_d})|H_d^0|^2 \\ & - [bH_u^0H_d^0 + \text{h.c.}] + \frac{1}{8}(g^2 + g_Y^2)(|H_u^0|^2 - |H_d^0|^2)^2 \,, \end{aligned} \tag{1.28}$$

where $m^2_{H_d}$, $m^2_{H_u}$, and b are the soft supersymmetry breaking parameters. There are two neutral Higgs components, H_u^0 and H_d^0. We have two mass eigenstate h^0 and H^0 by taking the linear combination of these scalar fields as

$$\begin{aligned} H_u^0 &= \frac{1}{\sqrt{2}}v_u + \frac{1}{\sqrt{2}}h^0\sin\beta + \frac{1}{\sqrt{2}}H^0\cos\beta + \dots \,, \\ H_d^0 &= \frac{1}{\sqrt{2}}v_d + \frac{1}{\sqrt{2}}h^0\cos\beta + \frac{1}{\sqrt{2}}H^0\sin\beta + \dots \,. \end{aligned} \tag{1.29}$$

The lightest Higgs boson h^0 is identified with the SM Higgs boson. v_u and v_d are respectively the VEVs for H_u^0 and H_d^0, and the VEV of the SM Higgs boson is given by $v^2 = v_u^2 + v_d^2$. The ratio of the VEVs is given by $\tan\beta = v_u/v_d$. At the tree-level, the quartic coupling of the SM Higgs in scalar potential is connected with the gauge coupling

$$\lambda = \frac{1}{4}(g^2 + g_Y^2)\cos^2 2\beta\,. \tag{1.30}$$

The mass of the SM Higgs boson is given by

$$m_{h^0}^2 = \frac{1}{2}\left(m_{A^0}^2 + m_Z^2 - \sqrt{(m_{A^0}^2 - m_Z^2)^2 + 4m_Z^2 m_{A^0}^2 \sin^2 2\beta}\right)\,, \tag{1.31}$$

with the CP-odd Higgs mass $m_{A^0}^2 = 2b/\sin 2\beta$. This tells us that the Higgs mass is bounded from above as

$$m_{h_0} < m_Z|\cos 2\beta|\,. \tag{1.32}$$

The mass of the observed Higgs boson is measured as 125.09 GeV at the LHC experiments [26], which is larger than the mass of Z-boson. However, it is possible to raise the upper bound by including radiative corrections [83, 84].

In order to realize the observed mass of the SM Higgs boson, we need to include additional contributions. One is to extend the MSSM such as the Next-MSSM [29], in which there is an additional contribution to the scalar potential from an additional singlet superfield. The others are the models with large quantum corrections: high-scale or split SUSY scenarios [85, 86], the additional vector-like matter scenarios [27, 28], or specific mass spectrum scenarios [29].

Extended Models and Gauge Coupling Unification

We discuss the extended models introducing additional quantum corrections in the last of this section. The gauge coupling unification works well in the MSSM with TeV-scale SUSY [87]. Fig. 1.1 compares the gauge coupling running in the SM (Black broken lines) and in the MSSM (Blue solid lines) with two-loop RGEs and one-loop threshold corrections. Here, we define $\alpha = g^2/4\pi$ with gauge coupling g. For blue lines, the SUSY breaking scale is set to be 1 TeV, the gaugino masses are set to be $M_2 = 300$ GeV and $M_3 = 1$ TeV, and the ratio of the Higgs VEVs is $\tan\beta = 3$. We see that the coupling unification is achieved in the MSSM and the unification scale lies around 10^{16} GeV. Since it is found that the coupling unification is not complete even in the MSSM, the mismatch of the couplings is interpreted as the threshold corrections at the GUT scale. In other words, the GUT scale mass spectrum contributes to splitting the gauge couplings

Anyway, we give a brief introduction of the extensions and show the unification works well even in the extended SUSY SM.

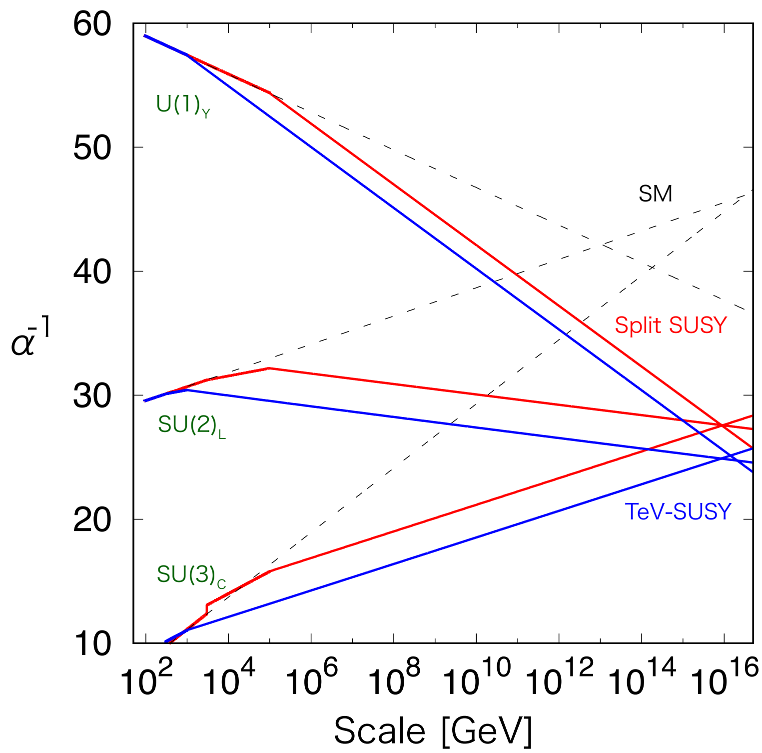


Figure 1.1: Gauge coupling unification in SM (Black broken lines), in TeV-scale MSSM (Blue solid lines), and in Split SUSY (Red solid lines).

Split SUSY Model

To begin with, we consider the split SUSY models [30, 88]. If there is no singlet superfield in the SUSY breaking sector, gauginos do not obtain tree-level masses. We consider the SUSY breaking spurion field X carries hidden charge. The SUSY breaking arises from the F-term VEV of X given as

$$\langle X \rangle = F_X \theta^2 \,. \tag{1.33}$$

Sfermion masses arise from the following higher-dimensional operator,

$$\int d^4\theta \frac{XX^\dagger}{\Lambda^2} \Phi^\dagger \Phi \,, \tag{1.34}$$

where Λ denotes the SUSY breaking mediation scale. For instance, $\Lambda \sim M_{\rm Pl}$ if the SUSY is broken by the (quantum or super-) gravity effect or Λ is comparable to the mass scale of messenger fields if the SUSY is dynamically broken in the hidden sector. Anyway, the sfermion masses are generated as $m_\phi^2 \sim F_X^2/\Lambda^2$. The gaugino masses and the scalar trilinear couplings (A-terms) are not generated because of the hidden charge conservation. In other words, the following operators are forbidden by hidden symmetry.

$$\int d^2\theta \frac{X}{\Lambda} \mathcal{W}^a \mathcal{W}^b \,, \quad \int d^2\theta \frac{X}{\Lambda} \lambda^{ijk} \Phi_i \Phi_j \Phi_k \,. \tag{1.35}$$

In this case, they are generated by anomaly mediated supersymmetry breaking (AMSB) [89, 90]. The gaugino masses and A-terms are generated with one-loop suppression

$$M_a(\mu) = -\frac{\beta(g_a^2)}{2g_a^2} m_{3/2} \,, \quad A_{ijk}(\mu) = -\frac{1}{2}(\gamma_i + \gamma_j + \gamma_k) m_{3/2} \lambda^{ijk}(\mu) \,. \tag{1.36}$$

Here, β is the beta function for a gauge coupling and γ_i is the anomalous dimension for the chiral superfield Φ_i. $m_{3/2}$ represents the mass of gravitino. Gauginos with a mass of a few TeV are compatible with the thermal DM scenario. The heavy sfermion and gravitino scenarios solve several problems: SUSY flavor and/or CP problems [91], cosmological problems (such as the Polonyi problem [92] and gravitino problem [93]).

Even in this scenario, the gauge coupling unification works well. Red lines in Fig. 1.1 show the gauge coupling running in the split SUSY model. We assume the mass spectrum of sparticles as follows. The sfermions are degenerate in mass ($M_S = 10^2$ TeV), the mass of wino is set to be $M_2 = 3$ TeV, and the ratio of the masses of wino and gluino $M_3/M_2 = 9$.

Vector-like Extensions of MSSM

Next, we introduce extra matters to the MSSM. If the extra matters are introduced in vector-like representations of the SM group, the model is anomaly-free. The low-scale gauge mediated supersymmetry breaking (GMSB) model requires to introduce extra vector-like supermultiplets [94]. Moreover, the extra matters in irreducible representations of the unified gauge group (such as $SU(5)$) ensure the coupling unification.

For a $\mathbf{5} + \overline{\mathbf{5}}$ extension, we introduce the following superfields.

$$\begin{aligned} D'(\mathbf{3},\mathbf{1})_{-1/3} \,, \quad \overline{D}'(\overline{\mathbf{3}},\mathbf{1})_{1/3} \,, \\ L'(\mathbf{1},\mathbf{2})_{-1/2} \,, \quad \overline{L}'(\mathbf{1},\mathbf{2})_{1/2} \,. \end{aligned} \tag{1.37}$$

Here, $(r_C, r_W)_Y$ represents $SU(3)_C$ r_C-plet, $SU(2)_L$ r_W-plet with hypercharge Y under the SM group. In a $\mathbf{10} + \overline{\mathbf{10}}$ extended model, additional superfields,

$$\begin{aligned} U'(\mathbf{3},\mathbf{1})_{2/3} \,, \quad \overline{U}'(\overline{\mathbf{3}},\mathbf{1})_{-2/3} \,, \\ Q'(\mathbf{3},\mathbf{2})_{-1/6} \,, \quad \overline{Q}'(\overline{\mathbf{3}},\mathbf{2})_{1/6} \,, \\ E'(\mathbf{1},\mathbf{1})_{1} \,, \quad \overline{E}'(\mathbf{1},\mathbf{1})_{-1} \,, \end{aligned} \tag{1.38}$$

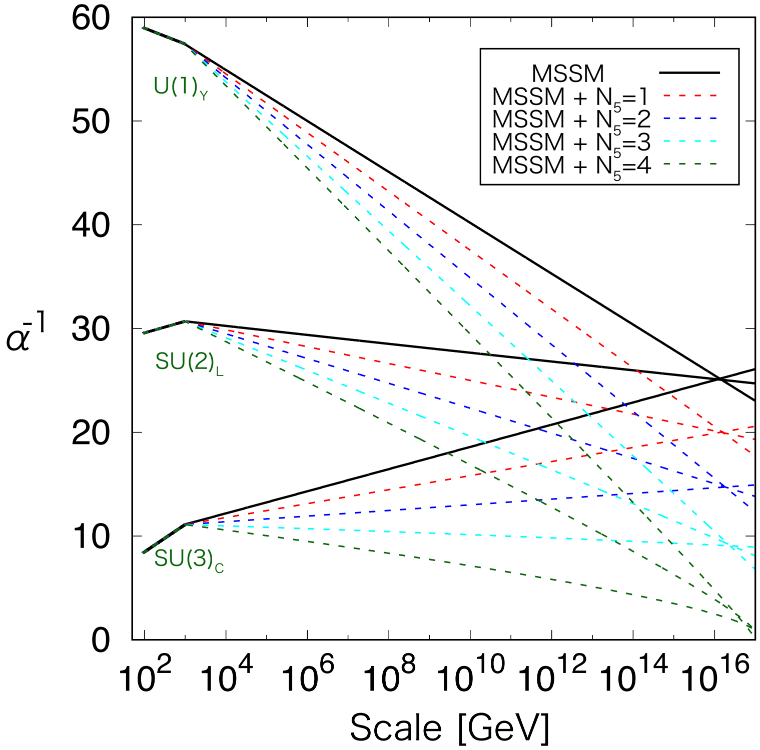


Figure 1.2: Gauge coupling running in MSSM with extra vector-like multiplets.

are introduced.

Vector-like scenarios have fruitful phenomenological consequences. For instance, if the mass scale of extra matters lies around several hundred GeV, they explain the 3σ discrepancy of muon anomalous magnetic moment between measured value [95] and the SM prediction (see Ref. [96]) within the SUSY SM [97, 98].

Fig. 1.2 shows the gauge coupling running in the vector-like extensions. N_5 in this figure denotes the number of $\mathbf{5} + \bar{\mathbf{5}}$ pairs. We assume that all sparticles and extra multiplets are degenerate in mass, and their mass scale is set to be 1 TeV. From this figure, we understand that the gauge couplings converge at the high-energy in the presence of several extra vector-like multiplets. Furthermore, the unified coupling becomes larger as we increase the number of extra matters. As we will see in Chapter 2, it implies that the proton decay rate via an extra gauge boson mediation will be enhanced [99].

Even if the mass scale of extra multiplet is higher, there are some interesting features. Considering that the mixture of the split SUSY and extra vector-like multiplet, LSP in this model is often bino [100]. In order to explain the correct relic density, bino should be nearly degenerate with other gauginos in mass. In this framework, the additional CP-violation could be also generated in gaugino mass parameters since gaugino masses arise from both of AMSB and GMSB contributions. This fact affects the predictions of nucleon and electron electric dipole moments (EDMs) [101], and hence the future EDM experiments have the potential to discriminate the simplest split-SUSY model and its extension.

In the present day, the mass of gluino is severely constrained by the LHC experiments. The lower bound for gluino mass is 1.9 TeV in a simplified mass spectrum [102]. Heavy gauginos and RGE considerations naïvely result in heavy colored scalars, and thus new particles may not be seen at the LHC experiments. However, if the soft parameters for the extra matters are much larger than those for the MSSM sparticles, the light stop with the mass below 1 TeV, the heavy gluino, and the observed Higgs boson are simultaneously explained in the context of gaugino mediation scenario [103].

Chapter 2

Grand Unification and Proton Decay

Grand unified theories (GUTs) explain several problems in the SM. For instance, there is no reason that the gauge anomalies for the SM gauge group are completely cancelled. Moreover, the $U(1)_Y$ charges for matter fermions are quantized in the SM, otherwise the hydrogen atom which is a bound state of electron and proton can be charged. These facts imply that the SM gauge group is a subgroup of the larger gauge (simple) group. Indeed, three gauge couplings at the weak scale unify at around 10^{16} GeV if the low scale supersymmetry is assumed [87].*

The framework of the grand unification gives explanations for neutrino masses and the baryon asymmetry of the Universe. The familiar scenario is the (type-I) see-saw mechanism and the standard Leptogenesis scenario. Furthermore, in the case that the unified gauge group is simple, quantization of hypercharges is explained since the hypercharge $U(1)$ is a subgroup of the unified group.

$SU(5)$, $SO(10)$, and E_6 are the candidates of the unified gauge group. The original non-SUSY $SU(5)$ GUT was investigated by H. Georgi and S. L. Glashow [32]. An $SU(5)$ is the rank-four group, which the SM gauge group is also the same rank group, and contains the complex representations. Supersymmetrization of this minimal $SU(5)$ GUT was proposed after that [105, 106].

New colored fields such as new gauge bosons and color-triplet Higgs multiplets are included at the unification scale. These fields might give rise to the baryon-number violating processes (such as nucleon decay). In general, nucleon decay is one of the prediction of the SUSY GUTs, and hence the SUSY GUT models can be tested by the nucleon decay searches.

*The coupling unification can be ameliorated even in the split SUSY scenarios [104]. The multi-TeV gauginos play an important role of the gauge coupling unification.

2.1 Supersymmetric Grand Unification Models

In this section, we review the SUSY $SU(5)$ GUT models, especially the minimal SUSY $SU(5)$ GUT and the missing partner $SU(5)$ model.

Minimal $SU(5)$

Now, let us consider the minimal embedding of the MSSM into the $SU(5)$ GUT model. All the MSSM chiral superfields listed in Table 1.2 are embedded into the chiral superfields in the $SU(5)$ representation. In particular, $\overline{\mathbf{5}}+\mathbf{10}$ which are respectively denoted by Φ and Ψ contain all MSSM matter superfields:

$$\Phi_A(\overline{\mathbf{5}}) = \begin{pmatrix} \overline{D}_\alpha \\ \epsilon_{rs}L^s \end{pmatrix}, \qquad \Psi^{AB}(\mathbf{10}) = \frac{1}{\sqrt{2}}\begin{pmatrix} \epsilon^{\alpha\beta\gamma}\overline{U}_\gamma & Q^{\alpha s} \\ -Q^{\beta r} & \epsilon^{rs}\overline{E} \end{pmatrix}, \tag{2.1}$$

where $A, B, \cdots = 1,2,\cdots 5$ denote $SU(5)$ indices, and $\alpha,\beta,\cdots = 1,2,3$ and $r,s,\cdots = 1,2$ are $SU(3)_C$ and $SU(2)_L$ indices, respectively.

The MSSM Higgs doublets H_u、H_d are embedded in the $\mathbf{5}+\overline{\mathbf{5}}$ representation,

$$H(\mathbf{5}) = \begin{pmatrix} H_C^\alpha \\ H_u^r \end{pmatrix}, \quad \overline{H}(\overline{\mathbf{5}}) = \begin{pmatrix} \overline{H}_{C\alpha} \\ \epsilon_{rs}H_d^s \end{pmatrix}, \tag{2.2}$$

with two color-triplet Higgs superfields $H_C, \overline{H}_C$. Furthermore, we need to introduce the Higgs superfield breaking $SU(5)$ to the SM gauge group. The Higgs field with the GUT breaking VEV should be the singlet under the SM gauge group. Among the non-trivial $SU(5)$ representations, it is known that $\mathbf{24}, \mathbf{75}, \mathbf{200}, \cdots$ contain the SM singlet fields. Thus, we use the adjoint ($\mathbf{24}$) representation chiral superfield $\Sigma(\mathbf{24})$ as the GUT-breaking one in the minimal $SU(5)$. The SM decomposition of $\Sigma(\mathbf{24})$ is given by

$$\Sigma^A{}_B(\mathbf{24}) = \begin{pmatrix} \Sigma_8 & \Sigma_{(3,2)} \\ \Sigma_{(3^*,2)} & \Sigma_3 \end{pmatrix} + \frac{1}{\sqrt{2}}\frac{1}{\sqrt{30}}\begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix}\Sigma_{24}\,. \tag{2.3}$$

The GUT-breaking VEV is parametrize as

$$\langle\Sigma\rangle = v_{24}\begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix}. \tag{2.4}$$

The generator corresponding to the $U(1)$ sub-algebra is normalized as $\mathrm{tr}(T_{U(1)}T_{U(1)}) = 1/2$ while the $U(1)_Y$ charges have already been given in Table 1.2 and the corresponding generator T_Y differs from the unified $U(1)$. The relation between the $U(1)$ generators is given by

$$T_{U(1)} = \sqrt{\frac{3}{5}} T_Y \,. \tag{2.5}$$

Since the normalization of the gauge boson is determined the kinetic term, the coupling and generator in each normalization satisfy $g_1 T_{U(1)} = g_Y T_Y$. Thus, the unified $U(1)$ coupling g_1 relates to the hypercharge coupling as follows.

$$g_1 = \sqrt{\frac{5}{3}} g_Y \,. \tag{2.6}$$

The $SU(5)$ vector superfield contains the MSSM vector superfields and additional vector superfields. The MSSM decomposition of the $SU(5)$ vector superfield $\mathcal{V}_5$ is given as

$$\mathcal{V}_5 = \begin{pmatrix} G^\alpha{}_\beta - \frac{2}{2\sqrt{15}} B\delta^\alpha{}_\beta & \frac{1}{\sqrt{2}} X^{\dagger\alpha}{}_r \\ \frac{1}{\sqrt{2}} X^s{}_\beta & W^s{}_r + \frac{3}{2\sqrt{15}} B\delta^s{}_r \end{pmatrix} , \tag{2.7}$$

where $G^\alpha{}_\beta, W^s{}_r$, and B are the MSSM vector superfields for $SU(3)_C, SU(2)_L$, and $U(1)_Y$, respectively. X and $X^\dagger$ are extra vector superfields which obtain the non-zero mass after the GUT breaking.

In the minimal SUSY $SU(5)$ GUT, the Kähler potential and the superpotential in the flavor basis of matter superfields are written as

$$\begin{aligned} \mathcal{K} = {} & \Phi_i^{\dagger A} (e^{-2g_5\mathcal{V}_5})^B{}_A \Phi_{iB} + \Psi^\dagger_{iAB} (e^{2g_5\mathcal{V}_5})^A{}_C (e^{2g_5\mathcal{V}_5})^B{}_D \Psi_i^{CD} \\ & + 2\Sigma^{\dagger A}{}_B (e^{-2g_5\mathcal{V}_5})^C{}_A (e^{2g_5\mathcal{V}_5})^B{}_D \Sigma^D{}_C + H_{\bar{\mathbf{5}}}^{\dagger A} (e^{-2g_5\mathcal{V}_5})^B{}_A H_{\bar{\mathbf{5}}B} \\ & + H^\dagger_{\mathbf{5}A} (e^{2g_5\mathcal{V}_5})^A{}_B H_{\mathbf{5}}^B \,. \end{aligned} \tag{2.8}$$

X and $X^\dagger$ acquire a heavy mass through the interactions to Σ in the Kähler potential, and their masses are denoted by $M_X = 5\sqrt{2} g_5 v_{24}$.

Next, we consider the superpotential in the minimal $SU(5)$. We assume the renormalizability and the R-parity for superfields, and then we have the following gauge invariant superpotential.

$$\begin{aligned} W = {} & \frac{f}{3}\mathrm{tr}\Sigma^3 + \frac{m_{24}}{2}\mathrm{tr}\Sigma^2 + \lambda \overline{H}_A (\Sigma^A{}_B + 3v_{24}\delta^A{}_B) H^B \\ & + \frac{h^{ij}}{4} \epsilon_{ABCDE} \Psi_i^{AB} \Psi_j^{CD} H^E + \sqrt{2} f^{ij} \Psi_i^{AB} \Phi_{jA} \overline{H}_B \,, \end{aligned} \tag{2.9}$$

where $i, j = 1, 2, 3$ represent flavor indices. h^{ij} and f^{ij} is complex 3×3 matrices: h^{ij} is a symmetric matrix while f^{ij} is an unconstrained matrix. Using flavor rotation for Φ and Ψ, $U(3) \times U(3)$ rotation, we make h^{ij} diagonalize as follows.

$$\begin{aligned} h^{ij} &= (y_u^i e^{i\varphi_i})\, \delta^{ij} \,, \\ f^{ij} &= (U^*_{\text{CKM}})_{ij}\, y_d^j \,. \end{aligned} \tag{2.10}$$

Here, y_u and y_d are diagonalized Yukawa matrices, and U_{CKM} is the CKM matrix. Since there remain two additional phase parameters, the phases φ_i $(i = 1, 2, 3)$ are constrained as $\varphi_1 + \varphi_2 + \varphi_3 = 0$. By the degrees of freedom of field re-parametrization, the mass eigenstates for components of Ψ_i are given by

$$Q_i = \begin{pmatrix} U_i \\ D_i \end{pmatrix} \to \begin{pmatrix} U_i \\ (U_{\text{CKM}})_{ij} D_j \end{pmatrix}, \quad \overline{U}_i \to e^{-i\varphi_i}\overline{U}_i \,, \quad \overline{E}_i \to (U_{\text{CKM}})_{ij}\overline{E}_j \,. \tag{2.11}$$

For component superfields of Φ_i, the mass eigenstates are identified as the gauge eigenstates.

We impose the supersymmetric condition after the GUT-breaking as follows.

$$\left.\frac{\partial W}{\partial \Sigma}\right|_{\Sigma = \langle \Sigma \rangle} = 30(-f v_{24}^2 + m_{24} v_{24}) = 0 \,, \tag{2.12}$$

and then, we have $v_{24} = m_{24}/f$.

After the adjoint Higgs superfield obtains the VEV, the interaction term among $H, \overline{H}$, and the adjoint Higgs Σ is

$$\begin{aligned} W_{h\text{-}\Sigma} =& \lambda \overline{H}_A (\Sigma^A{}_B + 3 v_{24} \delta^A{}_B) H^B \\ \longrightarrow & \lambda \overline{H}_A \begin{pmatrix} 5v_{24} & & & & \\ & 5v_{24} & & & \\ & & 5v_{24} & & \\ & & & 0 & \\ & & & & 0 \end{pmatrix} H^B \,. \end{aligned} \tag{2.13}$$

Since v_{24} is on the order of the GUT scale ($\sim 10^{16}$ GeV), we tune the gauge invariant mass term for H and $\overline{H}$. In fact, the supersymmetric mass for the color-triplet Higgs is given by $M_{H_C} = 5\lambda v_{24}$, on the other hand that for the MSSM Higgs doublets is zero. This fine-tuning problem for the GUT parameters is known as the doublet-triplet splitting problem.

The masses of the adjoint Higgs multiplet are also split after the $SU(5)$ breaking. The weak triplet Σ_3 and the color octet Σ_8 have a common mass denoted by $M_\Sigma = \frac{5}{2} f v_{24}$, and the SM singlet Σ_{24} has $M_{\Sigma_{24}} = M_\Sigma/5$. The off-diagonal chiral superfields $\Sigma_{(3,2)}$ and $\Sigma_{(3^*,2)}$ are absorbed by the massive vector superfields X and $X^\dagger$.

In the minimal $SU(5)$ GUT, there are the following two problems.

- Doublet-triplet splitting problem
- GUT relation for fermion masses

The first one, the doublet-triplet splitting problem, is the above-mentioned fine tuning problem among the superpotential mass parameters. In the minimal GUT model, the MSSM Yukawa matrices at the GUT scale is imposed the relation $Y_D = (Y_E)^T$ since the doublet leptons and the down-type quarks are included in the same superfield, $\Phi(\overline{\mathbf{5}})$. It is, however, known that the Yukawa couplings at the GUT scale which are evolved by the renormalization group equations (RGEs) do not satisfy the GUT relation for Y_D and Y_E . In order for solving the GUT relations for fermion masses, the following ways are considered.

- Higher-dimensional (Planck scale suppressed) operators
- Higher-dimensional representation fields
- Additional vector-like matters and their mixing with the MSSM fields

In particular, since a tensor product $\mathbf{10} \times \overline{\mathbf{5}}$ is decomposed into a sum $\mathbf{45} + \mathbf{5}$, the product $\Psi_i^{AB}\Phi_{jC}$ can couple to the $\overline{\mathbf{45}}$-Higgs H_{AB}^C as follows

$$y^{ij}\Psi_i^{AB}\Phi_{jC}H_{AB}^C\,. \tag{2.14}$$

This new interaction can solve the bottom-tau unification problem. However, in order to avoid many remnants at the low scale, $\overline{\mathbf{45}}$ should be introduced with vector-like pair superfield since it contains huge component fields.

Missing Partner $SU(5)$

Next, we briefly review the model which solves the doublet-triplet splitting problem and is called "Missing-partner Model". This model is proposed by Grinstein [107] and Masiero-Nanopoulos-Tamvakis-Yanagida [108]. Here, we show the missing partner model with Peccei-Quinn (PQ) symmetry [109], which is proposed by Ref. [110].

The particle contents of this model is given in Table 2.1. Ψ_i and Φ_i are the matter superfields in the minimal SUSY $SU(5)$ with the generation index $i = 1, 2, 3$. N_i denotes the superfields including the right-handed neutrino. H ($\overline{H}$) is the (anti-)fundamental Higgs superfield in the minimal SUSY $SU(5)$. The additional fields are $\overline{\Theta}(\mathbf{50}), \Theta(\overline{\mathbf{50}})$, and $\Xi(\mathbf{75})$. In fact, the next minimal representation including a field with the same quantum number as the color-triplets in $\mathbf{5} + \overline{\mathbf{5}}$ is $\mathbf{50} + \overline{\mathbf{50}}$. $\mathbf{50} + \overline{\mathbf{50}}$ has no weak doublet, and thus the MSSM Higgs doublets are still massless after the GUT breaking. Since a product of $\overline{\mathbf{50}}$ and $\mathbf{5}$ is decomposed as $\overline{\mathbf{50}} \otimes \mathbf{5} = \mathbf{75} \oplus \mathbf{175}'$, the additional field $\Xi(\mathbf{75})$ is needed for the gauge invariant superpotential. In this model, the breaking of the $SU(5)$ GUT is induced by the VEV of $\Xi(\mathbf{75})$. We also introduce singlet fields P and Q in order to break the $U(1)$ PQ symmetry.

Table 2.1: Peccei-Quinn Charges in Missing Partner Model

	Ψ_i	Φ_i	N_i	H	$\overline{H}$	H'	$\overline{H}'$
dim.	**10**	$\overline{\mathbf{5}}$	**1**	**5**	$\overline{\mathbf{5}}$	**5**	$\overline{\mathbf{5}}$
$U(1)_{\rm PQ}$	$\frac{\alpha}{2}$	$\frac{\beta}{2}$	$\frac{2\alpha-\beta}{2}$	$-\alpha$	$-\frac{\alpha+\beta}{2}$	$\frac{\alpha+\beta}{2}$	α

	Θ	$\overline{\Theta}$	Θ'	$\overline{\Theta}'$	Ξ	P	Q
dim.	$\overline{\mathbf{50}}$	**50**	$\overline{\mathbf{50}}$	**50**	**75**	**1**	**1**
$U(1)_{\rm PQ}$	α	$\frac{\alpha+\beta}{2}$	$-\frac{\alpha+\beta}{2}$	$-\alpha$	0	$-\frac{3\alpha+\beta}{2}$	$\frac{3(3\alpha+\beta)}{2}$

The $U(1)_{\rm PQ}$ charge assignment is also given in Table 2.1. Here, we assume that the PQ symmetry remains after the GUT breaking, that is, $\Xi(\mathbf{75})$ is $U(1)_{\rm PQ}$ neutral. Moreover, we also assume $3\alpha+\beta\neq 0$. This assumption prohibits the dangerous operators such as $\Phi_i\Psi_j\Psi_k\Psi_l$ after the GUT breaking. Otherwise, the operators give rise to the dimension-five proton decay, which is discussed in the next section.

The gauge invariant and PQ-invariant superpotential is given by

$$\begin{aligned} W = &\frac{1}{4}h_{ij}\epsilon_{ABCDE}\Psi_i^{[AB]}\Psi_j^{[CD]}H^E + \sqrt{2}f_{ij}\Psi_i^{[AB]}\Phi_{jA}\overline{H}_B \\ &+ g_H\epsilon_{ABCDE}H^A\Xi^{[BC]}_{[FG]}\Theta^{[DE][FG]} + g_{\overline{H}}\epsilon^{ABCDE}\overline{H}_A\Xi^{[FG]}_{[BC]}\overline{\Theta}_{[DE][FG]} \\ &+ g'_H\epsilon_{ABCDE}H'^A\Xi^{[BC]}_{[FG]}\Theta'^{[DE][FG]} + g'_{\overline{H}}\epsilon^{ABCDE}\overline{H}'_A\Xi^{[FG]}_{[BC]}\overline{\Theta}'_{[DE][FG]} \\ &+ M_1\overline{\Theta}_{[AB][CD]}\Theta'^{[AB][CD]} + M_2\overline{\Theta}'_{[AB][CD]}\Theta^{[AB][CD]} \\ &+ m_{75}\Xi^{[CD]}_{[AB]}\Xi^{[AB]}_{[CD]} - \frac{1}{3}\lambda_{75}\Xi^{[AB]}_{[EF]}\Xi^{[CD]}_{[AB]}\Xi^{[EF]}_{[CD]}\,. \end{aligned} \tag{2.15}$$

Here, $[AB]$ represents an anti-symmetric pair of indices. ϵ_{ABCDE} and ϵ^{ABCDE} are totally anti-symmetric tensor. We introduce two copies of $\mathbf{5}+\overline{\mathbf{5}}$ and $\mathbf{50}+\overline{\mathbf{50}}$ for the invariant mass terms of **50**s.

If there remain the additional fields, the gauge coupling will diverge between the GUT and the Planck scale, and then the model do not have any perturbative picture. Therefore, $M_1 = M_2 = M_{\rm Pl} = 2.4\times 10^{18}$ GeV is assumed in double missing partner models. The VEV of the **75**-dimensional Higgs superfield is given as follows.

$$\begin{aligned} &\left\langle \Xi^{(ab)}_{(cd)}\right\rangle = \frac{3}{2}v_{75}(\delta^a_c\delta^b_d - \delta^a_d\delta^b_c)\,, \quad \left\langle \Xi^{(\alpha\beta)}_{(\gamma\delta)}\right\rangle = \frac{1}{2}v_{75}(\delta^\alpha_\gamma\delta^\beta_\delta - \delta^\alpha_\delta\delta^\beta_\gamma)\,, \\ &\left\langle \Xi^{(\alpha a)}_{(\beta b)}\right\rangle = -\frac{1}{2}v_{75}\delta^\alpha_\beta\delta^a_b\,. \end{aligned} \tag{2.16}$$

Table 2.2: Mass splitting of **75** components

irreps.	Mass
$(\mathbf{1},\mathbf{1})_0$	$\frac{2}{5}M_\Xi$
$(\mathbf{3},\mathbf{1})_{-\frac{5}{3}}$, $(\overline{\mathbf{3}},\mathbf{1})_{\frac{5}{3}}$	$\frac{4}{5}M_\Xi$
$(\mathbf{3},\mathbf{2})_{\frac{5}{6}}$, $(\overline{\mathbf{3}},\mathbf{2})_{-\frac{5}{6}}$	0 (NG modes)
$(\overline{\mathbf{6}},\mathbf{2})_{\frac{5}{6}}$, $(\mathbf{6},\mathbf{2})_{-\frac{5}{6}}$	$\frac{2}{5}M_\Xi$
$(\mathbf{8},\mathbf{1})_0$	$\frac{1}{5}M_\Xi$
$(\mathbf{8},\mathbf{3})_0$	$M_\Xi = 5m_{75}$

Using the gauge invariant Kähler Potential normalized as

$$\mathcal{K} = \Xi^{\dagger(AB)}_{(CD)}(e^{2gV})^C_G(e^{2gV})^D_H(e^{-2gV})^E_A(e^{-2gV})^F_B\Xi^{(GH)}_{(EF)}\,, \tag{2.17}$$

the mass of X-boson is given by $M_X = 2\sqrt{6}gv_{\mathbf{75}}$. From the saddle point condition for super-potential, we obtain the following relation among the VEV $v_{\mathbf{75}}$ and Lagrangian parameters.

$$\frac{\partial W_{\mathbf{75}}}{\partial v_{\mathbf{75}}} = 0\,, \qquad \Leftrightarrow \qquad v_{\mathbf{75}} = \frac{3}{2}\frac{m_{75}}{\lambda_{75}}\,. \tag{2.18}$$

There is mass splitting among the components of **75** after the symmetry breaking. The masses are listed in Table 2.2. $(\mathbf{3},\mathbf{2})_{\frac{5}{6}}$ and $(\overline{\mathbf{3}},\mathbf{2})_{-\frac{5}{6}}$ are absorbed as longitudinal modes of the X-boson.

The mass terms for color triplets arise from integrating out **50**s as

$$W_{\text{Color}} = M_{H_C}H_C\overline{H}'_C + M_{\overline{H}_C}H'_C\overline{H}_C\,, \tag{2.19}$$

with

$$M_{H_C} = \frac{48g_Hg'_{\overline{H}}v^2_{\mathbf{75}}}{M_2}\,, \qquad M_{\overline{H}_C} = \frac{48g'_Hg_{\overline{H}}v^2_{\mathbf{75}}}{M_1}\,. \tag{2.20}$$

In this model ,the typical mass scale of the color-triplets is around 10^{15} GeV. This is because that we assume M_1 and M_2 are of the order of the Planck scale. On the other hand, there remain four massless $SU(2)_L$ Higgs doublets, $H_f, \overline{H}_f, H'_f$, and $\overline{H}'_f$.

At last of this subsection, we consider the breaking of the PQ symmetry. We use the radiatively breaking of the the PQ symmetry [111]. P and Q play a role of $U(1)_{\rm PQ}$ breaking Higgs fields. The invariant superpotential for P and Q under the $U(1)_{\rm PQ}$ symmetry is given by

$$W'' = \frac{f_{\rm Pl}}{M_{\rm Pl}} P^3 Q + g_P \overline{H}'_A H'^A P\,. \tag{2.21}$$

The scalar potential generated from W'' is

$$V(P,Q) = \frac{f_{\rm Pl}^2}{M^2}\left(|P|^6 + |3P^2 Q|^2\right). \tag{2.22}$$

Since this potential is considerably flat, the PQ symmetry is easily broken via the soft SUSY breaking term for P. Concretely speaking, if we add the negative soft mass term ($\sim -m^2$) for the P field, the potential minimum is shifted and then P and Q obtain the non-zero VEV as

$$\langle P\rangle \sim \langle Q\rangle \sim \sqrt{\frac{M_{\rm Pl} m}{f_{\rm Pl}}}\,. \tag{2.23}$$

If we take $f \sim 1$ and $m \sim 1$ TeV, then $\langle P\rangle \sim \langle Q\rangle \sim 10^{11}$ GeV. In addition, the PQ symmetry breaking gives non-zero masses to the additional Higgs doublets

$$M_{H'_f} = g_P \langle P\rangle \sim 10^{11}\ {\rm GeV}\,. \tag{2.24}$$

The right-handed neutrinos $N_i(\mathbf{1})$ play a role of removing an extra global $U(1)$ symmetry. In fact, the PQ charges are characterized α and β. The superpotential with the right-handed neutrinos must be the following form

$$W''' = k_{ij} N_i \Phi_{jA} H^A + \lambda_{Nij} N_i N_j P\,, \tag{2.25}$$

in order that the see-saw mechanism works well. From the first term, we determine the PQ charge for the right-handed neutrinos, and hence the second term is invariant under $U(1)_{\rm PQ}$ if and only if $\alpha = 3\beta$.

2.2 Nucleon Decay in SUSY GUTs

As we mentioned at the beginning of this section, the nucleon decay is the promising signal of the SUSY GUTs. In this thesis, since we impose the R-parity conservation, baryon-number violating dimension-four operators are forbidden. The dominant decay modes in the SUSY GUTs with the simple unified group are induced via the color-triplet mediation and the extra gauge boson mediation.

Dimension-five Operators

After we integrate out the color-triplet Higgs multiplets, we obtain the dimension-five operators in the superpotential as follows.

$$
\begin{aligned}
W = & -\frac{1}{M_{H_C}}(U_{CKM})^*_{kl} y^l_d y^i_u e^{i\varphi_i} \overline{U}_i \overline{E}_i Q_k L_l \\
& -\frac{y^i_u e^{i\varphi_i}}{M_{H_C}} \left((U_{CKM})^*_{kl} y^l_d \epsilon^{abc} \overline{U}_{ia} \overline{E}_i \overline{U}_{kb} \overline{D}_{lc} + \frac{1}{2} (U_{CKM})^*_{kl} y^l_d \epsilon_{abc} (Q^b_i Q^c_i)(Q^a_k L_l) \right) .
\end{aligned}
\tag{2.26}
$$

Here, we use the gauge basis for matter superfields in Eq. (2.26). The last two terms give rise to the baryon-number violating processes. In the minimal SUSY $SU(5)$ GUT, the rapid decay mode ($p \to K^+ + \overline{\nu}$) is generated by these operators. This is because the flavor conserved part is vanished due to the anti-symmetric tensor for color indices. If the SUSY spectra lie around a several TeV, the partial lifetime is $\tau(p \to K^+ + \overline{\nu}) \simeq 10^{31}$ years [37,38]. On the other hand, the current lower bound for this decay mode is $\tau(p \to K^+ + \overline{\nu}) \gtrsim 6.6 \times 10^{33}$ years given by the Super-Kamiokande experiment [39]. Thus, the minimal SUSY $SU(5)$ GUT with the low-scale SUSY is excluded by the Super-Kamiokande experiment.

In order to evade from the experimental bound, some ideas have been proposed, which suppress or prohibit the dimension-five operators. The following list shows the examples which relax the proton decay constraints.

1. Global Symmetries

   If the supersymmetric mass term for color-triplets is prohibited by global symmetries, the dimension-five operators are not generated. Indeed, the $p \to K^+ + \overline{\nu}$ decay mode is suppressed in the SUSY $SU(5)$ GUTs by imposing the Peccei-Quinn symmetry [110], the anomalous $U(1)$ symmetry [112], and so on.

2. Extradimensions

   In five-dimensional $\mathcal{N} = 1$ SUSY theories, the chiral superfields H and $\overline{H}$ are respectively embedded in hypermultiplets (H, H^c) and $(\overline{H}, \overline{H}^c)$ in terms of the four-dimensional $\mathcal{N} = 2$ SUSY. The mass terms only for HH^c and $\overline{H}\overline{H}^c$ are allowed by orbifolding [113]. Furthermore, this rapid decay operators are prohibit by the continuum global symmetry (such as the remaining $U(1)$ R-symmetry [114]) or discrete symmetries [115,116].

3. High-scale or Split SUSY

   In the split SUSY scenarios, since amplitudes for this decay mode are proportional to $M_{H_C} M_S$ with the mass of sfermions M_S, this mode dominates proton lifetime and it is detectable at the future experiment (Hyper-Kamiokande) [117–119].

As we have listed, the dimension-five decay mode strongly depends on the GUT models. Moreover, the Planck scale physics may also affect to the prediction for this decay mode if the SUSY breaking scale is lying around TeV (see Refs. [120,121]).

Dimension-six Operators

The dimension-six proton decay operators are generated via the extra gauge-boson exchange. In the minimal setup of SUSY $SU(5)$ GUTs, the X-boson interactions with the matter superfields are given by the following terms.

$$\mathcal{L}_X = \int d^4\theta \left(\mathcal{K}^{(0)}_{V_1} + \mathcal{K}^{(0)}_{V_2} + \mathcal{K}^{(0)}_{V_3} \right) , \tag{2.27}$$

with

$$\begin{aligned} \mathcal{K}^{(0)}_{V_1} &= -\sqrt{2} g_5 \epsilon^{rs} L^\dagger_{si} \overline{D}_{\alpha i} X^{\dagger \alpha}_{\ r} + \text{h.c.} , \\ \mathcal{K}^{(0)}_{V_2} &= -\sqrt{2} g_5 \epsilon_{\alpha\beta\gamma} e^{i\varphi_i} \overline{U}^{\dagger \gamma}_i Q^{r\beta}_i X^{\dagger \alpha}_{\ r} + \text{h.c.} , \\ \mathcal{K}^{(0)}_{V_3} &= -\sqrt{2} g_5 \epsilon^{sr} (U_{\text{CKM}})_{ij} Q^\dagger_{s\alpha i} \overline{E}_j X^{\dagger \alpha}_{\ r} + \text{h.c.} , \end{aligned} \tag{2.28}$$

and the baryon-number violating operators are effectively induced by integrating out the X boson at the low energy. The effective dimension-six operators are written as follows at the tree level; *

$$\mathcal{L}_{\text{dim.6}} = \int d^4\theta \left(\mathcal{K}^{(0)}_1 + \mathcal{K}^{(0)}_2 \right) , \tag{2.29}$$

with

$$\begin{aligned} \mathcal{K}^{(0)}_1 &= -e^{i\varphi_i} \frac{g_5^2}{M_X^2} \epsilon_{\alpha\beta\gamma} \epsilon_{rs} \overline{U}^{\dagger\alpha}_i \overline{D}^{\dagger\beta}_j Q^{r\gamma}_i L^s_j + \text{h.c.} , \\ \mathcal{K}^{(0)}_2 &= -e^{i\varphi_i} (U_{\text{CKM}})^*_{kj} \frac{g_5^2}{M_X^2} \epsilon_{\alpha\beta\gamma} \epsilon_{rs} \overline{E}^\dagger_j \overline{U}^{\alpha\dagger}_i Q^{r\beta}_k Q^{s\gamma}_i + \text{h.c.} . \end{aligned} \tag{2.30}$$

The dominant decay mode by these operators is that proton decays into neutral pion and positron. The Super-Kamiokande experiment has reported the current lower lifetime limit on this mode is 1.6×10^{34} years [40].

The effective Lagrangian for the dominant decay mode is

$$\mathcal{L}_p = \frac{g_5^2}{M_X^2} e^{i\phi_1} \epsilon_{\alpha\beta\gamma} \left((u^\alpha_R d^\beta_R)(u^\gamma_L e_L) + (1 + |U_{ud}|^2)(u^\alpha_L d^\beta_L)(u^\gamma_R e_R) \right) , \tag{2.31}$$

where U_{ud} is the $(1,1)$ component of the CKM matrix. The decay rate for this mode is given by

$$\Gamma(p \to \pi^0 + e^+) = \frac{m_p}{32\pi} \left(1 - \frac{m^2_{\pi^0}}{m^2_p} \right)^2 \sum_{I=1}^{2} |C^{(0)}_I(\mu) W^I_0(\mu)|^2 . \tag{2.32}$$

*Notice that the propagators of the vector superfields differ from those of canonically normalized gauge bosons by a factor $1/2$ under our convention for the kinetic terms of the vector superfields.

Here, m_p and m_{π^0} are respectively the masses of a proton and a neutral pion. $C_I^{(0)}$ are the Wilson coefficients while W_0^I represents the hadron matrix elements.

$$\begin{aligned}\epsilon_{\alpha\beta\gamma}\,\langle 0|\,(u_R^{\alpha} d_R^{\beta}) u_L^{\gamma}\,|p\rangle &= W_0^1\; P_L u_p\,,\\ \epsilon_{\alpha\beta\gamma}\,\langle 0|\,(u_L^{\alpha} d_L^{\beta}) u_R^{\gamma}\,|p\rangle &= W_0^2\; P_R u_p\,.\end{aligned} \tag{2.33}$$

μ represents the renormalization scale and the scale dependence is vanished up to the appropriate order in perturbation theory, that is, the μ-dependences of $C_I^{(0)}$ and W_0^I cancel with each other.

We estimate for $C_I^{(0)}(\mu)$ perturbatively, while we can not do for W_0^I. Since the QCD coupling becomes strong at the low-energy scale, the hadron matrix elements should be estimated by the non-perturbative way such as the lattice calculation. There are two ways of calculating the hadron matrix elements; one is the indirect method and another is the direct method. The direct method is a direct calculation of the three-point functions of the meson-operator-baryon, while the indirect method is based on the low-energy chiral perturbation theory and is an indirect calculation through the two-point functions. In our numerical analysis, we use the results from the lattice simulation with direct method and $N_f = 2+1$ dynamical domain-wall fermion [41].

$$\begin{aligned}W_0^1(\mu = 2\ \mathrm{GeV}) &= -0.103(23)(34)\ \mathrm{GeV}^2\,,\\ W_0^2(\mu = 2\ \mathrm{GeV}) &= 0.133(29)(28)\ \mathrm{GeV}^2\,.\end{aligned} \tag{2.34}$$

The first and second errors in W_0^I show the statistical and systematic errors, respectively. Ref. [41] has shown that the total error for W_0^1 (W_0^2) is 40% (30%).

While the hadron matrix elements are estimated at the hadronic scale ($\mu = 2$ GeV), the initial condition for the Wilson coefficients is given at the GUT scale ($\sim 2\times 10^{16}$ GeV). It is important to include the RGE effects in order to evaluate the low-energy observables. Indeed, the large hierarchy between the GUT and EW scales leads to a significant logarithmic corrections to the Wilson coefficients, while the strong coupling around the hadronic scale also generates large corrections to them. The former is called the short-range effects, on the other hand the latter is called the long-range effects.

Perturbative calculations for the Wilson coefficients have been already performed at partially two-loop level. For long-range effects, the two-loop RGEs by the strong coupling were derived in Ref. [46]. The short-range renormalization factors for effective operators are also estimated in several literature; in the SM [45] and in the SUSY SM [47]. The above authors have taken into account only the gauge interactions for the short-range factors. Indeed, the effective operators for the dominant decay mode include only the first generation quarks and leptons, and thus the Yukawa corrections would be negligible.

Anyway, the comparable corrections have not been included in the short-range renormalization effect, that is, the finite threshold corrections.

2.3 GUT Mass Spectrum Constraints

In the next leading order (NLO) calculations, couplings in the effective field theory (EFT) include effects of massive degrees of freedom via threshold corrections. In particular, a mass spectrum of GUT particles is constrained using low-energy coupling constants and their threshold corrections. In this section, we review the constraint on the GUT mass spectrum via the threshold corrections.

The gauge couplings in the EFT relate with the unified coupling at one-loop level as follows

$$\frac{1}{g_i^2(\mu)} = \frac{1}{g_5^2(\mu)} - \xi_i(\mu)\,. \tag{2.35}$$

Here, μ is the matching scale, which we set to be $\mu = 2 \times 10^{16}$ GeV, and ξ_i is the threshold correction for g_i. This threshold correction in the $\overline{\text{DR}}$ scheme [122] is obtained as [123,124]

$$\xi_i(\mu)\delta^{AB} = \frac{1}{48\pi^2}\left[-21\text{tr}\left(I_{iV}^{AB}\ln\frac{M_V}{\mu}\right) + 8\text{tr}\left(I_{iF}^{AB}\ln\frac{M_F}{\mu}\right) + \text{tr}\left(I_{iS}^{AB}\ln\frac{M_S}{\mu}\right)\right], \tag{2.36}$$

where M_V, M_F, and M_S are the mass matrices of a massive gauge boson, a massive Dirac fermion, and a massive real scalar boson. The first term includes contributions from not only the massive vector boson but also ghost and Nambu-Goldstone (NG) bosons. We use a shorthand notation $I_{iX}^{AB} \equiv T_{iX}^A T_{iX}^B$ ($X = V, F$, and S). T_{iX} is the generators for the unbroken symmetry. The subscript $X = V, F$, and S represent the vector boson, the Dirac fermion, and the scalar bosons, respectively.

In supersymmetric field theories, we can write the threshold correction in terms of superfields. A vector superfield consists of a gauge field and two Weyl fermions, while a chiral superfield does of a complex scalar and two Weyl fermions. All components composing a superfield have the same mass, so that mass matrices are proportional to the unit matrices. For vector superfields, the threshold correction is given by

$$4\pi\xi_i(\mu) = \frac{1}{2\pi}(-2C)\ln\frac{M_V}{\mu}\,. \tag{2.37}$$

where $C\delta^{AB} = \text{tr}\, I_{iV}^{AB}$. Here, of course, this threshold correction includes the contribution from the absorbed NG supermultiplets. For chiral superfields, the threshold correction is

$$4\pi\xi_i(\mu) = \frac{1}{2\pi}T\ln\frac{M_S}{\mu}\,, \tag{2.38}$$

with $T\delta^{AB} = \text{tr}\, I_{iS}^{AB}$.

Since the gauge couplings unify into a single one in the SUSY GUTs, we obtain two independent equations without g_5 from Eq. (2.35). We take two independent equations as

$$\begin{aligned} \frac{3}{g_2^2(\mu)} - \frac{2}{g_3^2(\mu)} - \frac{1}{g_1^2(\mu)} &= -3\xi_2(\mu) + 2\xi_3(\mu) + \xi_1(\mu)\,, \\ \frac{5}{g_1^2(\mu)} - \frac{3}{g_2^2(\mu)} - \frac{2}{g_3^2(\mu)} &= -5\xi_1(\mu) + 3\xi_2(\mu) + 2\xi_3(\mu)\,. \end{aligned} \tag{2.39}$$

The right-hand side of these equations depends only on the GUT mass spectrum. In the next chapter, we will show explicit forms in the minimal $SU(5)$ model and the missing partner $SU(5)$ model. The left-hand side is just the gauge couplings at the GUT scale, which are evolved using two-loop RGEs. We get them using low-energy observables and assumptions of a model at intermediate scale. In other words, since we know the values of the left-hand side under some assumptions, the GUT mass spectrum in the right-hand side is constrained.

Chapter 3

Threshold Corrections to Dimension-six Operators

As we mentioned in Chapter 2, the NLO calculations for dimension-six baryon-number violating processes have been already performed partially. Indeed, the finite threshold corrections have not been taken into account in the NLO calculations [45–47]. In this section, we discuss the finite corrections to the baryon-number violating dimension-six operators at the SUSY and GUT scales.

This chapter is based on Refs. [125, 126].

3.1 Threshold Correction at SUSY Scale

In this section, we derive threshold corrections of dimension-six operators at the SUSY scale. We concentrate only on gauge interactions, again.

Two-Point Function

Now, we estimate threshold corrections to the two-point function of fermions. After picking the UV divergence, we obtain the two-point function of SM fermions ψ including the one-loop finite corrections as follows:

$$i\Gamma^{\psi}_{\text{full}} = \left[1 - \frac{1}{16\pi^2}\sum_a g_a^2 C_a(\Phi) f(M_a^2, m_\phi^2) + (\text{SM contributions})\right] i\Gamma^{\psi}_0 \,. \tag{3.1}$$

Here, $\Gamma^{\psi}_0 \equiv \not{p} - m_\psi$ denotes the tree-level two-point function of ψ with four-momentum p^μ and tree-level mass m_ψ. M_a and m_ϕ are the masses of gaugino and the superpartners of ψ, respectively. The second term arises from the gaugino-sfermion loop and the third term describes the contribution from the SM loops. g_a and $C_a(\Phi)$ are the SM gauge coupling and the quadratic Casimir invariant, respectively. The loop function f is defined as

$$f(x,y) \equiv 2\int_0^1 ds\,(1-s)\ln\left[sx+(1-s)y\right]\,, \tag{3.2}$$

where all mass parameters are normalized by renormalization scale μ. It is easily found that $f(x,x) = \frac{1}{2}\ln x$, which corresponds to the case of degenerate masses.

In the SM, the one-loop corrected two-point function has the form

$$i\Gamma^{\psi}_{\text{SM}} = \left[1 - \lambda_\psi + (\text{SM contributions})\right] i\Gamma^{\psi}_{0}\,, \tag{3.3}$$

where λ_ψ denotes the one-loop threshold correction to the two-point function of ψ. Then, we get λ_ψ after matching the two-point functions in the two theories:

$$\lambda_\psi = \frac{1}{16\pi^2}\sum_a g_a^2 C_a(\Phi) f(M_a^2, m_\phi^2)\,. \tag{3.4}$$

Four-Fermi Vertices

The baryon-number violating dimension-six operators are induced by the non-renormalizable Kähler potential. The supersymmetric interaction term including four-Fermi operators is given by the following Kähler potential.

$$\begin{aligned} C^{ijkl}\int d^4\theta\, \Phi_i^\dagger \Phi_j^\dagger \Phi_k \Phi_l = &-\frac{C^{ijkl}}{4}\left[\Box(\phi_i^*\phi_j^*)\phi_k\phi_l - 2\partial_\mu(\phi_i^*\phi_j^*)\partial^\mu(\phi_k\phi_l) + \phi_i^*\phi_j^*\Box(\phi_k\phi_l)\right] \\ &- \frac{iC^{ijkl}}{2}(\phi_i^*\overline{\Psi}_j + \overline{\Psi}_i\phi_j^*)\gamma^\mu \overleftrightarrow{\partial_\mu}(\phi_k\Psi_l + \Psi_k\phi_l) \\ &- \frac{C^{ijkl}}{2}\overline{\Psi}_i\gamma^\mu P_L \Psi_l \overline{\Psi}_j \gamma_\mu P_L \Psi_k\,. \end{aligned} \tag{3.5}$$

Here, Φ_i ($\Phi_i^\dagger$) is a chiral (anti-chiral) superfield. Scalar and four-component spinor components of Φ_i are denoted as ϕ_i and Ψ_i, respectively. The symbols $\Box$ and $\overleftrightarrow{\partial_\mu}$ are respectively defined as $\Box = \partial^\mu\partial_\mu$ and $A\overleftrightarrow{\partial_\mu}B = A\partial_\mu B - \partial_\mu AB$. The roman indices i, j, k, l describe the gauge and flavor indices. C^{ijkl} denotes the Wilson coefficient of the operators.

Fig. 3.1 shows the one-loop diagrams giving the four-Fermi interactions after integrating out superpartners. In these figures, the blobs denote the operator given by the second line of Eq. (3.5). Only four diagrams in Fig. 3.1 contribute as finite corrections to a decay rate at the SUSY scale. This is because one of two scalar fields must originate from chiral superfields and another from the anti-chiral ones. The one-loop contributions shown in Fig. 3.1 are

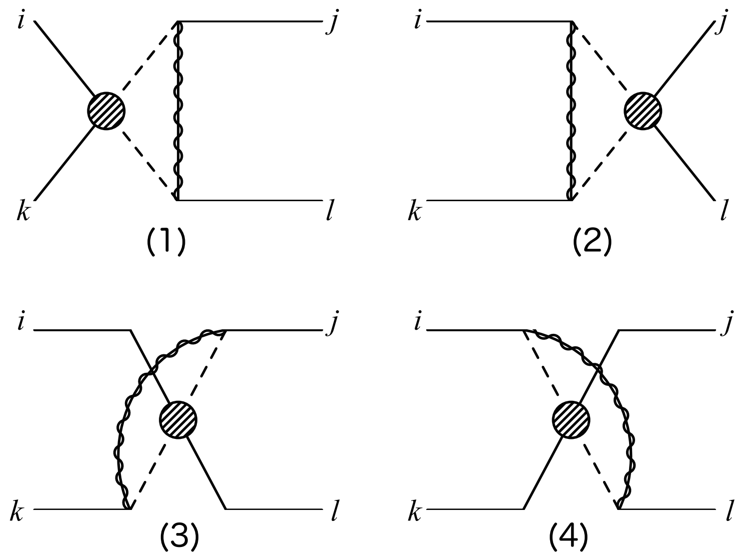


Figure 3.1: Four-Fermi interactions induced by one-loop diagrams. Blobs denote effective vertices including two fermions and two scalars, which are induced by the same Kähler potential. Solid and dashed lines describe fermion and scalar lines, respectively, while wavy-solid lines indicate gauginos.

given by;

$$\begin{aligned} i\mathcal{M}_1 &= -\frac{ig_a^2}{16\pi^2} C^{nimk} \sum_a (T^a_{jn} T^a_{ml}) F(m^2_{\phi_m}, m^2_{\phi_n}, M_a^2) \left\langle \mathcal{O}_{jlik} \right\rangle , \\ i\mathcal{M}_2 &= -\frac{ig_a^2}{16\pi^2} C^{jnlm} \sum_a (T^a_{in} T^a_{mk}) F(m^2_{\phi_m}, m^2_{\phi_n}, M_a^2) \left\langle \mathcal{O}_{jlik} \right\rangle , \\ i\mathcal{M}_3 &= \frac{ig_a^2}{16\pi^2} C^{nilm} \sum_a (T^a_{jn} T^a_{mk}) F(m^2_{\phi_m}, m^2_{\phi_n}, M_a^2) \left\langle \mathcal{O}_{jkil} \right\rangle , \\ i\mathcal{M}_4 &= -\frac{ig_a^2}{16\pi^2} C^{jnmk} \sum_a (T^a_{in} T^a_{ml}) F(m^2_{\phi_m}, m^2_{\phi_n}, M_a^2) \left\langle \mathcal{O}_{jkil} \right\rangle . \end{aligned} \tag{3.6}$$

Here, the subscript i for $\mathcal{M}_i$ corresponds to the label (i) of Fig. 3.1. M_a and m_{ϕ_m} represent the masses of gaugino and the superpartner of Ψ_m, respectively. We use a shorthand notation for operators $\mathcal{O}_{ijkl}$ defined by $\mathcal{O}_{ijkl} \equiv \overline{\Psi}_i \gamma^\mu P_L \Psi_j \overline{\Psi}_k \gamma_\mu P_L \Psi_l$. $\langle \ldots \rangle$ denotes the matrix element. We also define the loop function $F(x, y, z)$ as

$$F(x,y,z) = \frac{3}{4} + \frac{x^2(y-z)\ln x + y^2(z-x)\ln y + z^2(x-y)\ln z}{2(x-y)(y-z)(z-x)} . \tag{3.7}$$

Here, all masses are normalized by the renormalization scale μ, again.

Next, we concentrate on proton decay operators. However, this result is applicable to other operators consisting of two chiral and two anti-chiral superfields, for instance flavor-changing neutral currents.

Baryon-Number Violating Operators in SUSY SM

Let us now consider baryon-number violating operators in the SUSY SM. The Kähler potential for these operators is given by Eq. (2.29). The baryon-number violating four-Fermi operators are given by

$$\mathcal{L}_{\Delta B} = \sum_{i=1}^{2} C_{4F}^{(i)} \mathcal{O}_{4F}^{(i)} , \tag{3.8}$$

with operators

$$\begin{aligned}
\mathcal{O}_{4F}^{(1)} &= \epsilon_{\alpha\beta\gamma}\epsilon_{rs}(\overline{u}^{\alpha}\gamma^{\mu}P_L q^{r\gamma})(\overline{d}^{\beta}\gamma_{\mu}P_L l^{s}) , \\
\mathcal{O}_{4F}^{(2)} &= \epsilon_{\alpha\beta\gamma}\epsilon_{rs}(\overline{u}^{\alpha}\gamma^{\mu}P_L q^{s\gamma})(\overline{e}\gamma_{\mu}P_L q^{\beta r}) ,
\end{aligned} \tag{3.9}$$

and Wilson coefficients $C_{4F}^{(1)} = C_{4F}^{(2)} = g_5^2/2M_X^2$. The Wilson coefficients in the low-energy effective field theory (EFT) include threshold corrections: we simply redefine

$$C_{4F}^{(i)} \to (1 - \lambda_{\text{SUSY}}^{(i)})C_{4F}^{(i)} , \tag{3.10}$$

where $\lambda_{\text{SUSY}}^{(i)}$ denotes the threshold correction to $C_{4F}^{(i)}$.

After matching amplitudes in the SM and those in the SUSY SM, we find the threshold corrections $\lambda_{\text{SUSY}}^{(i)}$ at the SUSY scale as follows;

$$\begin{aligned}
\lambda_{\text{SUSY}}^{(1)} = &-\frac{1}{2}(\lambda_u + \lambda_q + \lambda_d + \lambda_l) - \frac{g_3^2}{16\pi^2}\left(\frac{1}{3}F(m_{\widetilde{u}}^2, m_{\widetilde{q}}^2, M_3^2) + \frac{1}{3}F(m_{\widetilde{d}}^2, m_{\widetilde{q}}^2, M_3^2)\right) \\
&- \frac{g_Y^2}{16\pi^2}\left(\frac{2}{9}F(m_{\widetilde{u}}^2, m_{\widetilde{q}}^2, M_1^2) - \frac{1}{9}F(m_{\widetilde{d}}^2, m_{\widetilde{q}}^2, M_1^2) - \frac{2}{3}F(m_{\widetilde{l}}^2, m_{\widetilde{u}}^2, M_1^2) + \frac{1}{3}F(m_{\widetilde{l}}^2, m_{\widetilde{d}}^2, M_1^2)\right) , \\
\lambda_{\text{SUSY}}^{(2)} = &-\frac{1}{2}(\lambda_u + \lambda_e + 2\lambda_q) - \frac{g_3^2}{16\pi^2}\frac{2}{3}F(m_{\widetilde{u}}^2, m_{\widetilde{q}}^2, M_3^2) \\
&- \frac{g_Y^2}{16\pi^2}\left(\frac{4}{9}F(m_{\widetilde{u}}^2, m_{\widetilde{q}}^2, M_1^2) - \frac{2}{3}F(m_{\widetilde{e}}^2, m_{\widetilde{q}}^2, M_1^2)\right) .
\end{aligned} \tag{3.11}$$

Here, m_ϕ ($\phi = \widetilde{q}, \widetilde{u}, \widetilde{d}, \widetilde{l}, \widetilde{e}$) denotes the sfermions mass while M_1 and M_3 denote the masses of bino and gluino, respectively. The loop function F is defined in Eq. (3.7), again. λ_ψ ($\psi = q, l, u, d, e$) is defined in Eq. (3.4), and represents the one-loop threshold corrections to the two-point functions of the SM chiral fermions.

3.2 Threshold Corrections at GUT scale

In supersymmetric theories, an effective Kähler potential is useful to derive radiative corrections. In order to evaluate the radiative corrections, we discuss the effective Kähler potential at the one-loop level, and evaluate the threshold corrections to the baryon-number violating operators.

First of all, let us discuss a general effective supersymmetric action $\Gamma[\Phi, \Phi^\dagger]$. which is the function of chiral superfield Φ, anti-chiral superfield $\Phi^\dagger$, and their derivatives. The general form of the effective supersymmetric action would be as follows,

$$\begin{aligned}\Gamma[\Phi,\Phi^\dagger] = &\int d^4x d^4\theta\ \mathcal{L}_{\text{eff}}(\Phi, \mathcal{D}_A\Phi, \mathcal{D}_A\mathcal{D}_B\Phi, \cdots, \Phi^\dagger, \mathcal{D}_A\Phi^\dagger, \mathcal{D}_A\mathcal{D}_B\Phi^\dagger, \cdots) \\ &+ \left\{ \int d^4x d^2\theta\ \mathcal{L}^{(c)}_{\text{eff}}(\Phi, \mathcal{D}_A\Phi, \mathcal{D}_A\mathcal{D}_B\Phi, \cdots) + \text{h.c.} \right\},\end{aligned} \tag{3.12}$$

where $\mathcal{D}_A$ is the superspace covariant derivative which consists of ∂_μ, $\mathcal{D}_\alpha$, and $\overline{\mathcal{D}}_{\dot\alpha}$. Here, we do not include vector superfields for simplicity. The perturbative corrections appear only in the D term due to the non-renormalization theorem [48]. The effective supersymmetric Lagrangian $\mathcal{L}_{\text{eff}}$ is divided into two parts under $\partial_\mu\Phi = 0$,

$$\mathcal{L}_{\text{eff}} = \mathcal{K}(\Phi, \Phi^\dagger) + \mathcal{F}(\mathcal{D}_\alpha\Phi, \mathcal{D}^2\Phi, \overline{\mathcal{D}}_{\dot\alpha}\Phi^\dagger, \overline{\mathcal{D}}^2\Phi^\dagger; \Phi, \Phi^\dagger), \tag{3.13}$$

where $\mathcal{K}(\Phi, \Phi^\dagger)$ is the effective Kähler potential and $\mathcal{F}(\mathcal{D}_\alpha\Phi, \mathcal{D}^2\Phi, \overline{\mathcal{D}}_{\dot\alpha}\Phi^\dagger, \overline{\mathcal{D}}^2\Phi^\dagger; \Phi, \Phi^\dagger)$ is called the effective auxiliary potential. While some diagrams may generate the terms including superfields on which more than three covariant derivatives act, we may always obtain the above form using algebra of super-covariant derivatives (D-algebra). * The effective auxiliary potential vanishes in the limit that $\mathcal{D}_\alpha\Phi = 0$ and $\overline{\mathcal{D}}_{\dot\alpha}\Phi^\dagger = 0$, so that the effective Kähler potential is equivalent to the effective Lagrangian in this limit.

Below, we study the threshold corrections to the baryon-number violating dimension-six operators at the GUT scale with the effective Kähler potential. First, we calculate the effective actions for constant fields in both full and effective theories at the one-loop level with the supergraph technique [56]. We adopt the modified dimensional reduction ($\overline{\text{DR}}$) scheme [122] as the renormalization scheme of the gauge coupling constants while we impose the on-shell condition for the mass on the massive vector superfield. We also introduce the IR cut off in order to control fictitious IR singularities. Then, we identify the effective Kähler potential for the baryon-number violating operators by taking $\mathcal{D}_\alpha\Phi = 0$ and $\overline{\mathcal{D}}_{\dot\alpha}\Phi^\dagger = 0$ together with the D-algebra. By matching the effective Kähler potentials in full and effective theories, we derive the one-loop threshold corrections to the Wilson corrections of the dimension-six operators.

*We give formulae for the D-algebra in Appendix C.1.

Radiative Corrections in the UV Theory

To begin with, we show the radiative corrections to the baryon-number violating dimension-six operators in the full theory, where the massive vector superfield is activated. The radiative corrections consist of the wave function renormalization of matter superfields, the vacuum polarization of the massive vector superfield, the vertex correction, and the box-like corrections. In this subsection, we show only the results of the supergraph calculation. Details of the calculations are given in Appendix E.

Two-Point Functions for Matter Superfields

First we study two-point functions for matter superfields at the one-loop level. The functions generally include UV divergences which are renormalized by the wave function renormalization factors. We estimate the factors in the $\overline{\mathrm{DR}}$ scheme, ignoring the contributions from the Yukawa interactions. The radiative corrections to the two-point functions via the gauge interactions are determined by the gauge groups, in the both of the full theory and the EFT.

In general, the renormalized two-point function for a chiral superfield Φ is defined as

$$\Gamma_{\Phi}^{\text{2-pt}} = \widetilde{\Gamma}_{\Phi}^{\text{2-pt}} + \delta_{Z_\Phi}\,, \tag{3.14}$$

with the un-renormalized two-point function $\widetilde{\Gamma}_{\Phi}^{\text{2-pt}}$ and a counter term $\delta_{Z_\phi} = Z_\Phi - 1$. The wave function renormalization constant for the matter superfield Z_Φ absorbs the UV divergent terms proportional to $1/\epsilon'$ in the $\overline{\mathrm{DR}}$ scheme: *

$$Z_\Phi = 1 + C_5(\Phi)\frac{g_5^2}{4\pi^2} \times \frac{1}{\epsilon'}\,, \quad Z_\Phi^{\text{EFT}} = 1 + \sum_{a=1}^{3} C_a(\Phi)\frac{g_a^2}{4\pi^2} \times \frac{1}{\epsilon'}\,. \tag{3.15}$$

Here, Z_Φ and Z_Φ^{EFT} denote the wave function renormalization factors in the full and the effective theories, respectively. g_3, g_2, and g_1 are the gauge couplings of $SU(3)_C, SU(2)_L$, and unified $U(1)_Y$ gauge symmetries, again. $C_5(\Phi)$ and $C_a(\Phi)$ ($a = 3,\,2,\,1$) are the quadratic Casimir of Φ in $SU(5)$, $SU(3)_C$, $SU(2)_L$, and GUT normalized $U(1)_Y$ gauge symmetries. †

We obtain the one-loop renormalized two-point function for a chiral superfield Φ in the full theory as follow.

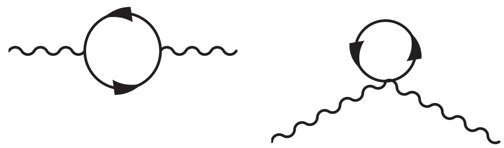
$$\Gamma_{\Phi}^{\text{2-pt;1-loop}} = \left(1 + a_\Phi f(M_X^2) + b_\Phi f(\mu_{\text{IR}}^2)\right)\Gamma_{\Phi 0}^{\text{2-pt}}\,. \tag{3.16}$$

Here, $\Gamma_{\Phi 0}^{\text{2-pt}}$ is the tree-level two-point function, and a_Φ and b_Φ are the constants obtained from the one-loop calculations,

$$a_\Phi = -\left(C_5(\Phi) - \sum_{a=1}^{3} C_a(\Phi)\right)\frac{g_5^2}{8\pi^2}\,, \quad b_\Phi = -\sum_{a=1}^{3} C_a(\Phi)\frac{g_5^2}{8\pi^2}\,. \tag{3.17}$$

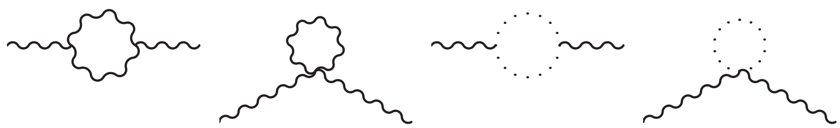
*$2/\epsilon' \equiv 2/\epsilon - \gamma + \ln 4\pi$ is defined and ϵ satisfies $\epsilon = 4 - d$ in the d-dimension momentum space. $\gamma = 0.57721\ldots$ is the Euler's constant.

†$C_1(\Phi)$ is given by $C_1(\Phi) = (Q_Y^\Phi)^2 \times (3/5)$, where Q_Y^Φ is hypercharge of Φ.



(a) (b) (c)

Figure 3.2: Diagrams of chiral multiplets for radiative correction to two-point function of superfield for massive vector superfield.



(a) (b) (c) (d)

Figure 3.3: Diagrams of gauge and ghost superfields for radiative correction to two-point function of superfield for massive vector superfield.

We set the mass of the MSSM vector superfields to be a non-zero value which is denoted by $\mu_{\rm IR}$ in order to regularize the IR divergence, as mentioned above. M_X represents the mass of the massive vector superfield. The function f in Eq. (3.16) is defined as

$$f(M^2) \equiv 1 - \ln \frac{M^2}{\mu^2}, \tag{3.18}$$

where μ denotes the renormalization scale in the $\overline{\rm DR}$ scheme. The two-point functions in the effective theory are derived by removing the massive vector contribution in Eq. (3.16) when $g_5 = g_3 = g_2 = g_1$.

Vacuum Polarization of Massive Vector Superfield

Next, we estimate radiative corrections to the propagator for the massive vector super field, which are known as vacuum polarization. Not only the MSSM superfields but the GUT-scale superfields such as the $SU(5)$-adjoint chiral superfield contribute to the vacuum polarization of the massive vector superfield.

The chiral superfields have three kinds of the contributions to the vacuum polarization, which is described in Fig. 3.2. The diagrams (a) and (b) are induced by the supergauge interaction $\Phi^\dagger V \Phi$ and $\Phi^\dagger V^2 \Phi$, respectively. Arrows in Fig. 3.2 (a) and (b) indicate chirality flow from an anti-chiral superfield into a chiral superfield. The diagram (c) is generated by the $SU(5)$-breaking Higgs superfield, which has interactions $\langle \Sigma^\dagger \rangle V^2 \Sigma$ and $\Sigma^\dagger V^2 \langle \Sigma \rangle$ after acquiring the VEV.

For the gauge sector, we have four-type diagrams to contribute to the two-point function of the massive vector superfield. Fig. 3.3 shows the diagrams. The diagrams (a) and (b) in Fig. 3.3 arise from the self interactions of the vector superfields. If the internal vector superfields in the diagram (b) are massless, the diagrams have no contribution to the two-point function in the $\overline{\text{DR}}$ scheme. The diagrams (c) and (d) show ghost contributions.

Finally, the two-point function of the massive vector superfield is in the form as below:

$$\Gamma_X^{(2)}(k^2) = k^2 - M_X^2 - \Sigma_X(k^2)\,, \tag{3.19}$$

where $\Sigma_X(k^2)$ is the renormalized vacuum polarization for massive vector superfield. The UV divergence in the one-loop corrections is absorbed by the wave function factor (Z_X) and mass (M_X) of the massive vector superfield. In this thesis, the on-shell condition for the mass of the massive vector superfield is imposed, so that this leads the equation $\Sigma_X(M_X^2) = 0$. This is because heavy particles are decoupled from $\Sigma_X(0)$ under the on-shell condition, if they have $SU(5)$ symmetric masses much larger than the mass of the massive vector superfield. * $\Sigma_X(0)$ will appear in the threshold correction to the baryon-number violating operators.

The counter term δZ_X is determined to absorb the UV divergence which arises from the gauge contributions and the matter contributions such as Figs. 3.2 and 3.3. We obtain

$$\delta Z_X = Z_X - 1 = \frac{g_5^2}{8\pi^2}\left(3C_2(G) - \sum_R I_G(R)\right) \times \frac{1}{\epsilon'}\,, \tag{3.20}$$

where $I_G(R)$ and $C_2(G)$ respectively represent a Dynkin index for an R-representation field and a quadratic Casimir invariant for the gauge group G. As expected, δZ_X is proportional to the one-loop beta function for the $SU(5)$ gauge coupling constant.

Let us now discuss the finite contributions to the vacuum polarization from various representations in $SU(5)$. First, fields in the irreducible representation of $SU(5)$ are decomposed into irreducible representations of the SM gauge groups. We give the SM decomposition of some $SU(5)$ representations in Appendix D.4. The vacuum polarization of the massive vector superfield is given by

$$\begin{aligned}\overline{\Sigma}_X(p^2) = \sum_{\text{Reps.}} \overline{\Sigma}_X^{\text{Rep.}}(p^2) - \frac{g_5^2}{16\pi^2}C_2(G)\left[\frac{5p^2}{2}A(p^2, M_X^2, 0) + B(p^2, M_X^2, 0)\right] \\ + (p^2\text{-independent terms})\,,\end{aligned} \tag{3.21}$$

with the vacuum polarization from a certain representation

$$\overline{\Sigma}_X^{\text{Rep.}}(p^2) = \frac{g_5^2}{16\pi^2}\sum_{i,j} b_{ij}B(p^2, M_i^2, M_j^2) + \frac{g_5^2 M_X^2}{16\pi^2}\sum_i a_i A(p^2, M_X^2, M_i^2)\,, \tag{3.22}$$

*The GUT-scale mass spectrum may be constrained using the gauge coupling unification [127,128]. In these Refs. , authors use the threshold correction to the gauge coupling constants at the GUT scale at the one-loop level so that the renormalization condition for the mass of the massive vector superfield does not appear there. We need the threshold correction at the two-loop level in order to get the constraint on the on-shell mass.

where the superscript "Rep." indicates the $SU(5)$ representation, such as $\mathbf{5}+\overline{\mathbf{5}}$, $\mathbf{10}+\overline{\mathbf{10}}$, $\mathbf{24}$, and so on. $i, j = 1, \cdots, N$ ($i, j = \overline{1}, \cdots, \overline{N}$) denote the labels of irreducible representations of the SM gauge groups (and its complex conjugated representation). The first term in Eq. (3.21) indicates the sum of the contributions from chiral superfields. The second term in Eq. (3.21) comes from the gauge sector illustrated by Fig. 3.3. In particular, the first and second terms in the square bracket arise from the gauge and ghost loops, respectively. The p^2-independent terms come from the diagrams Fig. 3.2 (b), Fig. 3.3 (b), and Fig. 3.3 (d). However, they do not give any corrections since we take the on-mass shell condition

$$\Sigma_X(p^2) = \overline{\Sigma}_X(p^2) - \overline{\Sigma}_X(M_X^2)\,, \tag{3.23}$$

for the vacuum polarization of the massive vector superfield. Coefficients a_i and b_{ij} in Eq. (3.22), which we call the vacuum polarization coefficients, are determined by the interactions between the massive vector superfield and the corresponding chiral superfields. The loop functions A and B in Eq. (3.22) are defined by

$$\begin{aligned} A(p^2, M_1^2, M_2^2) &\equiv \int_0^1 dx \ln \frac{\Delta}{\mu^2}\,, \\ B(p^2, M_1^2, M_2^2) &\equiv \int_0^1 dx \left[\Delta - (2\Delta + x(x-1)p^2)\ln\frac{\Delta}{\mu^2}\right]\,, \end{aligned} \tag{3.24}$$

where $\Delta = x(x-1)p^2 + xM_2^2 + (1-x)M_1^2$.

The first term in Eq. (3.22) comes from the gauge interaction between the massive vector and (anti-)chiral superfields. The second term in Eq. (3.22) arises from the interactions with the GUT-breaking VEV shown in Fig. 3.2 (c). The variables of the functions M_i denote the mass eigenvalues of chiral superfields in the loop. We show the vacuum polarization coefficients a_i and b_{ij} in Table 3.1 for some $SU(5)$ representations. The a_i and b_{ij} not listed in Table 3.1 are zero. Here, we list a_i and b_{ij} from $\mathbf{5}+\overline{\mathbf{5}}$, $\mathbf{10}+\overline{\mathbf{10}}$, $\mathbf{24}$, and $\mathbf{75}$ representations. Although the missing-partner model includes $\mathbf{50}+\overline{\mathbf{50}}$ pairs to induce the masses of color-triplet Higgs multiplets, their mass scales are set to be at the Planck scale in order to keep the theory in the perturbative regime. Hence, they do not contribute to the vacuum polarization of the massive vector superfield. In Appendix D.4, we display a_i and b_{ij} only from $SU(5)$ representations, whose Dynkin indices are smaller than the Dynkin index of the $\mathbf{75}$ representation.

As a concrete model, we show the vacuum polarization in the minimal SUSY $SU(5)$ GUT in the presence of the extra vector-like matters. The unified gauge coupling is enhanced in these models as we discussed in Section 1.3, so that it is important to estimate the size of the threshold corrections. For simplicity, we treat extra matters and the MSSM matters as

Table 3.1: Vacuum polarization coefficients b_{ij} and a_i. Coefficients which are not listed here are zero. a_0 is coefficient from NG and MSSM vector supermultiplets loops, while a_i ($i = 1, 2, \cdots$) is coefficient from massive vector and chiral supermultiplets loops. b_{ij} and a_i which are not listed here are zero.

Reps.	b_{ij}	a_i
$\mathbf{5}+\overline{\mathbf{5}}$	$b_{12} = b_{\overline{12}} = 1$	
$\mathbf{10}+\overline{\mathbf{10}}$	$b_{13} = b_{\overline{13}} = 1$ $b_{23} = b_{\overline{23}} = 2$	
$\mathbf{24}$	$b_{14} = b_{15} = 3/2$ $b_{24} = b_{25} = 8/3$ $b_{34} = b_{35} = 5/6$	$a_0 = 5/2$ $a_1 = 3$ $a_2 = 16/3$ $a_3 = 5/3$
$\mathbf{75}$	$b_{14} = b_{23} = 1/3$ $b_{16} = b_{25} = 2$ $b_{37} = b_{47} = 4/3$ $b_{38} = b_{48} = 2/3$ $b_{39} = b_{49} = 26/3$ $b_{58} = b_{68} = b_{59} = b_{69} = 6$	$a_0 = 5/2$ $a_1 = a_2 = 6$ $a_7 = 8/3$ $a_8 = 4/3$ $a_9 = 12$

massless fields.

$$
\begin{aligned}
\widetilde{\alpha}_5^{-1}\overline{\Sigma}_X(p^2) = {} & [(N_{\mathbf{5}} + N_{\overline{\mathbf{5}}}) + 3(N_{\mathbf{10}} + N_{\overline{\mathbf{10}}})]\, B(p^2, 0, 0) \\
& + \frac{25}{3} B(p^2, M_\Sigma^2, M_X^2) + \frac{5}{3} B(p^2, M_{\Sigma_{24}}^2, M_X^2) \\
& + 2B(p^2, M_{M_{H_C}}^2, 0) \\
& + \frac{5}{6} M_X^2 \left[3A(p^2, M_X^2, 0) + 10A(p^2, M_X^2, M_\Sigma^2) + 2A(p^2, M_X^2, M_{\Sigma_{24}})\right] \\
& - 5C_2(G)\frac{p^2}{2} A(p^2, M_X^2, 0) - C_2(G) B(p^2, M_X^2, 0) \\
& + (p^2\text{-independent terms})\,,
\end{aligned}
\tag{3.25}
$$

where N_r ($r = \mathbf{5}, \overline{\mathbf{5}}, \mathbf{10}, \overline{\mathbf{10}}$) denotes the number of massless superfields in r representation, and $\widetilde{\alpha}_5 = g_5^2/16\pi^2$. As we have seen, the first-three lines in Eq. (3.25) show the chiral superfield contributions illustrated in Fig. 3.2 (a). They consist of contributions from massless chiral superfields (the first line), the adjoint Higgs superfields (the second line), and the color-triplet Higgs superfield (the third line). In the second line, the first term is the sum of

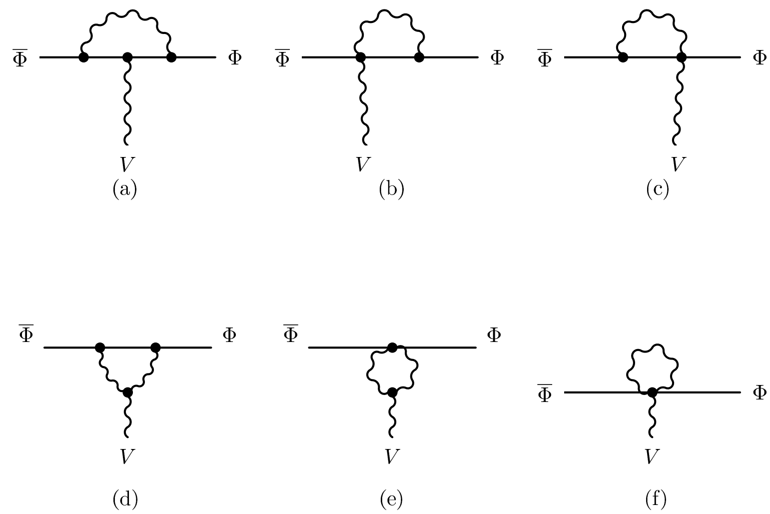


Figure 3.4: Diagrams for vertex correction

the color-octet and weak triplet contributions, and the second term is the singlet contribution. The fourth line is the contribution from the adjoint supermultiplets with GUT-breaking VEV, as in the second term of Eq. (3.22). We have already explained the last two lines in Eq. (3.21). In the case of the minimal SUSY $SU(5)$ GUT, $N_{\overline{\mathbf{5}}} = N_{\mathbf{10}} = 3$ and $N_{\mathbf{5}} = N_{\overline{\mathbf{10}}} = 0$.

The re-summed propagator $D_{XX}(p^2)$ of massive vector superfield in terms of the superfield notation is given by $D_{XX}(p^2) = -i/(2\Gamma_X^{(2)}(p^2))$. After the spontaneous symmetry breaking of the GUT gauge symmetry, the baryon-number violating dimension-six operators are induced by the massive vector superfield, and the coefficients are proportional to $1/M_X^2$. In order to match the full and the effective theories at the one-loop level, we need to take into account the one-loop corrections to the propagator of the massive vector superfield. Since the momenta of external fields in the baryon-number violating dimension-six operators are negligible compared with the mass of the vector superfield, we may set the momentum of internal massive vector superfield zero.

Vertex Corrections

Next, we show one-loop vertex corrections to the interactions of massive vector superfields with quark and lepton superfields. The tree-level interactions are given in Eq. (2.27).

Fig. 3.4 shows one-loop diagrams contributing to the vertex corrections. Since the supersymmetric gauge interactions in terms of the superfield formalism have the form $\Phi^\dagger e^{2gV}\Phi$, there exist diagrams which do not appear in calculation using component fields. Indeed, multi-vector interactions as $\Phi^\dagger V^n \Phi$ $(n \geq 3)$ also induce one-loop diagrams, which are illustrated in Fig. 3.4 (f). The diagram (a) has only the vertex $2g\Phi^\dagger V\Phi$, and the diagrams (b) and (c) include the vertex $2g^2\Phi^\dagger V^2\Phi$. The diagrams (d) and (e) include the three-point self interactions of vector superfields. Since the external vector superfield is for the broken gauge symmetry, two internal vector superfields must be massive and massless ones. The contribution from the diagram (f) is vanishing due to the superspace integral.

We now calculate the contributions from the diagrams (a)-(e) in Fig. 3.4. The momenta of all the external superfields are set to be $p^2 = 0$, for simplicity. In some diagrams, since they contain IR divergent contributions in this momentum assignment, the non-zero masses of the MSSM vector superfields ($\mu_{\rm IR}$) are introduced as IR regulators. Under this momentum assignment, we carry out loop momentum integrals and Grassmann integrals, and we discard the auxiliary terms. We expand the one-loop Kähler terms around $p^2 = 0$, and then we extract the dominant contributions around $p^2 = 0$. The vertex corrections to the gauge interactions between the MSSM matter superfields and the massive vector superfield are as follows:

$$\begin{aligned}
\mathcal{K}_{V_1}^{(1)} &= \left[-\frac{2}{5}C_1^{(\rm v)}(\mu_{\rm IR}) + \frac{21}{5}C_2^{(\rm v)}(\mu_{\rm IR}) + 5C_2^{(\rm v)}(M_X)\right]\mathcal{K}_{V_1}^{(0)}\,, \\
\mathcal{K}_{V_2}^{(1)} &= \left[\frac{12}{5}C_1^{(\rm v)}(\mu_{\rm IR}) - 2C_1^{(\rm v)}(M_X) + \frac{49}{5}C_2^{(\rm v)}(\mu_{\rm IR}) + 9C_2^{(\rm v)}(M_X)\right]\mathcal{K}_{V_2}^{(0)}\,, \\
\mathcal{K}_{V_3}^{(1)} &= \left[\frac{2}{5}C_1^{(\rm v)}(\mu_{\rm IR}) - 4C_1^{(\rm v)}(M_X) + \frac{29}{5}C_2^{(\rm v)}(\mu_{\rm IR}) + 13C_2^{(\rm v)}(M_X)\right]\mathcal{K}_{V_3}^{(0)}\,.
\end{aligned} \tag{3.26}$$

The contributions from the diagrams (d) and (e) in Fig. 3.4 are canceled each other. The coefficients $C_1^{(\rm v)}$ and $C_2^{(\rm v)}$ correspond to the correction from the diagram (a), and the ones from the diagrams (b) and (c) in Fig. 3.4, respectively. After the loop momentum and superspace integrals, we find that $C_1^{(\rm v)}$ and $C_2^{(\rm v)}$ are given by the functions of the mass of the internal vector superfield M,

$$C_1^{(\rm v)}(M) = -C_2^{(\rm v)}(M) \equiv \frac{1}{2}\frac{g_5^2}{16\pi^2}\left[\frac{2}{\epsilon'} + 1 - \ln\frac{M^2}{\mu^2}\right]. \tag{3.27}$$

These loop functions are the coefficients of the effective Kähler potential $\mathcal{K}_1^{(\rm v)}$ and $\mathcal{K}_2^{(\rm v)}$ defined in Appendix E in the limit that p^2 vanishes.

Now, we determine the renormalization constants for the vertices. One-loop renormalized vertex functions are given by

$$\begin{aligned}
\mathcal{K}_{V_1} &= \mathcal{K}_{V_1}^{(0)} + \mathcal{K}_{V_1}^{(1)} + \left(Z_L^{1/2}Z_D^{1/2}Z_X^{1/2}Z_{\mathcal{C}_{V_1}} - 1\right)\mathcal{K}_{V_1}^{(0)}\,, \\
\mathcal{K}_{V_2} &= \mathcal{K}_{V_2}^{(0)} + \mathcal{K}_{V_2}^{(1)} + \left(Z_U^{1/2}Z_Q^{1/2}Z_X^{1/2}Z_{\mathcal{C}_{V_2}} - 1\right)\mathcal{K}_{V_2}^{(0)}\,, \\
\mathcal{K}_{V_3} &= \mathcal{K}_{V_3}^{(0)} + \mathcal{K}_{V_3}^{(1)} + \left(Z_Q^{1/2}Z_E^{1/2}Z_X^{1/2}Z_{\mathcal{C}_{V_3}} - 1\right)\mathcal{K}_{V_3}^{(0)}\,.
\end{aligned} \tag{3.28}$$

When $\mathcal{K}_{V_n}$ ($n = 1, 2, 3$) are described as $\mathcal{K}_{V_n} = \mathcal{C}_{V_n}\mathcal{O}_{V_n}$ with the operators $\mathcal{O}_{V_n}$ and the Wilson coefficients $\mathcal{C}_{V_n}$, $Z_{\mathcal{C}_{V_n}}$ are defined to renormalize the UV divergences in $\mathcal{C}_{V_n}$. Then,

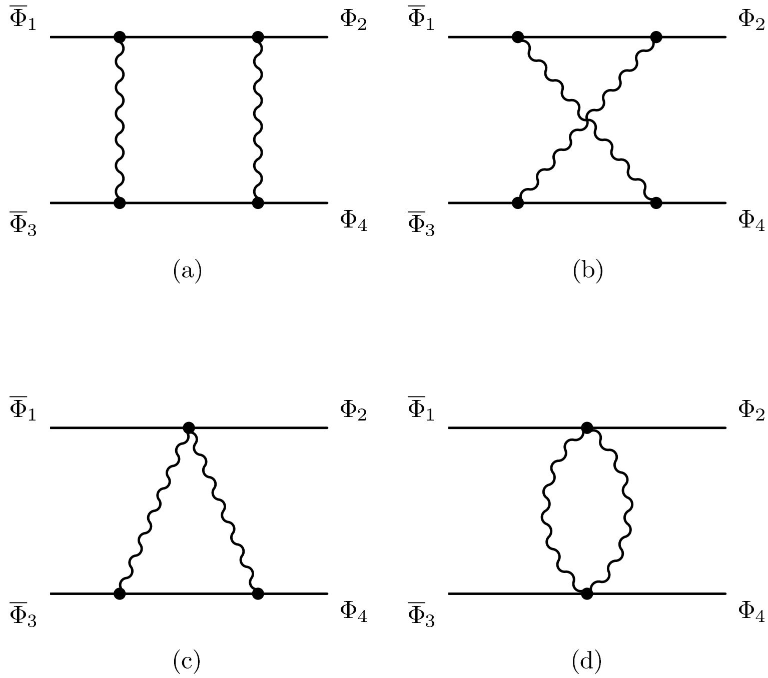


Figure 3.5: Box-like diagrams

we find the renormalization factors for $\mathcal{C}_{V_n}$ $(n = 1, 2, 3)$ as follows.

$$Z_{\mathcal{C}_{V_1}} = Z_{\mathcal{C}_{V_2}} = Z_{\mathcal{C}_{V_3}} = 1 - \frac{g_5^2}{16\pi^2}\left(3C_2(G) - \sum_R I_G(R)\right) \times \frac{1}{\epsilon'}, \tag{3.29}$$

which are consistent with the one-loop beta function for the gauge coupling.

Box-like Corrections

The box-like diagrams contribute to the radiative corrections of the dimension-six operators. Fig. 3.5 shows all type of the box-like diagrams; we refer to the diagram (a) as the box diagram, the diagram (b) as the crossing box diagram, and the diagram (c) as the triangle diagram. The diagram (d) vanishes due to the superspace integral. Thus, it is sufficient that we evaluate the diagrams (a)-(c) in Fig. 3.5. In these figures, one of two internal gauge superfield lines must be massive since we focus on the baryon-number violating operators. As is the case in the vertex corrections, we set all momenta of the external superfields to be $p^2 = 0$ and the fictitious masses of the MSSM vector superfields to be μ_{IR}, and we remove the auxiliary terms. For this momentum assignment, we find that the box diagram (a) vanishes while the crossing box diagram (b) and the triangle diagram (c) are given by the following

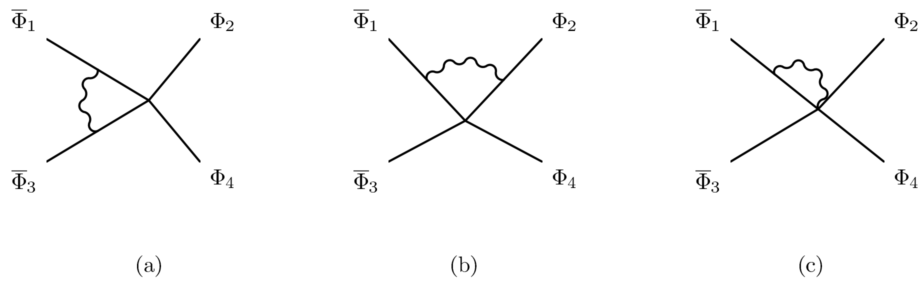


Figure 3.6: Radiative corrections in EFT

functions:

$$C_{\text{cross}}(M_X) = -C_{\text{triangle}}(M_X) \equiv -\frac{1}{2}\frac{g_5^2}{16\pi^2}\ln\frac{M_X^2}{\mu_{\text{IR}}^2}\,. \tag{3.30}$$

These loop functions correspond to the coefficients in the effective Kähler potential $\mathcal{K}_{\text{cross}}$ and $\mathcal{K}_{\text{triangle}}$ defined in Appendix E in the limit that p^2 vanishes.

In SUSY $SU(5)$ GUTs, the baryon-number violating dimension-six operators are generated at the tree level in Eq. (2.29). The one-loop radiative corrections from the box-like diagrams are written by C_{cross} and C_{triangle}:

$$\begin{aligned}
\mathcal{K}_1^{(\text{Box})} &= \left[-\frac{18}{5}C_{\text{cross}}(M_X) + \frac{14}{5}C_{\text{triangle}}(M_X)\right]\mathcal{K}_1^{(0)}\,, \\
\mathcal{K}_2^{(\text{Box})} &= \left[-\frac{14}{5}C_{\text{cross}}(M_X) + \frac{22}{5}C_{\text{triangle}}(M_X)\right]\mathcal{K}_2^{(0)}\,.
\end{aligned} \tag{3.31}$$

Radiative Corrections in EFT

Now we consider the radiative correction to the higher-dimensional Kähler terms in the EFT. There are three kinds of contributions to the radiative correction. The first one is the diagram (a) in Fig. 3.6, where a vector superfield is attached to two chiral superfields or two anti-chiral superfields. The second one is the diagram (b), in which a vector superfield is attached to both a chiral and an anti-chiral superfield. The third one is the radiative corrections induced by the gauge interaction of the composite operators.

We adopt the same momentum assignment which we used in the full theory. After the loop momentum and the superspace integrals, we derive the one-loop corrections as

$$\begin{aligned}
\mathcal{K}_1^{(1):\text{eff}} &= \left[\left(\frac{16}{3}g_3^2 + \frac{4}{15}g_1^2\right)C_1^{\text{EFT}}(\mu_{\text{IR}}) + \left(\frac{32}{3}g_3^2 + \frac{8}{15}g_1^2\right)C_2^{\text{EFT}}(\mu_{\text{IR}})\right]\mathcal{K}_1^{(0)}\,, \\
\mathcal{K}_2^{(1):\text{eff}} &= \left[\left(\frac{16}{3}g_3^2 + \frac{4}{15}g_1^2\right)C_1^{\text{EFT}}(\mu_{\text{IR}}) + \left(\frac{32}{3}g_3^2 + \frac{8}{15}g_1^2\right)C_2^{\text{EFT}}(\mu_{\text{IR}})\right]\mathcal{K}_2^{(0)}\,.
\end{aligned} \tag{3.32}$$

Here, the diagram (a) vanishes while the diagrams (b) and (c) are given by $C_1^{\rm EFT}(\mu_{\rm IR})$ and $C_2^{\rm EFT}(\mu_{\rm IR})$, respectively:

$$C_1^{\rm EFT}(\mu_{\rm IR}) = -C_2^{\rm EFT}(\mu_{\rm IR}) \equiv \frac{1}{16\pi^2}\frac{1}{2}\left(\frac{2}{\epsilon'} + 1 - \ln\frac{\mu_{\rm IR}^2}{\mu^2}\right) . \tag{3.33}$$

These functions correspond to the coefficients defined in Eq. (E.17) in the limit: p^2 vanishes. The effective Kähler potentials up to the one-loop level are described as

$$\begin{aligned} \mathcal{K}_1^{\rm eff} &= \mathcal{K}_1^{(1):\rm eff} + \mathcal{K}_1^{(0)} + \left(Z_{\mathcal{C}_1} Z_U^{\rm EFT\,1/2} Z_Q^{\rm EFT\,1/2} Z_D^{\rm EFT\,1/2} Z_L^{\rm EFT\,1/2} - 1\right)\mathcal{K}_1^{(0)} , \\ \mathcal{K}_2^{\rm eff} &= \mathcal{K}_2^{(1):\rm eff} + \mathcal{K}_2^{(0)} + \left(Z_{\mathcal{C}_2} Z_E^{\rm EFT\,1/2} Z_U^{\rm EFT\,1/2} Z_Q^{\rm EFT} - 1\right)\mathcal{K}_2^{(0)} . \end{aligned} \tag{3.34}$$

The logarithmic divergences are absorbed by the counter terms of $\mathcal{C}_A$, and then we have

$$\begin{aligned} Z_{\mathcal{C}_1} &= 1 - \frac{2}{16\pi^2\epsilon'}\left(\frac{11}{30}g_1^2 + \frac{3}{2}g_2^2 + \frac{4}{3}g_3^2\right) , \\ Z_{\mathcal{C}_2} &= 1 - \frac{2}{16\pi^2\epsilon'}\left(\frac{23}{30}g_1^2 + \frac{3}{2}g_2^2 + \frac{4}{3}g_3^2\right) . \end{aligned} \tag{3.35}$$

These are consistent with the results of Ref. [43]. In the next section, we determine the threshold corrections for the wave functions and the Wilson coefficients of the dimension-six baryon-number violating operators by matching the full and effective theories.

Matching Conditions for Dimension-Six Operators

In the previous subsections, we have shown the radiative corrections to two-, three-, and four-point vertex functions in the SUSY $SU(5)$ GUTs and we have shown also the radiative corrections to the Wilson coefficients of the dimension-six operators in the EFT. Now, we determine the threshold corrections by matching the amplitudes in the EFT and those in the full theory.

First, let us discuss the threshold corrections to the two-point functions for matter superfields. As we have seen in Eq. (3.16), the one-loop two-point functions are divided into two parts: one is linear to $f(M_X^2)$ and the other is linear to $f(\mu_{\rm IR}^2)$. The latter is the contribution from the MSSM gauge interactions, and the former is the contribution from the broken gauge interaction in $SU(5)$. On the other hand, the two-point functions in the EFT at the GUT scale have the form;

$$\Gamma_\Phi^{\text{2-pt;eff}} = (1-\lambda_\Phi)\left(1 + b_\Phi f\left(\mu_{\rm IR}^2\right)\right)\Gamma_{\Phi\,0}^{\text{2-pt}} . \tag{3.36}$$

Here, the chiral superfield in the EFT is given by $(1-\lambda_\Phi/2)\Phi$ (Φ is in the full theory). λ_Φ is determined so as to match the two-point function in the EFT and that in the full theory:

$$\lambda_\Phi(\mu) = \frac{g_5^2}{16\pi^2}\hat{\lambda}_\Phi f\left(M_X^2\right) , \tag{3.37}$$

where $(\hat{\lambda}_Q, \hat{\lambda}_U, \hat{\lambda}_D, \hat{\lambda}_L, \hat{\lambda}_E) = (3, 4, 2, 3, 6)$ is defined.

Next, we determine the threshold corrections for the baryon-number violating dimension-six operators. The two-point functions of the matter superfields in the full theory and the EFT are matched above, and we have determined the threshold corrections to the renormalizable kinetic terms. For a matter superfield Φ, the renormalizable kinetic term has the form $(1-\lambda_\Phi)\Phi^\dagger\Phi$ in the EFT. The finite corrections to the two-point functions in the EFT appear in the correction to the Wilson coefficients of higher-dimensional operators. The Wilson coefficients of higher-dimensional operators themselves also include the finite corrections. Thus, we redefine the effective Kähler potentials $\mathcal{K}_I^{\rm eff}$ $(I = 1, 2)$ as the ones with threshold corrections up to the one-loop level as follows:

$$\begin{aligned}
\mathcal{K}_1^{\rm eff} &= \mathcal{K}_1^{(1):\rm eff} + \left(1 - \lambda_1 - \frac{1}{2}(\lambda_U + \lambda_Q + \lambda_D + \lambda_L)\right)\mathcal{K}_1^{(0)} \\
&\quad + \left(Z_{\mathcal{C}_1} Z_U^{{\rm EFT}\,1/2} Z_Q^{{\rm EFT}\,1/2} Z_D^{{\rm EFT}\,1/2} Z_L^{{\rm EFT}\,1/2} - 1\right)\mathcal{K}_1^{(0)}, \\
\mathcal{K}_2^{\rm eff} &= \mathcal{K}_2^{(1):\rm eff} + \left(1 - \lambda_2 - \frac{1}{2}(\lambda_E + \lambda_U + 2\lambda_Q)\right)\mathcal{K}_2^{(0)} \\
&\quad + \left(Z_{\mathcal{C}_2} Z_E^{{\rm EFT}\,1/2} Z_U^{{\rm EFT}\,1/2} Z_Q^{\rm EFT} - 1\right)\mathcal{K}_2^{(0)}.
\end{aligned} \tag{3.38}$$

λ_1 and λ_2 are the threshold corrections to the Wilson coefficients for the baryon-number violating operators.

In the full theory (the SUSY $SU(5)$ GUTs), we have computed the effective Kähler potential for the dimension-six operators at the one-loop level,

$$\begin{aligned}
\mathcal{K}_1^{\rm full} &= -\frac{1}{2}\frac{1}{M_X^2 + \Sigma(0)}\mathcal{C}_{V_2}\mathcal{C}_{V_1}\mathcal{O}^{(1)} + \mathcal{K}_1^{(\rm Box)}, \\
\mathcal{K}_2^{\rm full} &= -\frac{1}{2}\frac{1}{M_X^2 + \Sigma(0)}\mathcal{C}_{V_2}\mathcal{C}_{V_3}\mathcal{O}^{(2)} + \mathcal{K}_2^{(\rm Box)}.
\end{aligned} \tag{3.39}$$

The first terms include the vacuum polarization of the massive vector superfield $\Sigma(0)$ and the one-loop effective couplings $\mathcal{C}_{V_1}, \mathcal{C}_{V_2}$, and $\mathcal{C}_{V_3}$ which are defined in Eq. (3.28). The second terms are the box corrections defined in Eq. (3.31).

There are IR divergences in $\mathcal{K}_I^{\rm full}$ and $\mathcal{K}_I^{\rm eff}$ $(I = 1, 2)$, which are represented by μ_{IR}. The divergences are absorbed by the operators $\mathcal{O}^{(I)}$. * Then, we divide the effective Kähler potentials into the coefficients $\mathcal{C}_I$ and the renormalized operator $\mathcal{O}_r^{(I)}$:

$$\mathcal{K}_I^{\rm full} = \mathcal{C}_I^{\rm full}\mathcal{O}_r^{(I)}, \quad \mathcal{K}_I^{\rm eff} = \mathcal{C}_I^{\rm eff}\mathcal{O}_r^{(I)}. \tag{3.40}$$

*Since the IR divergent terms from the box-like diagrams are proportional to $\ln M_X^2/\mu_{\rm IR}^2$, we divide this into $\ln M_X^2/\mu^2 + \ln \mu^2/\mu_{\rm IR}^2$ where μ denotes the renormalization scale, and then the IR divergent terms are absorbed by the operators.

The one-loop coefficients in the full theory are given by

$$\begin{aligned}\frac{\mathcal{C}_1^{(1):\text{full}}}{\mathcal{C}_1^{(0)}} &= \frac{M_X^2}{M_X^2+\Sigma(0)} - \frac{g_5^2}{16\pi^2}\left[6+6\left(1-\ln\frac{M_X^2}{\mu^2}\right)-\frac{16}{5}\ln\frac{M_X^2}{\mu^2}\right],\\ \frac{\mathcal{C}_2^{(1):\text{full}}}{\mathcal{C}_2^{(0)}} &= \frac{M_X^2}{M_X^2+\Sigma(0)} - \frac{g_5^2}{16\pi^2}\left[\frac{32}{5}+8\left(1-\ln\frac{M_X^2}{\mu^2}\right)-\frac{18}{5}\ln\frac{M_X^2}{\mu^2}\right],\end{aligned} \tag{3.41}$$

where $\mathcal{C}_1^{(0)}$ and $\mathcal{C}_2^{(0)}$ are the tree-level ones: $\mathcal{C}_1^{(0)} = \mathcal{C}_2^{(0)} = -g_5^2/M_X^2$. The first-two terms in square brackets arise from the vertex corrections, while the third term is the box correction. The vertex corrections are divided into two parts: the first term is the massless vector contribution and the second term is the massive vector contribution.

In the EFT, we find the one-loop corrected coefficients

$$\begin{aligned}\frac{\mathcal{C}_1^{(1):\text{eff}}}{\mathcal{C}_1^{(0)}} &= 1-\lambda_1-\frac{6g_5^2}{16\pi^2}\left(1-\ln\frac{M_X^2}{\mu^2}\right)-\frac{14}{5}\frac{g_5^2}{16\pi^2},\\ \frac{\mathcal{C}_2^{(1):\text{eff}}}{\mathcal{C}_2^{(0)}} &= 1-\lambda_2-\frac{8g_5^2}{16\pi^2}\left(1-\ln\frac{M_X^2}{\mu^2}\right)-\frac{14}{5}\frac{g_5^2}{16\pi^2}.\end{aligned} \tag{3.42}$$

Here, M_X-dependent terms come from the threshold corrections to the wave function for matter superfields.

We assume that the matching scale is $\mu = M_{\text{GUT}}(\simeq M_X)$, where the unification $g_1 = g_2 = g_3 = g_5$ is achieved. By comparing the amplitudes obtained in the full and effective theories, we determine the threshold corrections to the Wilson coefficients of dimension-six operators λ_1 and λ_2 at the one-loop level:

$$\begin{aligned}\lambda_1 &= \frac{\Sigma(0)}{M_X^2+\Sigma(0)}+\frac{g_5^2}{16\pi^2}\frac{16}{5}\left(1-\ln\frac{M_X^2}{\mu^2}\right),\\ \lambda_2 &= \frac{\Sigma(0)}{M_X^2+\Sigma(0)}+\frac{g_5^2}{16\pi^2}\frac{18}{5}\left(1-\ln\frac{M_X^2}{\mu^2}\right).\end{aligned} \tag{3.43}$$

We find that the corrections to the wave function for the matter superfield and the vertices of the massive vector superfield are canceled with each other as expected from the Ward identity and that the threshold corrections come from the corrections to the vacuum polarization and the box-like contributions.

3.3 Numerical Results

In this section, we estimate the size of threshold corrections at the SUSY scale and the GUT scale.

SUSY Scale Threshold Corrections

To begin with, we evaluate the threshold effects at the SUSY scale in the split SUSY scenario. As mentioned in Section 1.3, the heavy sfermion scenarios make the constraint on a dimension-five proton decay mild. We assume that all sparticles except gauginos are degenerate in mass M_S. Gaugino masses are set to be as follows: bino and wino are degenerate in mass $M_1 = M_2 = 3$ TeV, and we treat the ratio of the gluino and bino masses as a free parameter. In the numerical estimation, we set $M_3/M_1 = 1, 3$, and 9. We also choose $\tan\beta = 3$ since a small $\tan\beta$ is preferred to get the observed Higgs mass in the heavy sfermion scenario [30, 86, 129]. The matching scale for the proton decay amplitudes is set to be the sfermion mass scale M_S.

We define the ratio of decay rates with and without SUSY threshold corrections in order to estimate their effects. The ratio is approximately written as

$$R \equiv \frac{A_S^{(1)2} + (1+|U_{ud}|^2)^2 A_S^{(2)2}\Big|_{\mathrm{w}}}{A_S^{(1)2} + (1+|U_{ud}|^2)^2 A_S^{(2)2}\Big|_{\mathrm{w/o}}}\,, \tag{3.44}$$

with the short-distance renormalization factors

$$A_S^{(I)} \equiv \frac{\mathcal{C}_I(m_Z)}{\mathcal{C}_I(M_{\mathrm{GUT}})}\,, \tag{3.45}$$

where $\mathcal{C}_I(\mu)$ $(I = 1, 2)$ are the Wilson coefficients of the dimension-six operators at renormalization scale μ. The denominator of Eq. (3.44) includes only the threshold corrections at the GUT scale, on the other hand the numerator of Eq. (3.44) includes threshold corrections at both GUT and SUSY scales.

We see these threshold effects at the SUSY scale in Fig. 3.7, where the minimal SU(5) model with all mass parameters and the GUT scale fixed at 2×10^{16} GeV is considered. Let us add a few comments.

First, if all sparticles are degenerate in mass, there is no contribution from the threshold corrections. This is because the loop functions f and F behave as follows ($\mu = M_S$)

$$f(M_S^2, M_S^2) \sim \frac{1}{2}\ln\frac{M_S^2}{\mu^2} \to 0\,, \quad F(M_S^2, M_S^2, M_S^2) \sim -\frac{1}{2}\ln\frac{M_S^2}{\mu^2} \to 0\,, \tag{3.46}$$

Second, the ratio R approaches a constant in the limit of decoupled sfermions since these loop functions behave as

$$f(M_a^2, M_S^2) \sim -\frac{1}{4}\left(1 - 2\ln\frac{M_S^2}{\mu^2}\right), \quad F(M_S^2, M_S^2, M_a^2) \sim \frac{1}{4}\left(1 - 2\ln\frac{M_S^2}{\mu^2}\right), \tag{3.47}$$

in the limit of $M_S \gg M_a$. In Fig. 3.7, the ratio R is enhanced when $M_3/M_1 = 9$ and $M_S \sim O(10)$ TeV. In this case, the sfermion mass is smaller than the gluino mass is 27 TeV. In this

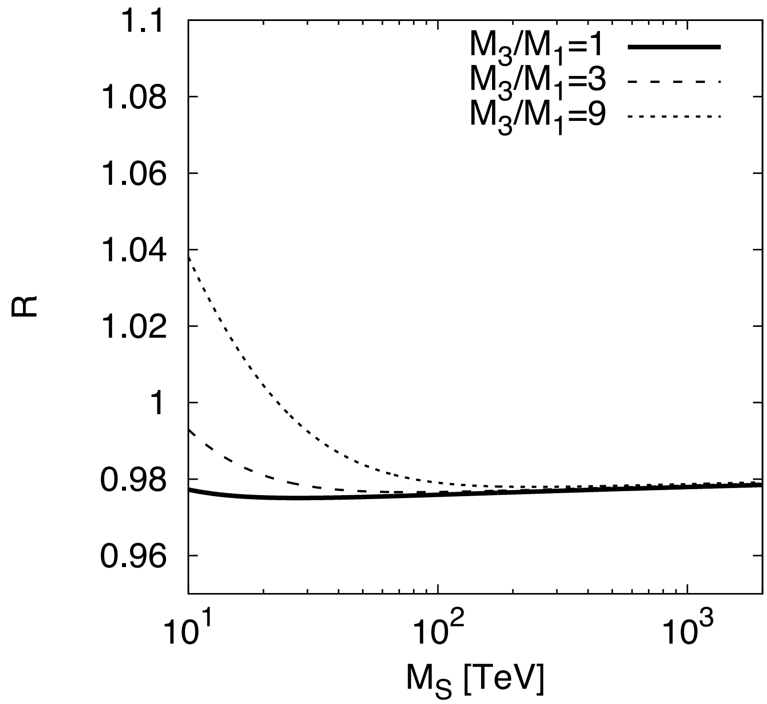


Figure 3.7: Ratio of proton decay rates with and without threshold correction at SUSY scale. At GUT scale, we assume the minimal SUSY $SU(5)$ GUT model, and each decay rates includes threshold correction at GUT scale. Bino mass M_1 is set to be 3 TeV. Solid, broken, and dotted lines respectively indicate $M_3/M_1 = 1,\ 3$, and 9.

case we should take the matching scale somewhere between the sfermion and gaugino mass in order to minimize the 2-loop corrections.

As a result, we conclude that there is only a few % correction to proton lifetime in the split SUSY scenario.

Minimal $SU(5)$ with Extra Matters

Now, we give numerical results of the short-distance renormalization factor including threshold corrections in the minimal SUSY $SU(5)$ GUT and its vector-like extension. In the minimal SUSY $SU(5)$ GUT, the X supermultiplets, the color-triplet Higgs supermultiplets, and the adjoint Higgs multiplets acquire heavy mass through the VEV of the adjoint Higgs multiplet. First, we set the masses of the GUT particles to be degenerate in mass 2.0×10^{16} GeV since they are model-dependent parameters. * The dependence of the threshold correction

*The GUT mass spectrum may be constrained by the gauge coupling unification. In this subsection, we do not use this constraint. However, we consider the constrained mass spectrum in the next subsection.

on the GUT scale mass spectrum is shown later. The threshold corrections in the minimal SUSY $SU(5)$ GUT are divided into two parts: the one comes from the vacuum polarization of the massive vector superfield as

$$\lambda_1|_{\text{vac.}} = \lambda_2|_{\text{vac.}} = \frac{\Sigma(0)}{M_X^2 + \Sigma(0)} = -3.68 \times 10^{-2}\,, \tag{3.48}$$

another one comes from the box-type diagram:

$$\begin{aligned} \lambda_1|_{\text{vert.}} &= \frac{g_5^2}{16\pi^2}\frac{16}{5}\left(1 - \ln\frac{M_X^2}{\mu^2}\right) = 1.03 \times 10^{-2}\,, \\ \lambda_2|_{\text{vert.}} &= \frac{g_5^2}{16\pi^2}\frac{18}{5}\left(1 - \ln\frac{M_X^2}{\mu^2}\right) = 1.15 \times 10^{-2}\,. \end{aligned} \tag{3.49}$$

Then, by combining these contribution we obtain the numerical values of threshold corrections as

$$\lambda_1(M_{\text{GUT}}) = -2.66 \times 10^{-2}\,, \quad \lambda_2(M_{\text{GUT}}) = -2.53 \times 10^{-2}\,, \tag{3.50}$$

where we assume that all sparticle masses are set to be $M_{\text{SUSY}} = 1$ TeV. We set the renormalization scale at which we match the amplitudes in the full theory and the EFT to $M_{\text{GUT}} = 2.0 \times 10^{16}$ GeV.

As in the previous subsection, the short-distance renormalization factors of Wilson coefficients $A_S^{(I):\text{SUSY}}$ which include two-loop RGEs in the SUSY SM and threshold corrections are defined by

$$A_S^{(I):\text{SUSY}} \equiv \frac{\mathcal{C}_I(M_{\text{SUSY}})}{\mathcal{C}_I(M_{\text{GUT}})}\,, \tag{3.51}$$

These numerical factors are obtained by the RGEs at the two-loop level *

$$A_S^{(1):\text{SUSY}} = 2.025\,, \quad A_S^{(2):\text{SUSY}} = 2.118\,. \tag{3.52}$$

We have also evaluated the short-distance renormalization factor to the partial decay rate ($p \to e^+ + \pi^0$). We define the ratio of the short-distance renormalization factor with and without the threshold correction to the Wilson coefficients of the dimension-six operators as Eq. (3.44). The denominator and numerator correspond to the short-distance enhancement factor of the nucleon decay rate without and with threshold corrections, respectively. We obtain $R = 1.052$ in the minimal SUSY $SU(5)$ GUT, that is, there is about 5% enhancement compared with the short-distance renormalization factor without threshold corrections.

*The RGEs for the gauge and Yukawa coupling constants and the Wilson coefficients for the baryon-number violating dimension-six operators are summarized in Appendix G.

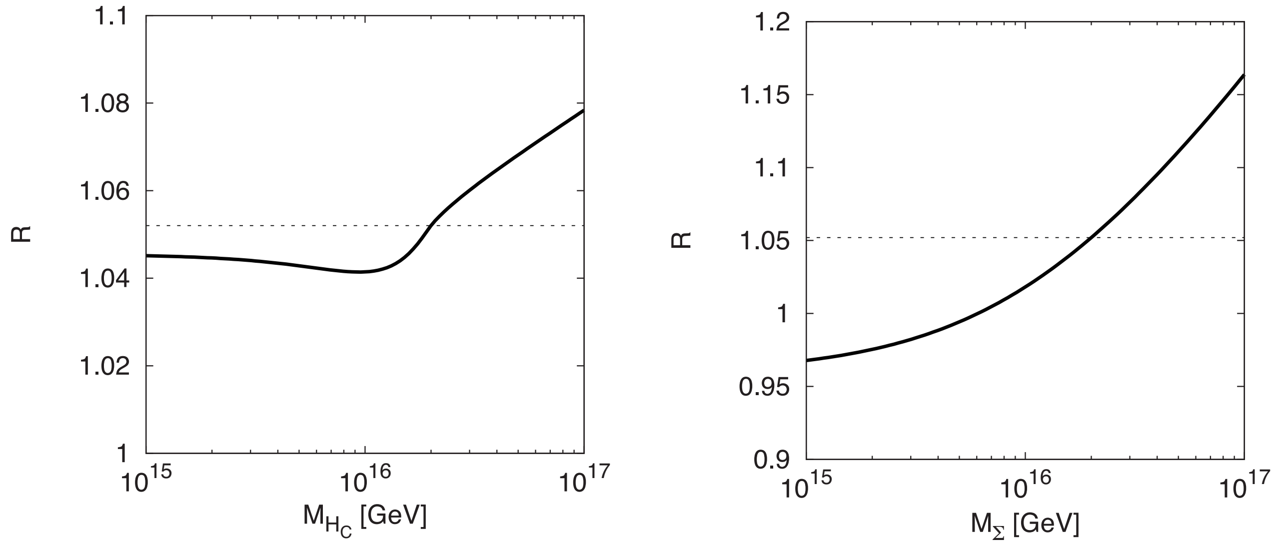


Figure 3.8: GUT-scale particle mass dependence on renormalization factor of proton decay. Left panel shows M_{H_C} dependence for $M_\Sigma = M_X = 2 \times 10^{16}$ GeV. Right panel shows M_Σ dependence for $M_{H_C} = M_X = 2 \times 10^{16}$ GeV. Dotted line shows the degenerate mass case in each panels.

We note that the mass relation between M_Σ and $M_{\Sigma_{24}}$ is $M_\Sigma = 5M_{\Sigma_{24}}$ in the minimal SUSY $SU(5)$ GUT. When we adopt this mass relation and we assume the masses of the GUT particles are set to be $M_X = M_\Sigma = 2.0 \times 10^{16}$ GeV, we have

$$A_S^{(1):\text{SUSY}} = 2.014\,, \qquad A_S^{(2):\text{SUSY}} = 2.107\,, \tag{3.53}$$

and then, we obtain $R = 1.041$.

In Fig. 3.8, we describe the heavy mass dependence on the ratio of the short-distance renormalization factor in the minimal SUSY $SU(5)$ GUT. Here, we set the mass of the component fields of the adjoint Higgs multiplet to be degenerate in M_Σ, that is, we set $M_{\Sigma_{24}} = M_\Sigma$, for simplicity. The left panel of Fig. 3.8 shows the color-triplet Higgs mass (M_{H_C}) dependence of the ratio with the fixed adjoint Higgs mass $M_\Sigma = 2.0 \times 10^{16}$ GeV. The right panel of Fig. 3.8 shows the adjoint Higgs mass (M_Σ) dependence of the ratio with the fixed color-triplet Higgs mass $M_{H_C} = 2.0 \times 10^{16}$ GeV. Since, in a large M_{H_C} region, the vacuum polarization behaves as $\Sigma \sim \frac{1}{2} M_X^2 (\frac{1}{2} - \ln M_{H_C}^2 / \mu^2)$, the decay rate of proton is slightly enhanced in this region.

In the SUSY $SU(5)$ GUT with light vector-like matter scenario, the threshold corrections to the Wilson coefficients of the dimension-six operators are enhanced since the unified gauge coupling becomes large. This large unified coupling leads to the large renormalization effect to the Wilson coefficients of the dimension-six operators.

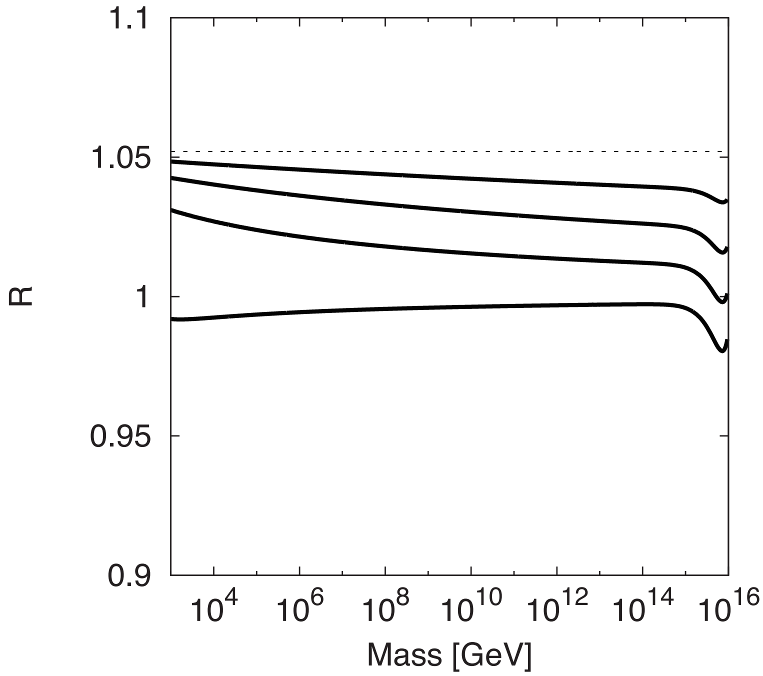


Figure 3.9: Ratio of short-distance renormalization effects with and without threshold effect in the minimal SUSY $SU(5)$ GUT with light vector-like matters. We take $n_{\mathbf{5}} = 1, \cdots, 4$ in solid lines from top to bottom. The case of the minimal SUSY $SU(5)$ with no light vector-like matter is shown in dotted line.

Fig. 3.9 shows the ratio of the short-distance renormalization factors in the extra vector-like matter scenario. The horizontal line and the vertical line indicate the mass scale of the vector-like matters and the ratio of the short-distance renormalization effect, respectively. The solid lines correspond to the case that the number of $\mathbf{5} + \overline{\mathbf{5}}$ vector-like matters is set to be $n_{\mathbf{5}} = 1, \cdots, 4$ from top to bottom without $\mathbf{10} + \overline{\mathbf{10}}$ vector-like matter. In this estimation, we assume the masses of the heavy multiplets and the GUT scale are set to be 2.0×10^{16} GeV. If the mass (number) of the vector-like superfields is sufficiently light (large), the unified gauge coupling at the GUT scale becomes larger. However, the additional contribution from the vector-like matters cancels with the gauge contributions. In fact, the vacuum polarization $\Sigma_X(0)$ from the vector-like matters is proportional to

$$\Sigma_X(0)|_{\text{vector-like}} \propto \frac{g_5^2(N_{\mathbf{5}} + N_{\overline{\mathbf{5}}})}{16\pi^2} M_X^2 \left(1 - \ln \frac{M_X}{\mu}\right) . \tag{3.54}$$

Here, we neglect the vector-like mass dependence since we are interested in the case of the sufficiently small vector-like masses. Therefore, the additional positive contribution from the vector-like matters cancels with the negative contribution $\lambda_i|_{\text{vec.}}$ $(i = 1, 2)$ in Eq. (3.48) when we set $M_X = \mu = 2.0 \times 10^{16}$ GeV.

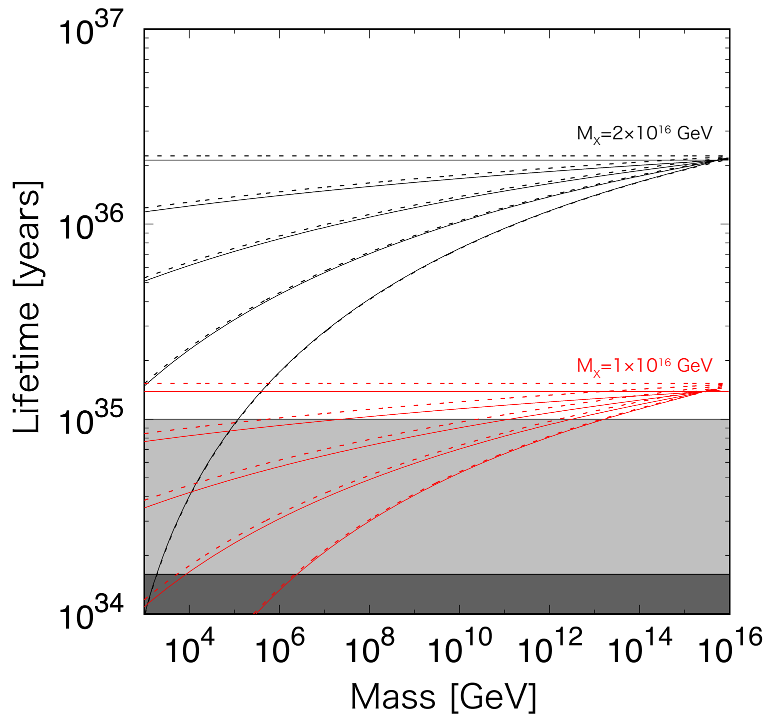


Figure 3.10: Partial proton lifetime ($p \to \pi^0 + e^+$) in vector-like extension scenario. In solid (dotted) lines, we take $n_5 = 0, 1, \cdots, 4$ with (without) threshold corrections at GUT scale. Deep gray (gray) region corresponds to experimental excluded region by Super-Kamiokande (future sensitivity by Hyper-Kamiokande).

In Fig. 3.10, we show the partial proton lifetime ($p \to \pi^0 + e^+$) in the minimal SUSY $SU(5)$ and its vector-like extension. In this evaluation, we assume the masses of the GUT spectrum except the massive vector superfield are set to be the same mass (2.0×10^{16} GeV). The mass of massive vector superfield is set to be $M_X = 2.0 \times 10^{16}$ GeV in black lines and $M_X = 1.0 \times 10^{16}$ GeV in red lines. The deep gray region is excluded by the present lower bound on this decay mode at the Super-Kamiokande ($\tau(p \to \pi^0 + e^+) > 1.6 \times 10^{34}$ years). The gray region corresponds to the future sensitivity on this decay mode by the Hyper-Kamiokande ($\tau(p \to \pi^0 + e^+) > 1.0 \times 10^{35}$ years). The unified coupling becomes large at the GUT scale due to the extra matters, and hence the proton lifetime is suppressed in the extra matter extensions [99].

Minimal vs. Missing Partner $SU(5)$

Let us consider now the effect of the additional fields at the GUT scale. The precise calculation for proton lifetime depends on the unified coupling g_5 and the mass spectrum at the

GUT scale. In particular, as we have shown in Section 2.3, the GUT mass spectrum and the value of the unified gauge coupling are connected through the threshold corrections to the gauge couplings at the GUT scale [127, 128]. Thus by requiring gauge coupling unification we get a constraint on the mass spectrum of the GUT particles through threshold corrections. At the GUT scale, the one-loop matching conditions for gauge couplings are given as in Eq. (2.35). The threshold corrections ξ_i for the gauge couplings depend on the details of the GUT particles spectrum. For the minimal renormalizable SUSY $SU(5)$ GUT, the mass of color-triplet Higgs multiplets M_{H_C} and the combination of the mass parameters $M_X^2 M_\Sigma$ are determined by [127, 128]

$$\frac{3}{g_2^2(\mu)} - \frac{2}{g_3^2(\mu)} - \frac{1}{g_1^2(\mu)} = \frac{1}{8\pi^2}\frac{12}{5}\ln\frac{M_{H_C}}{\mu}, \tag{3.55}$$

$$\frac{5}{g_1^2(\mu)} - \frac{3}{g_2^2(\mu)} - \frac{2}{g_3^2(\mu)} = \frac{1}{8\pi^2}12\ln\frac{M_X^2 M_\Sigma}{\mu^3}. \tag{3.56}$$

The right-hand sides of Eqs. (3.55) and (3.56) are thus determined by the low-energy gauge couplings and the SUSY mass spectrum. Notice that M_X, which enters in the expression for the $D = 6$ proton lifetime, cannot be determined since Eq. (3.56) constrains only the combination $M_X^2 M_\Sigma$. We treat the mass of the massive vector superfield as a free parameter in the following numerical calculation, and we estimate the gauge couplings at the matching scale (*i.e.* GUT scale), which we take $\mu = 2 \times 10^{16}$ GeV, by using the two-loop RGEs for gauge couplings. After we fix M_X, we determine the mass of the color triplet M_{H_C} from Eq. (3.55) and that of the adjoint Higgs M_Σ from Eq. (3.56). One of Eqs. (2.35) (for example g_1) is then used to get g_5 at the GUT scale.

In the missing-partner $SU(5)$ model, the combinations of the GUT masses are constrained as [110];

$$\frac{3}{g_2^2(\mu)} - \frac{2}{g_3^2(\mu)} - \frac{1}{g_1^2(\mu)} = \frac{1}{8\pi^2}\left(\frac{12}{5}\ln\frac{M_{H_C}M_{\overline{H}_C}}{M_{H'_f}\mu} + 6\ln\frac{2^6}{5^5}\right), \tag{3.57}$$

$$\frac{5}{g_1^2(\mu)} - \frac{3}{g_2^2(\mu)} - \frac{2}{g_3^2(\mu)} = \frac{1}{8\pi^2}\left(12\ln\frac{M_X^2 M_\Xi}{\mu^3} + 54\ln\frac{5}{4}\right). \tag{3.58}$$

Here, M_{H_C} and $M_{\overline{H}_C}$ correspond to the mass of the color-triplet Higgs multiplets, while $M_{H'_f}$ denotes the mass of the extra Higgs doublet induced by the breaking of the PQ symmetry. M_Ξ is defined as the mass of the component fields $(\mathbf{8}, \mathbf{3})_0$ in Table 2.2. The constant terms arise from the mass splitting of the component fields of the **75**-dimensional Higgs multiplet as we have shown in Table 2.2.

In the following analysis for the missing-partner model, we determine the combinations $M_{H_C}M_{\overline{H}_C}/M_{H'_f}$ and $M_X^2 M_\Xi$ from Eqs. (3.57)-(3.58). As in the case of the minimal $SU(5)$, for a given sparticle mass spectrum, M_X cannot be determined: Eq. (3.58) gives only a relation

Table 3.2: Threshold effects on the partial proton decay rate. For simplicity, we assume that all superparticles are degenerate in mass $M_S = 1$ TeV.

	Minimal $SU(5)$		Missing-Partner	
M_X	1.0×10^{16} GeV	2.0×10^{16} GeV	1.0×10^{16} GeV	2.0×10^{16} GeV
$A_S^{(1)}$	2.070	1.968	1.301	1.269
$A_S^{(2)}$	2.162	2.059	1.352	1.295
R	1.10	0.994	0.429	0.394
g_5	0.697	0.713	0.938	1.198
$\tau(p \to e^+\pi^0)$ [years]	1.38×10^{35}	2.23×10^{36}	1.08×10^{35}	7.09×10^{35}

between M_X and M_Ξ. We also define the unified coupling g_5 as in the minimal $SU(5)$. For simplicity, we take the typical mass scale for the color-triplets, $M_{H_C} = M_{\overline{H}_C} = 10^{15}$ GeV, as shown in Chapter 2, so that $M_{H'_f}$ is given by Eq. (3.57) at the matching scale $\mu = M_{\rm GUT} = 2 \times 10^{16}$ GeV.

We list numerical results in the minimal SUSY $SU(5)$ and the missing-partner model in Table 3.2. Here, we assume that all SUSY particles are degenerate in mass, $M_S = 1$ TeV, for simplicity. Since proton lifetime strongly depends on the mass of the X boson, we display the results for two choices of $M_X = 1.0 \times 10^{16}$ GeV and $M_X = 2.0 \times 10^{16}$ GeV. By using the central values for gauge couplings at the EW scale, the mass parameters determined by Eqs. (3.55)-(3.58) are obtained as follows:

$$M_{H_C} = 6.35 \times 10^{15}\ \text{GeV}\,, \qquad (M_X^2 M_{\Sigma_{24}})^{1/3} = 1.48 \times 10^{16}\ \text{GeV}\,, \tag{3.59}$$

for the minimal SUSY $SU(5)$,

$$\frac{M_{H_C} M_{\overline{H}_C}}{M_{H'_f}} = 1.06 \times 10^{20}\ \text{GeV}\,, \qquad (M_X^2 M_\Xi)^{1/3} = 5.43 \times 10^{15}\ \text{GeV}\,, \tag{3.60}$$

for the missing-partner model. In the missing partner model, we get $M_{H'_f} \sim 10^{10}$ GeV using the above constraint and the assumption $M_{H_C(\overline{H}_C)} = 10^{15}$ GeV. This result is consistent with the intermediate PQ breaking [111].

The quantities $A_S^{(i)}$ $(i = 1, 2)$ in Table 3.2 show the short-range renormalization factors with the threshold corrections defined by Eq. (3.45). For each M_X, we get the threshold corrections in the minimal $SU(5)$ model as

$$\begin{aligned} &\lambda^{(1)} = -4.94 \times 10^{-2}\,, \quad \lambda^{(2)} = -4.65 \times 10^{-2}\,, \qquad (\text{for } M_X = 1.0 \times 10^{16}\ \text{GeV})\,, \\ &\lambda^{(1)} = 1.98 \times 10^{-3}\,, \quad \lambda^{(2)} = 3.26 \times 10^{-3}\,, \qquad (\text{for } M_X = 2.0 \times 10^{16}\ \text{GeV})\,, \end{aligned} \tag{3.61}$$

and in the missing-partner model as

$$\begin{aligned} \lambda^{(1)} &= 0.340\,, \quad \lambda^{(2)} = 0.346\,, \qquad (\text{for } M_X = 1.0 \times 10^{16}\,\text{GeV})\,, \\ \lambda^{(1)} &= 0.369\,, \quad \lambda^{(2)} = 0.373\,, \qquad (\text{for } M_X = 2.0 \times 10^{16}\,\text{GeV})\,. \end{aligned} \tag{3.62}$$

When we estimate the partial proton lifetime in each model, we use the proton decay matrix elements calculated by the lattice simulation at 2 GeV [41]. Note that the unified coupling g_5 in the missing-partner model is larger than the one in the minimal $SU(5)$. This is due to the following two reasons:

1. The combination $M_X^2 M_\Xi$ in the missing-partner model is slightly smaller than in the minimal $SU(5)$ due to the constant term present in the right-hand side of Eq. (3.58) but not of Eq. (3.57).
2. There are many components of **75** contributing to the threshold correction for the gauge couplings.

In Table 3.2 we see that threshold effects are negligible in the minimal SU(5) model ($R \sim 1$), but suppress the proton decay rate by an approximate factor 0.4 in the missing-partner model. A much bigger effect comes from the variation of the mass of massive vector superfield M_X, and is unfortunately not under control. This change of the lifetime τ with M_X in Table 3.2 can be understood by the approximate tree level relation $\tau \propto (M_X/g_5)^4$.

The threshold corrections from vertex and box contributions depend only on M_X and g_5 in the context of the SUSY $SU(5)$ GUTs, and these values are of order 10^{-2} as we have shown in the previous subsection. Let us consider the model dependence of the threshold effect, which appears in the vacuum polarization of the massive vector superfield. When the heavy spectrum is degenerate, the dominant (first term) contribution to the vacuum polarization in Eq. (3.21) is proportional to the one-loop beta function for the unified gauge coupling. In the minimal SUSY $SU(5)$, the contribution from the gauge supermultiplets dominates the vacuum polarization, that is the theory is asymptotically-free. Since there exist many massive fields in the missing-partner model, the contribution from the chiral supermultiplets is enhanced, and thus the vacuum polarization is dominated by the chiral supermultiplets in the model. Therefore, the resulting threshold effects vary among GUT models due to the relative sign between the contributions from the gauge and the chiral supermultiplets. Notice that we, of course, include other contributions, such as vertex contributions, box contributions, and vacuum polarizations which arise from interactions including GUT breaking VEV, though they are subdominant in the missing partner model.

If we estimate the masses of the Higgs multiplets by using Eqs. (3.57)-(3.58) with M_X lying around 10^{16} GeV, all components can be lighter than the massive vector superfield in the missing-partner model. The **75**-dimensional Higgs multiplet has a number of component fields with mass different from the matching (GUT) scale, and thus there can be a large contribution of the threshold correction to the proton lifetime. As a result, the proton decay rate in the missing-partner model is suppressed about 60% with $M_X = 2.0 \times 10^{16}$ GeV as we show in Table 3.2.

Chapter 4

Conclusion and Discussions

In this thesis we have discussed next-leading order (NLO) corrections to the baryon-number violating operators in the context of the supersymmetric grand unified theories. In particular, at the sparticle mass scale and the GUT scale, we have derived the one-loop threshold corrections to the Wilson coefficients causing $p \to e^+\pi^0$ mode. The derivation has been focused only on the gauge interactions where two-loop RGEs for proton decay operators have already been studied.

We have considered the specific supersymmetric models which explain the observed Higgs mass. The split SUSY models have the hierarchical mass spectrum among gauginos and sfermions. Since the threshold corrections depend on the mass spectrum, it has been expected that the corrections at the SUSY scale can be significant in the NLO calculations. In the SUSY SM with extra vector-like matters, the unified coupling is larger, and then the corrections at the GUT scale has been also expected to become larger.

For the split SUSY models, in the limit of the degenerate sfermions, there is not so much correction to the baryon-number violating operators at the SUSY scale against our expectation. In fact, the correction to proton decay rate is a few percents with degenerate sfermions. However, it might be possible to enhance the correction if the hierarchical sfermion masses are realized. While we have focused on the split SUSY model in the numerical simulation, the analytic formula Eq. (3.11) is applicable to the generic SUSY spectrum. Furthermore, it is straightforward to extend our formula to the other operators which generate four-Fermi interactions, such as flavor-changing neutral currents.

We have also investigated the threshold effects on the proton lifetime in the minimal SUSY $SU(5)$ GUT with extra vector-like matters. We have found that the threshold corrections give tiny effects in spite of the large unified coupling. This is because there is a cancellation between vacuum polarizations of the massive vector superfield from extra vector-like superfields and gauge supermultiplets.

We have also studied the effects from the higher-dimensional representations at the GUT scale. In this thesis, we have focused on the specific GUT model called the missing-partner model. In this model, the Higgs sector effectively contains the **75**-dimensional Higgs multiplet and two pairs of $\mathbf{5}+\overline{\mathbf{5}}$. We have determined the mass spectrum at the GUT scale using the low-energy gauge coupling constants with fixed some mass parameters such as the mass of the massive vector superfield and the color-triplet chiral superfields. We have also determined the unified gauge coupling including threshold correction, so that the mass differences among the components of **75**-dimensional Higgs make the coupling large. As a result, due to the small masses and the large coupling, the contributions of all these multiplets sum up to a correction of about 60% to the proton lifetime.

In this study, we have not included the corrections from Yukawa coupling. The reasons are the following: no two-loop RGE analysis with Yukawa couplings for the baryon-number violating processes is available, and the effect is not expected to be sizable due to the smallness of Yukawa couplings except the top Yukawa coupling.

Moreover, we have not considered the effect of higher-dimensional operators with the Planck-scale suppression. In the missing-partner model, we assume $\mathbf{50}+\overline{\mathbf{50}}$ pairs have the mass around the Planck scale. There might be the Planck-suppressed operators in general, so that they would affect the gauge couplings and the mass spectrum about a few percent at the GUT scale. The corrections strongly depend on the couplings of the higher-dimensional operators. However, since the UV description of the supersymmetric grand unified theories is not available, we cannot include the correct effects from the Planck-scale physics so far.

We have focused attention only on the SUSY $SU(5)$ GUT models in this study. We expect the extension to the other unification group such as $SO(10)$ and E_6 is straightforward. In particular, we give the explicit formulae of the supergraph calculations in Appendix E. Hence, what to do is decomposing the GUT interactions and calculating the numerical factor for each supergraphs.

Last but not least, we notice remaining uncertainties in the precise determination of the proton lifetime. One of them is the hadron matrix elements. The matrix elements of $p \to \pi^0 e^+$ is evaluated with lattice QCD, and they have at present around 30% uncertainty [41], and below 20% uncertainty with improved statistics [130]. We hope that this uncertainty will be reduced in future studies.

Another is the next-to-next-to leading order (NNLO) calculations. There exists the additional loop suppression in the NNLO calculations such as three-loop RGEs and two-loop threshold corrections. The loop factor at the GUT scale, that is $g_5^2(M_{\rm GUT})/16\pi^2$, becomes 3.3×10^{-3} to 1.5×10^{-2} corresponding to the number of vector-like matters being $N_5 = 0$ to $N_5 = 4$. The loop factor in the missing partner model is 9.1×10^{-2} using the unified coupling $g_5 = 1.198$ as shown in Table 3.2. Thus, the NNLO calculations should be much smaller than the uncertainty of the matrix elements with lattice QCD.

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Finally, I owe my deepest gratitude to my parents for their endless support.

Appendices

Appendix A

Convention and Notations

Convention

We use the metric on the four-dimensional Minkowski spacetime which is given by

$$\eta^{\mu\nu} = \mathrm{diag}(+,-,-,-)\,. \tag{A.1}$$

Here, Greek indices ν, ν run $(0,1,2,3)$ with $\mu = 0$ being the time coordinate. We also use the definition of the totally anti-symmetric tensor as $\epsilon^{0123} = 1$.

Angular momentum operators J_i and boost operators K_i are given by

$$\begin{aligned} J_i &\equiv \epsilon_{ijk}\mathcal{M}^{jk}\,, \\ K_i &\equiv \mathcal{M}_{i0} = -\mathcal{M}_{0i}\,, \end{aligned} \tag{A.2}$$

where $\mathcal{M}_{\mu\nu}$ is the generator of the Lorentz group. Commutation relations of them are given by

$$\begin{aligned} [J_i, J_j] &= i\epsilon_{ijk}J_k\,, \\ [J_i, K_j] &= i\epsilon_{ijk}K_k\,, \\ [K_i, K_j] &= -i\epsilon_{ijk}J_k\,. \end{aligned} \tag{A.3}$$

Now we define the following spins

$$A_i \equiv \frac{1}{2}(J_i + iK_i)\,, \qquad B_i \equiv \frac{1}{2}(J_i - iK_i)\,. \tag{A.4}$$

These spins, of course, satisfy $\mathbf{A}^* = \mathbf{B}$ and the following commutation relations.

$$\begin{aligned} [A_i, A_j] &= i\epsilon_{ijk}A_k\,, \\ [B_i, B_j] &= i\epsilon_{ijk}B_k\,, \\ [A_i, B_j] &= 0\,. \end{aligned} \tag{A.5}$$

Thus, **A** and **B** spins satisfy the $SU(2)$ Lie algebra. The arbitrary finite-dimensional representation of the Lorentz group is characterized by the size of spins **A** and **B**. The dimension of the representation space is $(2A+1)(2B+1)$. The fields corresponding to the simplest representation are $SL(2,C)$-spinors $(A,B) = (1/2,0),(0,1/2)$.

$SL(2,C)$-spinors are denoted by

$$\chi_\alpha\,, \qquad \chi^\dagger_{\dot\alpha}\,. \tag{A.6}$$

We refer to $\chi_\alpha(1/2,0)$ as the left-handed spinor, whereas we do to $\chi^\dagger_{\dot\alpha}(0,1/2)$ as the right-handed spinor. Since **A** and **B** are complex conjugates of each other, we write

$$(\chi_\alpha)^\dagger = \chi^\dagger_{\dot\alpha}\,. \tag{A.7}$$

Hereafter, we use the short-hand notation for spinor contractions

$$\chi\eta = \chi^\alpha\eta_\alpha\,, \quad \chi^\dagger\eta^\dagger = \chi^\dagger_{\dot\alpha}\eta^{\dagger\dot\alpha}\,. \tag{A.8}$$

We use the totally anti-symmetric tensor $\epsilon^{\alpha\beta}$ to raise lower indices of $SL(2,C)$-spinors, and vice versa. Since $\epsilon^{\alpha\beta}$ is an invariant tensor of $SL(2,C)$, we define the components as follows without distinction between dotted and un-dotted indices.

$$\epsilon^{12} = \epsilon_{21} = 1\,, \quad \epsilon_{12} = \epsilon^{21} = -1\,, \quad \epsilon_{11} = \epsilon_{22} = 0\,. \tag{A.9}$$

The Grassmann derivative and the Grassmann integral measure are defined as:

$$\frac{\partial}{\partial\theta^\alpha}\theta^\beta = \delta^\beta_\alpha\,, \qquad \frac{\partial}{\partial\theta^\dagger_{\dot\alpha}}\theta^\dagger_{\dot\beta} = \delta^{\dot\alpha}_{\dot\beta}\,, \tag{A.10}$$

$$d^2\theta = -\frac{1}{4}d\theta^\alpha d\theta^\beta\epsilon_{\alpha\beta}\,, \qquad d^2\theta^\dagger = -\frac{1}{4}d\theta^\dagger_{\dot\alpha}d\theta^\dagger_{\dot\beta}\epsilon^{\dot\alpha\dot\beta}\,. \tag{A.11}$$

Then, the integral of the products of the Grassmannian coordinate $\theta\theta \equiv \theta^\alpha\theta_\alpha$, $\theta^{\dagger\dot\alpha}\theta^\dagger_{\dot\alpha}$ are obtained as the following form;

$$\int d^2\theta(\theta\theta) = 1\,, \qquad \int d^2\theta^\dagger(\theta^\dagger\theta^\dagger) = 1\,. \tag{A.12}$$

Spinor Relations

Gamma matrices in chiral representation are given by

$$\gamma^\mu \equiv \begin{pmatrix} 0 & \sigma^\mu_{\alpha\dot\beta} \\ \overline{\sigma}^{\mu\dot\alpha\beta} & 0 \end{pmatrix}\,, \quad \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -\delta^\beta_\alpha & 0 \\ 0 & \delta^{\dot\alpha}_{\dot\beta} \end{pmatrix}\,. \tag{A.13}$$

Chiral projection operators are defined as

$$\begin{cases} P_L \equiv \dfrac{1}{2}(1-\gamma_5) = \begin{pmatrix} \delta_\alpha^\beta & 0 \\ 0 & 0 \end{pmatrix} \\ P_R \equiv \dfrac{1}{2}(1+\gamma_5) = \begin{pmatrix} 0 & 0 \\ 0 & \delta^{\dot\alpha}_{\dot\beta} \end{pmatrix} \end{cases} . \tag{A.14}$$

Now, we divide Dirac spinor $\Psi(x)$ into two mass-degenerate, opposite charged Weyl spinors $\chi_\alpha(x), \eta_\alpha(x)$ as:

$$\Psi(x) = \begin{pmatrix} \chi_\alpha(x) \\ \eta^{\dagger\dot\alpha}(x) \end{pmatrix} . \tag{A.15}$$

The Hermitian conjugate of Weyl-spinor is defined as:

$$(\psi_\alpha)^\dagger = (\psi^\dagger)_{\dot\alpha} \equiv \psi^\dagger_{\dot\alpha} , \qquad (\psi^{\dagger\dot\alpha})^\dagger = \psi^\alpha . \tag{A.16}$$

These relation means that the Hermite conjugate of the left-handed Weyl spinor is the right-handed Weyl spinor, and vice versa. Then, we have

$$\overline{\Psi}(x) = \Psi^\dagger(x) A = (\eta^\beta,\, \chi^\dagger_{\dot\beta}) , \quad \Psi^C(x) = C\overline{\Psi}^T(x) = \begin{pmatrix} \eta_\alpha \\ \chi^{\dagger\dot\alpha} \end{pmatrix} . \tag{A.17}$$

Then, we also have

$$\Psi_L(x) = P_L\Psi(x) = \begin{pmatrix} \chi_\alpha(x) \\ 0 \end{pmatrix} , \quad \Psi_R(x) = P_R\Psi(x) = \begin{pmatrix} 0 \\ \eta^{\dagger\dot\alpha}(x) \end{pmatrix} . \tag{A.18}$$

We are able to divide all of the Dirac bilinear forms into the bilinear forms in terms of the Weyl spinors:

$$\begin{aligned} \overline{\Psi^i} P_L \Psi_j &= \eta^i \chi_j , \quad \overline{\Psi^i} P_R \Psi_j = \chi^{\dagger i} \eta^\dagger_j , \\ \overline{\Psi^i} P_L \Psi^C_j &= \eta^i \eta_j , \quad \overline{\Psi^i} P_R \Psi^C_j = \chi^{\dagger i} \chi^\dagger_j , \\ \overline{\Psi^{iC}} P_L \Psi_j &= \chi^i \chi_j , \quad \overline{\Psi^{iC}} P_R \Psi_j = \eta^{\dagger i} \eta^\dagger_j , \end{aligned} \tag{A.19}$$

$$\overline{\Psi^i} \gamma^\mu P_L \Psi_j = \chi^{\dagger i} \overline{\sigma}^\mu \chi_j , \qquad \overline{\Psi^i} \gamma^\mu P_R \Psi_j = \eta^i \sigma^\mu \eta^\dagger_j . \tag{A.20}$$

Thus, we have the transformation between the bilinear forms of the Dirac spinor and the Weyl spinors:

$$\begin{aligned} \overline{\Psi^i} \Psi_j &= \eta^i \chi_j + \chi^{\dagger i} \eta^\dagger_j , \qquad \overline{\Psi^i} \gamma_5 \Psi_j = -\eta^i \chi_j + \chi^{\dagger i} \eta^\dagger_j , \\ \overline{\Psi^i} \gamma^\mu \Psi_j &= \chi^{\dagger i} \overline{\sigma}^\mu \chi_j + \eta^i \sigma^\mu \eta^\dagger_j , \quad \overline{\Psi^i} \gamma^\mu \gamma_5 \Psi_j = -\chi^{\dagger i} \overline{\sigma}^\mu \chi_j + \eta^i \sigma^\mu \eta^\dagger_j . \end{aligned} \tag{A.21}$$

We obtain the following relations by using formulae for the Pauli matrices.

$$\begin{aligned}
(\chi^{\dagger i}\overline{\sigma}^{\mu}\chi_j)(\eta^k\sigma_\mu\eta_l^\dagger) &= \chi_{\dot{\alpha}}^{\dagger i}(\overline{\sigma}^{\mu})^{\dot{\alpha}\alpha}\chi_{j\alpha}\eta^{k\beta}(\sigma_\mu)_{\beta\dot{\beta}}(\eta_l^\dagger)^{\dot{\beta}} \\
&= 2(\chi_{\dot{\alpha}}^{\dagger i}\eta_l^{\dagger\dot{\alpha}})(\chi_{j\alpha}\eta^{k\alpha})\,, \\
(\chi^{\dagger i}\overline{\sigma}^{\mu}\chi_j)(\chi^{k\dagger}\overline{\sigma}_\mu\chi_l) &= 2(\epsilon^{\dot{\alpha}\dot{\beta}}\chi_{\dot{\alpha}}^{i\dagger}\chi_{\dot{\beta}}^{\dagger k})(\epsilon^{\alpha\beta}\chi_{j\alpha}\chi_{l\beta})\,, \\
(\eta^i\sigma^\mu\eta_j^\dagger)(\eta^k\sigma_\mu\eta_l^\dagger) &= 2(\epsilon_{\dot{\alpha}\dot{\beta}}(\eta_j^\dagger)^{\dot{\alpha}}(\eta_l^\dagger)^{\dot{\beta}})(\epsilon_{\alpha\beta}\eta^{i\alpha}\eta_{k\beta})\,.
\end{aligned} \tag{A.22}$$

In terms of Dirac spinor relation, we have

$$\begin{aligned}
(\overline{\Psi}^i\gamma^\mu P_L\Psi_j)(\overline{\Psi}^k\gamma_\mu P_L\Psi_l) &= 2(\overline{\Psi}^k P_R\Psi^{iC})(\overline{\Psi^C}_j P_L\Psi_l)\,, \\
&= 2(\overline{\Psi}^i P_R\Psi^{kC})(\overline{\Psi^C}_l P_L\Psi_j) \\
(\overline{\Psi}^i\gamma^\mu P_R\Psi_j)(\overline{\Psi}^k\gamma_\mu P_L\Psi_l) &= 2(\overline{\Psi}^i P_R\Psi_l)(\overline{\Psi}^k P_L\Psi_j)\,.
\end{aligned} \tag{A.23}$$

Group Theoretical Constants

In this section, we summarize group theoretical constants. We consider both a simple group and a product group. A generator acting on a field Φ is denoted by T^A with A running over the number of generators. The Dynkin index $I(\Phi)$ and the quadratic Casimir invariant $C(\Phi)$ are defined by

$$\begin{aligned}
\sum_A T_a^A T_a^A &= C_a(\Phi)\mathbf{1}\,, \\
\mathrm{tr}(T_a^A T_a^B) &= I_a(\Phi)\delta^{AB}\,.
\end{aligned} \tag{A.24}$$

Here, subscript a denotes the label of the group G: $a = G$ if G is simple and a runs over all simple groups G_i if G is not simple and $G = G_1 \times G_2 \times \cdots \times G_n$. Through this thesis, by convention, the Dynkin index of fundamental representations is normalized as $I = 1/2$ for an $SU(N)$ group. In the text, we often use the following notation for the quadratic Casimir invariant of the adjoint representation,

$$f_{AMN}f_{BMN} = C(a)\delta_{AB}\,. \tag{A.25}$$

For the SM gauge group and the unified group, we also use the following shorthand notations in the text. The subscripts $i = 1, 2$, and 3 of $I_i(\Phi)$ and $C_i(\Phi)$ respectively represent the GUT-normalized $U(1)_Y$, $SU(2)_L$, and $SU(3)_C$. $I_5(\Phi)$ and $C_5(\Phi)$ respectively represent the Dynkin index and the quadratic Casimir invariant for a field Φ in an irreducible representation of $SU(5)$.

Appendix B

Input Parameters

In this appendix, we present input values for numerical simulations.

B.1 Electroweak Scale

First, the SM gauge couplings at the EW scale are shown in Table B.1. These couplings are estimated in the $\overline{\text{MS}}$ scheme. The $SU(2)_L$ and $U(1)_Y$ couplings, g and g_Y, are given as

$$\frac{g^2(m_Z)}{4\pi} = \frac{\alpha_{\text{EM}}(m_Z)}{\sin^2\theta_W(m_Z)}, \quad \frac{g_Y^2(m_Z)}{4\pi} = \frac{\alpha_{\text{EM}}(m_Z)}{1-\sin^2\theta_W(m_Z)}. \tag{B.1}$$

The relations between $\overline{\text{MS}}$ and $\overline{\text{DR}}$ couplings will be shown later.

We evaluate the Yukawa coupling matrices at the EW scale by using the quark and lepton mass matrices as follows,

$$\frac{v}{\sqrt{2}}Y_u = M_u\,, \quad \frac{v}{\sqrt{2}}Y_d U_{\text{CKM}} = M_d\,, \quad \frac{v}{\sqrt{2}}Y_e = M_e\,. \tag{B.2}$$

Here, the diagonal elements of the quark and lepton mass matrices M_u, M_d, and M_e are listed in Table B.2. In Table B.2, the light quark masses, m_u, m_d, and m_s, are $\overline{\text{MS}}$ masses at 2 GeV. The charm quark mass m_c and bottom quark mass m_b are estimated at each mass scale as $\overline{\text{MS}}$ masses. We treat the top quark mass m_t as a pole mass. We also show the transformation from the $\overline{\text{MS}}$ and the pole masses to $\overline{\text{DR}}$ masses, and show the short-range renormalization factor for quark masses later. U_{CKM} is the CKM matrix parameterized as

$$U_{\text{CKM}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{B.3}$$

Table B.1: Electroweak Parameters

	Symbol	Value
Fine structure constant	$\alpha_{\mathrm{EM}}^{-1}(m_Z)$	127.940 [57]
Weak angle	$\sin^2\theta_W(m_Z)$	0.23126 ± 0.0008 [57]
Strong coupling constant	$\alpha_S(m_Z)$	0.118 ± 0.007 [57]
Z boson mass	m_Z	91.1876 ± 0.0021 GeV [57]
W boson mass	m_W	80.385 ± 0.015 GeV [57]
Higgs boson mass	m_h	125.09 ± 0.21 (stat.) ± 0.11 (syst.) GeV [26]

where $c_{ij} = \cos\theta_{ij}, s_{ij} = \sin\theta_{ij}$, and the CKM phase δ. In our calculation, we used the Wolfenstein parameterization [131].

$$s_{12} \equiv \lambda, \;\; s_{23} \equiv A\lambda^2, \;\; s_{13}e^{i\delta} \equiv \frac{A\lambda^3(\bar{\rho}+i\bar{\eta})\sqrt{1-A^2\lambda^4}}{\sqrt{1-\lambda^2}[1-A^2\lambda^4(\bar{\rho}+i\bar{\eta})]}. \tag{B.4}$$

These values have been measured by various experiments.

$$\begin{aligned} \lambda = 0.22535 \pm 0.00065, \;\; A = 0.811^{+0.022}_{-0.012}, \\ \bar{\rho} = 0.131^{+0.026}_{-0.013}, \;\; \bar{\eta} = 0.345^{+0.013}_{-0.014}. \end{aligned} \tag{B.5}$$

Scheme Transformations

The gauge couplings in $\overline{\mathrm{MS}}$ and $\overline{\mathrm{DR}}$ schemes relate to each other. This relation is easily derived from the finite correction to gauge couplings with the unification requirement. Indeed, the finite corrections in $\overline{\mathrm{MS}}$ and $\overline{\mathrm{DR}}$ differ in only mass-independent term. The formula is given as follows [132]:

$$\frac{1}{\alpha_i^{\overline{\mathrm{DR}}}(\mu)} = \frac{1}{\alpha_i^{\overline{\mathrm{MS}}}(\mu)} - \frac{C_i}{12\pi}, \tag{B.6}$$

where μ denotes the renormalization scale and

$$(C_1, C_2, C_3) = (0, 2, 3)\,. \tag{B.7}$$

For quark masses, we convert $\overline{\mathrm{MS}}$ masses into $\overline{\mathrm{DR}}$ ones since we use the $\overline{\mathrm{DR}}$ renormalization scheme in the supersymmetric models. By comparing the pole masses and the running

Table B.2: Masses of Quarks and Leptons

Symbols	Values
$m_u(2\ \text{GeV})^{\overline{\text{MS}}}$	$2.3^{+0.7}_{-0.5}$ MeV
$m_d(2\ \text{GeV})^{\overline{\text{MS}}}$	$4.8^{+0.7}_{-0.3}$ MeV
m_e	$0.510998928(11)$ MeV
$m_c(m_c)^{\overline{\text{MS}}}$	1.275 ± 0.025 GeV
$m_s(2\ \text{GeV})^{\overline{\text{MS}}}$	95.0 ± 5 MeV
m_μ	$105.6583715(35)$ MeV
m_t	$173.07 \pm 0.6 \pm 0.8$ GeV
$m_b(m_b)^{\overline{\text{MS}}}$	4.18 ± 0.03 GeV
m_τ	$1.77682(16)$ GeV

masses (in the $\overline{\text{MS}}$ and $\overline{\text{DR}}$ renormalization schemes), and then we get the $\overline{\text{DR}}$ mass for quarks at one-loop level in terms of $\overline{\text{MS}}$ couplings and masses [133],

$$m_i^{\overline{\text{DR}}}(\mu) = m_i^{\overline{\text{MS}}}(\mu)\left(1 - \frac{\alpha_S^{\overline{\text{MS}}}(\mu)}{3\pi}\right) . \tag{B.8}$$

The one-loop scheme transformation between the pole mass and the $\overline{\text{DR}}$ mass for top quark is given by [134]

$$m_t^{\text{pole}} = m_t^{\overline{\text{DR}}}(m_t^{\overline{\text{DR}}})\left(1 - \frac{5}{3\pi}\alpha_S^{\overline{\text{DR}}}(m_t^{\overline{\text{DR}}})\right) . \tag{B.9}$$

Short-range Renormalization Factor

Below the EW scale, the quark mass m_q develops at the one-loop level as follows

$$m_q(\mu) = \left(\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}\right)^{-4/b_S} m_q(\mu_0) , \tag{B.10}$$

where μ and μ_0 are the renormalization scales, and b_S is the coefficient of the β-function defined as

$$b_S = -\left(11 - \frac{2}{3}N_q\right) . \tag{B.11}$$

Here, N_q is the number of quarks.

Appendix C

Supergraph and Feynman Rule on Superspace

In this appendix, external superfields have the external momenta and fermionic coordinates $\theta \equiv (\theta_\alpha, \theta^\dagger_{\dot\alpha})$. If external superfields have the momenta (the size of the momenta is denoted as p) which are the same (opposite) direction as the chirality arrow, we denote the external chiral and anti-chiral superfields as $\Phi(p,\theta)$ and $\Phi^\dagger(p,\theta)$ ($\Phi(-p,\theta)$ and $\Phi^\dagger(-p,\theta)$), respectively.

C.1 D-Algebra

We will give the formulae for the super-covariant derivatives, which are called "D-algebra". By using these formulae, we obtain not the full effective action but "the effective Kähler potential" in this thesis. In momentum space, the anti-commutation relations are given as follows:

$$\begin{aligned} \left\{\mathcal{D}_\alpha(p)\,,\mathcal{D}_\beta(p)\right\} &= 0\,, \quad \left\{\overline{\mathcal{D}}_{\dot\alpha}(p)\,,\overline{\mathcal{D}}_{\dot\beta}(p)\right\} = 0\,, \\ \left\{\mathcal{D}_\alpha(p)\,,\overline{\mathcal{D}}_{\dot\beta}(p)\right\} &= -2\sigma^\mu_{\alpha\dot\beta} p_\mu\,, \quad \left\{\mathcal{D}^\alpha(p)\,,\overline{\mathcal{D}}^{\dot\beta}(p)\right\} = -2\overline{\sigma}^{\dot\beta\alpha}_\mu p^\mu\,, \end{aligned} \tag{C.1}$$

where we substitute $-ip_\mu$ for the usual derivative ∂_μ in momentum space. Now, we do not write down the momentum-dependence of the super-covariant derivative $\mathcal{D}(p)$ explicitly below. We use $p_{\alpha\dot\alpha} = (\sigma^\mu)_{\alpha\dot\alpha} p_\mu$ and $p^{\dot\alpha\alpha} = (\sigma^\mu)^{\dot\alpha\alpha} p_\mu$ as shorthand notation. δ denotes the delta function for the superspace coordinates.

$$\delta \equiv \delta(\theta) = \theta^2 \theta^{\dagger 2}\,. \tag{C.2}$$

Now we find easily the formulae for simple D-algebra:

$$\delta F \delta = 0\,, \tag{C.3}$$

with $F = 1, \mathcal{D}^\alpha, \mathcal{D}^2, \mathcal{D}^\alpha\overline{\mathcal{D}}^{\dot\alpha}, \mathcal{D}^\alpha\overline{\mathcal{D}}^2$, and these Hermite conjugates. This is because the fermionic delta function has two θs and two $\theta^\dagger$s. Projection operators are defined as follows:

$$\begin{aligned} P_1 &= \frac{1}{16}\frac{\mathcal{D}^2\overline{\mathcal{D}}^2}{p^2}\,, \quad P_2 = \frac{1}{16}\frac{\overline{\mathcal{D}}^2\mathcal{D}^2}{p^2}\,, \\ P_+ &= \frac{1}{4}\frac{\mathcal{D}^2}{\sqrt{-p^2}}\,, \quad P_- = \frac{1}{4}\frac{\overline{\mathcal{D}}^2}{\sqrt{-p^2}}\,, \quad P_T = -\frac{1}{8}\frac{\mathcal{D}^\alpha\overline{\mathcal{D}}^2\mathcal{D}_\alpha}{p^2}\,. \end{aligned} \tag{C.4}$$

Formulae with four derivatives

There are non-zero contributions from the D-algebra with the scalar-type index, which means that all of the spinor indices are contracted.

$$\delta\mathcal{D}^2\overline{\mathcal{D}}^2\delta = \delta\overline{\mathcal{D}}^2\mathcal{D}^2\delta = \delta\overline{\mathcal{D}}_{\dot\beta}\mathcal{D}^2\overline{\mathcal{D}}^{\dot\beta}\delta = \delta\mathcal{D}^\alpha\overline{\mathcal{D}}^2\mathcal{D}_\alpha\delta = 16\delta\,. \tag{C.5}$$

We also obtain the epsilon-tensor type contributions: at first, using the following relation

$$\mathcal{D}_\alpha\mathcal{D}_\beta = \frac{1}{2}\epsilon_{\alpha\beta}\mathcal{D}^2\,, \quad \overline{\mathcal{D}}_{\dot\alpha}\overline{\mathcal{D}}_{\dot\beta} = \frac{1}{2}\epsilon_{\dot\alpha\dot\beta}\overline{\mathcal{D}}^2\,, \tag{C.6}$$

we obtain

$$\begin{aligned} \delta\mathcal{D}_\alpha\overline{\mathcal{D}}^2\mathcal{D}_\beta\delta &= \delta\mathcal{D}_\alpha\mathcal{D}_\beta\overline{\mathcal{D}}^2\delta = \delta\overline{\mathcal{D}}^2\mathcal{D}_\alpha\mathcal{D}_\beta\delta = 8\epsilon_{\alpha\beta}\delta\,, \\ \delta\overline{\mathcal{D}}_{\dot\alpha}\mathcal{D}^2\overline{\mathcal{D}}_{\dot\beta}\delta &= \delta\overline{\mathcal{D}}_{\dot\alpha}\overline{\mathcal{D}}_{\dot\beta}\mathcal{D}^2\delta = \delta\mathcal{D}^2\overline{\mathcal{D}}_{\dot\alpha}\overline{\mathcal{D}}_{\dot\beta}\delta = 8\epsilon_{\dot\alpha\dot\beta}\delta\,. \end{aligned} \tag{C.7}$$

Formulae with five derivatives

Using the anti-commutation relation, we obtain

$$\overline{\mathcal{D}}_{\dot\alpha}\mathcal{D}^2 = 4p_{\alpha\dot\alpha}\mathcal{D}^\alpha + \mathcal{D}^2\overline{\mathcal{D}}_{\dot\alpha}\,, \tag{C.8}$$

then,

$$\overline{\mathcal{D}}_{\dot\alpha}\mathcal{D}^2\overline{\mathcal{D}}^2 = 4p_{\alpha\dot\alpha}\mathcal{D}^\alpha\overline{\mathcal{D}}^2\,. \tag{C.9}$$

Formulae with six derivatives

We find

$$\delta\mathcal{D}^\alpha\overline{\mathcal{D}}^{\dot\beta}\mathcal{D}^2\overline{\mathcal{D}}^2\delta = 32p^{\dot\beta\alpha}\delta\,, \quad \delta\overline{\mathcal{D}}^{\dot\beta}\mathcal{D}^\alpha\overline{\mathcal{D}}^2\mathcal{D}^2\delta = 32p^{\dot\beta\alpha}\delta\,, \tag{C.10}$$

and $\delta\mathcal{D}^\alpha\overline{\mathcal{D}}^2\mathcal{D}^\beta\overline{\mathcal{D}}^2\delta = 0$. Futhermore,

$$\overline{\mathcal{D}}^2(p)\mathcal{D}^2(p)\overline{\mathcal{D}}^2(p) = 16p^2\overline{\mathcal{D}}^2(p)\,, \quad \mathcal{D}^2(p)\overline{\mathcal{D}}^2(p)\mathcal{D}^2(p) = 16p^2\mathcal{D}^2(p)\,. \tag{C.11}$$

External Superfields with Covariant Derivatives

In a full quantum action, there are terms with the covariant derivatives acting on external fields. To obtain the effective Kähler potential, we have to divide them into the terms with and without covariant derivatives. The terms with covariant derivatives are vanished in the limit of $\mathcal{D}_\alpha\Phi = 0$ and $\overline{\mathcal{D}}_{\dot{\beta}}\Phi^\dagger = 0$. Here, we give

$$\begin{aligned} \mathcal{D}^\alpha\overline{\mathcal{D}}^{\dot{\beta}}(\Phi(p)\Phi^\dagger(q)) &= -2q^{\dot{\beta}\alpha}\Phi(p)\Phi^\dagger(q) + \mathcal{D}^\alpha\Phi(p)\overline{\mathcal{D}}^{\dot{\beta}}\Phi^\dagger(q)\,, \\ \overline{\mathcal{D}}^{\dot{\beta}}\mathcal{D}^\alpha(\Phi(p)\Phi^\dagger(q)) &= -2p^{\dot{\beta}\alpha}\Phi(p)\Phi^\dagger(q) + \overline{\mathcal{D}}^{\dot{\beta}}\Phi^\dagger(q)\mathcal{D}^\alpha\Phi(p)\,. \end{aligned} \tag{C.12}$$

The last terms correspond to the auxiliary potential. The products of external superfields with projection operators are divided as follows.

$$\begin{aligned} \overline{\mathcal{D}}^2\mathcal{D}^2(\Phi(p)\Phi(q)) &= 16(q+p)^2\Phi(q)\Phi(p)\,, \\ \mathcal{D}^2\overline{\mathcal{D}}^2(\Phi^\dagger(p)\Phi^\dagger(q)) &= 16(q+p)^2\Phi^\dagger(q)\Phi^\dagger(p)\,, \\ \overline{\mathcal{D}}^2\mathcal{D}^2(\Phi(p)\Phi^\dagger(q)) &= 16p^2\Phi(p)\Phi^\dagger(q) + 8p_{\alpha\dot{\beta}}\mathcal{D}^\alpha\Phi(p)\overline{\mathcal{D}}^{\dot{\beta}}\Phi^\dagger(q) + \mathcal{D}^2\Phi(p)\overline{\mathcal{D}}^2\Phi^\dagger(q)\,. \end{aligned} \tag{C.13}$$

In order to obtain the last line, we use the relation $\overline{\mathcal{D}}_{\dot{\beta}}\mathcal{D}^2\Phi(p) = 4p_{\alpha\dot{\beta}}\mathcal{D}^\alpha\Phi(p)$.

For the product of chiral and antichiral superfields projected on the transverse mode, we also decomposed it as the following form:

$$\begin{aligned} &\mathcal{D}^\alpha\overline{\mathcal{D}}^2\mathcal{D}_\alpha[\Phi(p)\Phi^\dagger(q)] \\ &= -2(q-p)_{\alpha\dot{\beta}}\mathcal{D}^\alpha\Phi(p)\overline{\mathcal{D}}^{\dot{\beta}}\Phi^\dagger(q) - 8p\cdot q\Phi(p)\Phi^\dagger(q) + \mathcal{D}^2\Phi(p)\overline{\mathcal{D}}^2\Phi^\dagger(q)\,. \end{aligned} \tag{C.14}$$

Here, we use $P_T\Phi = 0$ ($P_T\Phi^\dagger = 0$). Indeed, the chiral superfield projected on the transverse mode is exactly zero due to the property of the projection operator, $P_1 + P_2 + P_T = 1$. The dot product is defined as $p\cdot q = p^\mu q_\mu$.

We also have the gauge interactions among chiral, anti-chiral, and vector superfields. From these interactions, we get the quantum action in which superspace covariant derivatives act on the vector superfield. In terms of the effective Kähler potential, we must neglect the terms vanished as $\mathcal{D}_\alpha\Phi = 0$ and $\overline{\mathcal{D}}_{\dot{\beta}}\Phi^\dagger = 0$.

$$\begin{aligned} &\int d^4\theta\Phi^\dagger(-p)\Phi(k)\overline{\mathcal{D}}^{\dot{\beta}}\mathcal{D}^\alpha V(q) \\ &= \int d^4\theta\left[-2p^{\dot{\beta}\alpha}\Phi^\dagger(-p)\Phi(k) + \overline{\mathcal{D}}^{\dot{\beta}}\Phi^\dagger(-p)\mathcal{D}^\alpha\Phi(k)\right]V(q)\,, \end{aligned} \tag{C.15}$$

$$\begin{aligned} &\int d^4\theta\Phi^\dagger(-p)\Phi(k)\mathcal{D}^\alpha\overline{\mathcal{D}}^{\dot{\beta}}V(q) \\ &= \int d^4\theta\left[-2k^{\dot{\beta}\alpha}\Phi^\dagger(-p)\Phi(k) + \mathcal{D}^\alpha\Phi(k)\overline{\mathcal{D}}^{\dot{\beta}}\Phi^\dagger(-p)\right]V(q)\,. \end{aligned} \tag{C.16}$$

The terms with projection operators acting on the vector superfield are decomposed as follows.

$$
\begin{aligned}
&\int d^4\theta \Phi^\dagger(-p)\Phi(k)\frac{1}{16}\mathcal{D}^2\overline{\mathcal{D}}^2 V(q) \\
&= \frac{1}{16}\int d^4\theta \left[\overline{\mathcal{D}}^2\Phi^\dagger(-p)\mathcal{D}^2\Phi(k) + 2\overline{\mathcal{D}}_{\dot\beta}\Phi^\dagger(-p)\overline{\mathcal{D}}^{\dot\beta}\mathcal{D}^2\Phi(k) + 16k^2\Phi^\dagger(-p)\Phi(k)\right] V(q)\,, \\
&\int d^4\theta \Phi^\dagger(-p)\Phi(k)\frac{1}{16}\overline{\mathcal{D}}^2\mathcal{D}^2 V(q) \\
&= \frac{1}{16}\int d^4\theta \left[\overline{\mathcal{D}}^2\Phi^\dagger(-p)\mathcal{D}^2\Phi(k) + 2\mathcal{D}^\alpha\Phi(k)\mathcal{D}_\alpha\overline{\mathcal{D}}^2\Phi^\dagger(-p) + 16p^2\Phi^\dagger(-p)\Phi(k)\right] V(q)\,, \\
&\int d^4\theta \Phi^\dagger(-p)\Phi(k)\left(-\frac{1}{8}\mathcal{D}^\alpha\overline{\mathcal{D}}^2\mathcal{D}_\alpha\right) V(q) \\
&= \frac{1}{2}(k-p)^{\dot\beta\alpha}\int d^4\theta\overline{\mathcal{D}}_{\dot\beta}\Phi^\dagger(-p)\mathcal{D}_\alpha\Phi(k)V(q) + (q^2-p^2-k^2)\int d^4\theta\Phi^\dagger(-p)\Phi(k)V(q)\,.
\end{aligned} \tag{C.17}
$$

C.2 Feynman Rule on Superspace

Here, we summarize the Feynman rule on the superspace. First, we give the Feynman-rule on the coordinate space for the generic super-Yang-Mills with matter chiral superfield.*

1. Propagator:

   (a) Chiral Superfields with mass m

$$
\begin{aligned}
\Phi\Phi :&\quad \frac{m}{4}\frac{i\mathcal{D}^2}{\Box(\Box+m^2)}\delta^{(8)}(z-z') \\
\Phi^\dagger\Phi^\dagger :&\quad \frac{m}{4}\frac{i\overline{\mathcal{D}}^2}{\Box(\Box+m^2)}\delta^{(8)}(z-z') \\
\Phi\Phi^\dagger :&\quad -\frac{i}{(\Box+m^2)}\delta^{(8)}(z-z')
\end{aligned} \tag{C.18}
$$

   (b) Vector Superfields

$$
VV:\quad \frac{1}{2}\left(\frac{i}{\Box+m^2}P_T + \frac{i}{\Box+m^2/\alpha}(P_1+P_2)\right)\delta^{(8)}(z-z') \tag{C.19}
$$

   with the gauge parameter α. If we take the Fermi-Feynman gauge ($\alpha = 1$),

$$
VV:\quad \frac{1}{2}\frac{i}{\Box+m^2}\delta^{(8)}(z-z') \tag{C.20}
$$

*See Refs. [56, 135] for details of supergraphs.

Here, superspace coordinates are denoted by $z = (x, \theta, \theta^\dagger)$ and $z' = (x', \theta', \theta'^\dagger)$. We prepare a propagator of massive vector superfields for spontaneously broken gauge theories.

2. Vertices: In addition to the vertex rule read from the Lagrangian directly as the usual Feynman-rule, we act $-\overline{\mathcal{D}}^2/4$ ($-\mathcal{D}^2/4$) on the corresponding propagators for each chiral (anti-chiral) line leaving from vertices. For the superpotential vertices (purely chiral or anti-chiral vertices), we omit one $-\overline{\mathcal{D}}^2/4$ or $-\mathcal{D}^2/4$ among the corresponding propagators.

3. Carrying out integral for fermionic coordinates $\int d^4\theta$ at each vertices.

4. Finally, multiplying the symmetry factor.

In the momentum space, the Feynman rule on the superspace is given by the following procedure. We substitute $-ip_\mu$ for the spacetime derivative ∂_μ.

1. Propagators:

   (a) Chiral Superfields

$$\begin{aligned} \Phi\Phi : &\quad \frac{m}{4}\frac{i\mathcal{D}^2}{p^2(p^2-m^2)}\delta^{(4)}(\theta-\theta')\,, \\ \Phi^\dagger\Phi^\dagger : &\quad \frac{m}{4}\frac{i\overline{\mathcal{D}}^2}{p^2(p^2-m^2)}\delta^{(4)}(\theta-\theta')\,, \\ \Phi\Phi^\dagger : &\quad \frac{i}{p^2-m^2}\delta^{(4)}(\theta-\theta')\,. \end{aligned} \tag{C.21}$$

   (b) Vector Superfields

$$VV : \quad -\frac{1}{2}\left(\frac{i}{p^2-m^2}P_T + \frac{i}{p^2-m^2/\alpha}(P_1+P_2)\right)\delta^{(4)}(\theta-\theta')\,. \tag{C.22}$$

   For the Fermi-Feynman gauge ($\alpha = 1$),

$$VV : \quad -\frac{1}{2}\frac{i}{p^2-m^2}\delta^{(4)}(\theta-\theta')\,. \tag{C.23}$$

2. Carrying out the fermionic integrals $\int d^4\theta$ at each vertices, the loop integrals at each loops , and multiplying $(2\pi)^4\delta(\sum k_{\text{ext}})$ for each external lines with external momentum k_{ext}.

3. Calculating one-particle irreducible graphs in order to obtain the effective action, and multiplying

$$\int \frac{d^4 k_i}{(2\pi)^4} \Psi(k_i)\,, \tag{C.24}$$

for the external superfields Ψ with external momentum k_i.

Of course, the rule for vertices is prescribed in the same way as the coordinate Feynman-rule, and the symmetry factor is also multiplied.

Appendix D

Decomposition of $SU(5)$ Interactions

D.1 Interactions of Vector Superfields

In Super-Yang-Mills (SYM) theories, the renormalizable Lagrangian is written as

$$\mathcal{L}_{\text{SYM}} = \frac{1}{8g^2}\text{tr}\int d^2\theta\ \mathcal{W}^\alpha \mathcal{W}_\alpha + \text{h.c.}\,, \tag{D.1}$$

where the field strength chiral superfield $\mathcal{W}^\alpha$ is given in Eq. (1.19). "tr" represents a trace over the gauge indices. The Lagrangian is expanded in the vector superfield V as

$$\begin{aligned}\frac{1}{8g^2}\text{tr}\int d^2\theta\ \mathcal{W}^\alpha \mathcal{W}_\alpha = -\frac{1}{8}\text{tr}\int d^4\theta\ \Big[&-V\mathcal{D}^\alpha\overline{\mathcal{D}}^2\mathcal{D}_\alpha V + 2gV\mathcal{D}^\alpha\overline{\mathcal{D}}^2[V,\mathcal{D}_\alpha V] \\ &+g^2[V,\mathcal{D}^\alpha V]\overline{\mathcal{D}}^2[V,\mathcal{D}_\alpha V] + \frac{4g^2}{3}\mathcal{D}_\alpha V\overline{\mathcal{D}}^2[V,[V,\mathcal{D}_\alpha V]] + \cdots\Big]\,.\end{aligned} \tag{D.2}$$

The decomposition of the $SU(5)$ vector superfield $\mathcal{V}_5$ is given by Eq. (2.7). As mentioned in the text, we denote $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$ vector superfields in the MSSM with G, W, and B, again. The kinetic terms of the vector superfields in the $SU(5)$ GUTs are given into the following form;

$$\mathcal{L}_{VV} = 2\,\text{tr}\int d^4\theta\ G\Box P_T G + 2\,\text{tr}\int d^4\theta\ W\Box P_T W + \int d^4\theta\ B\Box P_T B + 2\int d^4\theta\ X^\dagger\Box P_T X\,, \tag{D.3}$$

where X denotes the massive vector superfield associated with the broken $SU(5)$ generators. Here, $P_T(\equiv \mathcal{D}^\alpha\overline{\mathcal{D}}^2\mathcal{D}_\alpha/(8\Box))$ is the projection operator to the transverse mode ($P_T^2 = P_T$).

From the second term of Eq. (D.2), the three-point interaction terms between X and MSSM vector superfields are obtained as

$$\mathcal{L}_{X\text{-3 pt}} = \int d^4\theta\left[\delta_r^s(T^a)_\alpha^\beta(\mathcal{K}_{XG}^a)_{s\beta}^{r\alpha} + (t^a)_r^s\delta_\alpha^\beta(\mathcal{K}_{XW}^a)_{s\beta}^{r\alpha} + \frac{5}{2\sqrt{15}}\delta_r^s\delta_\alpha^\beta(\mathcal{K}_{XB})_{s\beta}^{r\alpha}\right], \tag{D.4}$$

where

$$(\mathcal{K}^a_{XV})^{r\alpha}_{s\beta} \equiv \frac{g_5}{4}\left[X^r_\beta(\overline{\mathcal{D}}^2\mathcal{D}X^\dagger)^\alpha_s\mathcal{D}V^a + V^a(\overline{\mathcal{D}}^2\mathcal{D}X)^r_\beta(\mathcal{D}X^\dagger)^\alpha_s + X^{\dagger\alpha}{}_s(\overline{\mathcal{D}}^2\mathcal{D}V^a)\mathcal{D}X^r_\beta \right. \\ \left. -X^r_\beta(\overline{\mathcal{D}}^2\mathcal{D}V^a)(\mathcal{D}X^\dagger)^\alpha_s - V^a(\overline{\mathcal{D}}^2\mathcal{D}X^\dagger)^\alpha_s\mathcal{D}X^r_\beta - X^{\dagger\alpha}{}_s(\overline{\mathcal{D}}^2\mathcal{D}X)^r_\beta\mathcal{D}V^a\right] . \tag{D.5}$$

Here, spinor indices of covariant derivatives $\mathcal{D}$ and $\overline{\mathcal{D}}$ are contracted like ${}^\alpha{}_\alpha$ or ${}_{\dot\alpha}{}^{\dot\alpha}$. The four-point self interaction of X is given as

$$\mathcal{L}_{X\text{-4 pt}} = -\frac{g_5^2}{48}\int d^4\theta\left[(\overline{\mathcal{D}}^2\mathcal{D}X^{\dagger\alpha}{}_r)(X^r_\beta X^{\dagger\beta}{}_s(\mathcal{D}X)^s_\alpha - 2X^r_\beta(\mathcal{D}X^\dagger)^\beta_s X^s_\alpha + (\mathcal{D}X)^r_\beta X^{\dagger\beta}{}_s X^s_\alpha) \right. \\ \left. + (\overline{\mathcal{D}}^2\mathcal{D}X^r_\alpha)(X^{\dagger\alpha}{}_s X^s_\beta(\mathcal{D}X^\dagger)^\beta_r - 2X^{\dagger\alpha}{}_s(\mathcal{D}X)^s_\beta X^{\dagger\beta}{}_r + (\mathcal{D}X^\dagger)^\alpha_s X^s_\beta X^{\dagger\beta}{}_r)\right] . \tag{D.6}$$

D.2 Vector-Ghost Interactions

The Lagrangian for the massless Fadeev-Popov ghost chiral superfields, which are denoted by b and c, is given by

$$\mathcal{L}_{\text{FP}} = 2\,\text{tr}\int d^4\theta(b+b^\dagger)\mathcal{L}_{gV}\left[(c+c^\dagger) + \coth(\mathcal{L}_{gV})(c-c^\dagger)\right] , \tag{D.7}$$

where $\mathcal{L}_A B$ is the Lie derivative ($\mathcal{L}_A B \equiv [A, B]$). Therefore, the kinetic terms for ghost fields in the $SU(5)$ GUTs are obtained as

$$\mathcal{L}_{\text{ghost}} = 2\int d^4\theta\left[\text{tr}(b_3^\dagger c_3 - b_3 c_3^\dagger) + \text{tr}(b_2^\dagger c_2 - b_2 c_2^\dagger)\right] + \int d^4\theta(b_1^\dagger c_1 - b_1 c_1^\dagger) \\ + \int d^4\theta\left[(b_X^\dagger c_X - b_X c_X^\dagger) + (b_{X^\dagger}^\dagger c_{X^\dagger} - b_{X^\dagger} c_{X^\dagger}^\dagger)\right] , \tag{D.8}$$

where the ghost multiplets are decomposed in a similar way to the gauge multiplets as

$$b = \begin{pmatrix} b_3 - \frac{2}{\sqrt{30}}b_1 & \frac{1}{\sqrt{2}}b_X \\ \frac{1}{\sqrt{2}}b_{X^\dagger} & b_2 + \frac{3}{\sqrt{30}}b_1 \end{pmatrix} , \quad c = \begin{pmatrix} c_3 - \frac{2}{\sqrt{30}}c_1 & \frac{1}{\sqrt{2}}c_X \\ \frac{1}{\sqrt{2}}c_{X^\dagger} & c_2 + \frac{3}{\sqrt{30}}c_1 \end{pmatrix} . \tag{D.9}$$

After spontaneously breaking of the GUT group by the adjoint Higgs chiral superfield, there exist kinetic mixing terms between X and the Nambu-Goldstone chiral superfields $\Sigma_{(3,2)}$ and $\Sigma_{(3^*,2)}$. By using the supersymmetric R_ξ-gauge [136], we remove the kinetic mixing terms, and we find the mass terms for the ghost chiral superfields [136] as:

$$\mathcal{L}_{\text{ghost mass}} = \int d^4\theta\left[(b_X + b_X^\dagger)\frac{M_X^2}{\xi\Box}(c_X - c_X^\dagger) + (b_{X^\dagger} + b_{X^\dagger}^\dagger)\frac{M_X^2}{\xi\Box}(c_{X^\dagger} - c_{X^\dagger}^\dagger)\right] . \tag{D.10}$$

We note that the terms such as $b_X c_X$ and $b_X^\dagger c_X^\dagger$ vanish by the superspace integral since these are chiral (or anti-chiral) superfields. Then, the propagator for massive ghost superfields is modified as

$$\Delta_{bc} = \frac{i}{k^2}\delta^4(\theta_1 - \theta_2) \to \frac{i}{k^2}\frac{1}{1-\frac{M_X^2}{\xi k^2}}\delta^4(\theta_1-\theta_2)\,. \tag{D.11}$$

In the evaluation of the self energy of X, we need interaction terms for X and the massive ghosts. In general, three-point and four-point interaction terms of ghost superfields and vector superfields are obtained from Eq. (D.7) as follows,

$$\mathcal{L}_{bVc} = \mathrm{tr}\int d^4\theta \left\{ 2g\,(b+b^\dagger)\left[V,(c+c^\dagger)\right] + \frac{2g^2}{3}(b+b^\dagger)[V,[V,(c-c^\dagger)]] + \mathcal{O}(V^3)\right\}\,. \tag{D.12}$$

Then, the interaction terms between X and the ghosts are given by:

$$\mathcal{L}_{bXc} = \int d^4\theta \left[\delta_r^s (T^a)_\alpha^\beta (\mathcal{K}^a_{bcG})^{r\alpha}_{s\beta} - (t^a)^s_r \delta_\alpha^\beta (\mathcal{K}^a_{bcW})^{r\alpha}_{s\beta} - \frac{5}{2\sqrt{15}}\delta_r^s\delta_\alpha^\beta(\mathcal{K}_{bcB})^{r\alpha}_{s\beta}\right]\,, \tag{D.13}$$

and

$$\begin{aligned}\mathcal{L}_{bX^2c} = &-\frac{g_5^2}{6}\int d^4\theta(\delta^{\beta\delta}_{\alpha\gamma}\delta^{tr}_{su} + \delta^{\delta\beta}_{\alpha\gamma}\delta^{rt}_{su})X_r^{\dagger\alpha}X^s_\beta \\ &\times\left[(b_{X^\dagger})^\gamma_t(c^\dagger_{X^\dagger})^u_\delta - (b_X^\dagger)^\gamma_t(c_X)^u_\delta - (b^\dagger_{X^\dagger})^u_\delta(c_{X^\dagger})^\gamma_t + (b_X)^u_\delta(c_X^\dagger)^\gamma_t\right]\,.\end{aligned} \tag{D.14}$$

Here, we define $\delta^{\beta\delta}_{\alpha\gamma} \equiv \delta^\beta_\alpha\delta^\delta_\gamma$ and $\delta^{tr}_{su} \equiv \delta^t_s\delta^r_u$. In the three-point interactions, we define the term $(\mathcal{K}^a_{bcV})^{r\alpha}_{s\beta}$ as:

$$\begin{aligned}(\mathcal{K}^a_{bcV})^{r\alpha}_{s\beta} \equiv &((b_X + b_{X^\dagger})^r_\beta X^{\dagger\alpha}_s - X^r_\beta(b_{X^\dagger}+b_X^\dagger)^\alpha_s)(c_V+c_V^\dagger)^a \\ &+ (b_V+b_V^\dagger)^a(X^r_\beta(c_{X^\dagger}+c_X^\dagger)^\alpha_s - (c_X + c^\dagger_{X^\dagger})^r_\beta X^{\dagger\alpha}_s)\,.\end{aligned} \tag{D.15}$$

D.3 Gauge Interactions of Matter Superfields

Now, we summarize the gauge interactions of the matter and Higgs multiplets in SUSY $SU(5)$ GUTs. The renormalizable Kähler potential in the $SU(5)$ GUTs is given as:

$$\begin{aligned}\mathcal{K} = &\Phi^{\dagger A}(e^{-2g_5\mathcal{V}_5})_A^B\Phi_B + \Psi^\dagger_{AB}(e^{2g_5\mathcal{V}_5})^A_C(e^{2g_5\mathcal{V}_5})^B_D\Psi^{CD} \\ &+ 2\Sigma^{\dagger A}_B(e^{-2g_5\mathcal{V}_5})^C_A(e^{2g_5\mathcal{V}_5})^B_D\Sigma^D_C + H^{\dagger A}_{\bar{5}}(e^{-2g_5\mathcal{V}_5})^B_A H_{\bar{5}B} + H^\dagger_{5A}(e^{2g_5\mathcal{V}_5})^A_B H^B_5\,.\end{aligned} \tag{D.16}$$

The three-point gauge interaction of the $\overline{\mathbf{5}}$ representation matter field Φ is given as

$$\begin{aligned}\mathcal{K}_{\Phi^\dagger V \Phi} =& -g_5 \overline{D}^\dagger \left(2G - \frac{2}{\sqrt{15}}B\right)\overline{D} + g_5 L^\dagger \left(2W - \frac{3}{\sqrt{15}}B\right) L \\ & -\sqrt{2} g_5 \left[\overline{D}^\dagger (X \cdot L) + \text{h.c.}\right] .\end{aligned} \tag{D.17}$$

For the four-point vertices, we only use the interactions which include only one X,

$$\mathcal{K}_{\Phi^\dagger V^2 \Phi} \ni \sqrt{2} g_5^2 \left(\overline{D}^\dagger G(X\cdot L) + \frac{1}{\sqrt{60}}\overline{D}^\dagger B(X \cdot L) + \overline{D}^\dagger (WX \cdot L)\right) + \text{h.c.}. \tag{D.18}$$

Here, $(A \cdot B) \equiv \epsilon_{rs} A^r B^s$. We also obtain the relevant gauge interactions from the **10** representation matter field Ψ,

$$\begin{aligned}\mathcal{K}_{\Psi^\dagger V \Psi} =& -g_5 \overline{U}^\dagger \left(2G + \frac{4}{\sqrt{15}}B\right)\overline{U} + g_5 Q^\dagger \left(2G + 2W + \frac{1}{\sqrt{15}}B\right) Q \\ & + g_5 \frac{6}{\sqrt{15}} \overline{E}^\dagger B \overline{E} + \sqrt{2} g_5 \left[[Q^\dagger X \overline{U}] - (Q^\dagger \cdot X^\dagger)\overline{E} + \text{h.c.}\right] ,\end{aligned} \tag{D.19}$$

$$\begin{aligned}\mathcal{K}_{\Psi^\dagger V^2 \Psi} \ni& \sqrt{2} g_5^2 \Bigg[[(GQ^\dagger)X\overline{U}] - [Q^\dagger X (G\overline{U})] - \frac{3}{\sqrt{60}} B [Q^\dagger X \overline{U}] + [(WQ^\dagger) X \overline{U}] \\ & + \overline{E}^\dagger \left((X\cdot GQ) + (X \cdot WQ) + \frac{7}{\sqrt{60}}(X \cdot BQ)\right)\Bigg] + \text{h.c.},\end{aligned} \tag{D.20}$$

where $[ABC] \equiv \epsilon^{\alpha\beta\gamma} A_\alpha B_\beta C_\gamma$ or $\epsilon_{\alpha\beta\gamma} A^\alpha B^\beta C^\gamma$.

There are also the three- and four-point interactions of X with Higgs multiplets. One of them comes from the interaction of the anti-fundamental Higgs superfield $\overline{H} = (H_{\overline{C}}, H_d)$,

$$\begin{aligned}\mathcal{K}_{\overline{H}^\dagger X \overline{H}} =& -\sqrt{2} g_5 \left[H_{\overline{C}}^\dagger (X \cdot H_d) + \text{h.c.}\right] \\ & + g_5^2 \Bigg[\sqrt{2}\left(H_{\overline{C}}^\dagger G (X \cdot H_d) + \frac{1}{2\sqrt{15}} H_{\overline{C}}^\dagger B (X \cdot H_d) + H_{\overline{C}}^\dagger (WX \cdot H_d)\right) + \text{h.c.} \\ & + H_{\overline{C}}^{\dagger\alpha} X_\alpha^r X_r^{\dagger\beta} H_{\overline{C}\beta} + (X^\dagger \cdot H_d^\dagger)(X \cdot H_d)\Bigg] .\end{aligned} \tag{D.21}$$

Another one comes from the fundamental Higgs superfield $H = (H_C, H_u)$,

$$\begin{aligned}\mathcal{K}_{H^\dagger X H} =& \sqrt{2} g_5 \left[H_u^\dagger X H_C + \text{h.c.}\right] \\ & + g_5^2 \Bigg[\sqrt{2}\left(H_u^\dagger X G H_C + \frac{1}{2\sqrt{15}} H_u^\dagger X B H_C + H_u^\dagger W X H_C\right) + \text{h.c.} \\ & + H_{C\alpha}^\dagger X_r^{\dagger\alpha} X_\beta^r H_C^\beta + H_{ur}^\dagger X_\alpha^r X_s^{\dagger\alpha} H_u^s\Bigg] .\end{aligned} \tag{D.22}$$

The adjoint Higgs superfield is decomposed as

$$\Sigma = \begin{pmatrix} \Sigma_8 - \frac{2}{\sqrt{60}}\Sigma_{24} & \frac{1}{\sqrt{2}}\Sigma_{(3,2)} \\ \frac{1}{\sqrt{2}}\Sigma_{(3^*,2)} & \Sigma_3 + \frac{3}{\sqrt{60}}\Sigma_{24} \end{pmatrix} . \tag{D.23}$$

In our calculation, we need the interaction terms of X with the GUT-breaking Higgs superfield,

$$\begin{aligned} \mathcal{K}_{\Sigma^\dagger X \Sigma} =& -2g_5\left[\Sigma^\dagger_{(3^*,2)}X\Sigma_8 - \Sigma^\dagger_8 X\Sigma_{(3,2)}\right] - 2g_5\left[\Sigma^\dagger_3 X\Sigma_{(3,2)} - \Sigma^\dagger_{(3^*,2)}X\Sigma_3\right] \\ &- \frac{5}{\sqrt{15}}g_5\left[\Sigma^\dagger_{(3^*,2)}X\Sigma_{24} - \Sigma^\dagger_{24}X\Sigma_{(3,2)}\right] + \text{h.c.}\,, \end{aligned} \tag{D.24}$$

$$\begin{aligned} \mathcal{K}_{\Sigma^\dagger X^\dagger X\Sigma} =& g_5^2\Big\{2X^\dagger(\Sigma^\dagger_8\Sigma_8 + \Sigma_8\Sigma^\dagger_8)X + 2X^\dagger(\Sigma^\dagger_3\Sigma_3 + \Sigma_3\Sigma^\dagger_3)X + \frac{5}{3}X^\dagger\Sigma^\dagger_{24}\Sigma_{24}X \\ &+ \frac{10}{\sqrt{15}}(\Sigma^\dagger_{24}X^\dagger X\Sigma_8 - \Sigma^\dagger_{24}X^\dagger X\Sigma_3 + \text{h.c.}) \\ &- 2\left(2\Sigma^\dagger_8 X^\dagger X\Sigma_3 + (\Sigma^\dagger_{(3^*,2)})^\alpha_r X^s_\alpha X^r_\beta(\Sigma_{(3,2)})^\beta_s + \text{h.c.}\right) \\ &+(\delta^{ru}_{st}\delta^{\alpha\gamma}_{\delta\beta} + \delta^{ru}_{ts}\delta^{\alpha\gamma}_{\beta\delta})X^t_\gamma X^{\dagger\delta}_u\left((\Sigma^\dagger_{(3,2)})^s_\alpha(\Sigma_{(3,2)})^\beta_r + (\Sigma^\dagger_{(3^*,2)})^\beta_r(\Sigma_{(3^*,2)})^s_\alpha\right)\Big\} . \end{aligned} \tag{D.25}$$

Finally, we consider the interactions of X with the GUT-breaking Higgs superfields. We have used these interactions in the calculation of a_i in Table 3.1.

$$\begin{aligned} \mathcal{K}_{v_{24}X} = -\sqrt{2}g_5 M_X\Big\{&\Sigma_{(3,2)}\left[GX - WX + \frac{5}{\sqrt{60}}BX\right] \\ &+\Sigma_{(3^*,2)}\left[GX^\dagger - WX^\dagger + \frac{5}{\sqrt{60}}BX^\dagger\right]\Big\} + \text{ h.c.}\,. \end{aligned} \tag{D.26}$$

Here, the mass of the extra vector superfield X and $X^\dagger$ is $M_X = 5\sqrt{2}g_5 v_{24}$.

In the missing partner model, **75** representation $\Xi^{[AB]}_{[CD]}$ breaks $SU(5)$ into the SM gauge group. For a **75** representation $\Xi^{[AB]}_{[CD]}$, the Kähler potential is given by Eq. (2.17). The GUT-breaking VEV of $\Xi^{(AB)}_{(CD)}$ is given in Eq. (2.16). The extra vector superfield becomes massive with $M_X = 2\sqrt{6}g_5 v_{\mathbf{75}}$ after we substitute the VEV for Ξ and $\Xi^\dagger$, while the interaction terms proportional to the VEV are

$$\begin{aligned}
\mathcal{K}_{v_{75}X} = & g_5 M_X \left\{ -X^\dagger X(\phi_8 + \phi_8^\dagger) + \frac{4}{\sqrt{3}} X^\dagger X(\phi_7 + \phi_7^\dagger) + \sqrt{6} X^\dagger X(\phi_9 + \phi_9^\dagger) \right\} \\
& - \sqrt{2} g_5 M_X \left\{ -G(X \cdot (\phi_3 + \phi_4^\dagger)) + X(W \cdot (\phi_3 + \phi_4^\dagger)) + \frac{5}{\sqrt{60}} B(X \cdot (\phi_3 + \phi_4^\dagger)) \right\} + \text{h.c.} \\
& + \frac{\sqrt{6}}{2} g_5 M_X \epsilon_{ab} \epsilon^{\alpha\beta\gamma} X_\alpha^a X_\beta^b (\phi_1 + \phi_2^\dagger)_\gamma + \text{h.c.}\,.
\end{aligned} \tag{D.27}$$

The components ϕ_i ($i = 1, \text{-}, 9$) of Ξ are given in Appendix D.4.

D.4 Field Decomposition of $SU(5)$ Representations

The Kähler potential for Φ in the $SU(5)$ irreducible representation is decomposed as follows;

$$\mathcal{K} = \Phi^\dagger \Phi = \sum_i \phi_i^\dagger \phi_i. \tag{D.28}$$

Here, ϕ_i transforms as an irreducible representation under the SM gauge groups. $\Phi^\dagger$ and $\phi^\dagger$ denote the anti-chiral superfields. In this section, we turn off gauge interactions for simplicity.

In Table D.1, we give the SM decomposition of the $SU(5)$ irreducible representations, whose Dynkin indices are below that of **75** representation. In this table, square brackets $[\cdots]$ represent antisymmetric indices while braces $\{\cdots\}$ represent symmetric indices.

We summarize the vacuum polarization coefficients b_{ij}, defined in Eq. (3.21), from the above-mentioned irreducible representations. In Table D.2, b_{ij} from each representations are shown except those from $\mathbf{5} + \overline{\mathbf{5}}, \mathbf{10} + \overline{\mathbf{10}}, \mathbf{24}$, and $\mathbf{75}$ which have been already shown in Table 3.1. As a check, the sum of b_{ij} in a representation divided by its Dynkin index is a representation independent constant number.

Table D.1: SM decomposition of the $SU(5)$ irreducible multiplets. $(r_C, r_W)_Y$ denotes a representation transforming as $SU(3)_C$ r_C-plet and $SU(2)_L$ r_W-plet with hypercharge Y.

$SU(5)$ representation	labels and SM representations
$\Phi^A(\mathbf{5})$	$\phi_1^a = (\mathbf{1},\mathbf{2})_{\frac{1}{2}}, \quad \phi_2^\alpha = (\mathbf{3},\mathbf{1})_{-\frac{1}{3}}$
$\Phi^{[AB]}(\mathbf{10})$	$\phi_1^{[ab]} = (\mathbf{1},\mathbf{1})_1, \quad \phi_2^{[\alpha\beta]} = (\overline{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}}, \quad \phi_3^{a\alpha} = (\mathbf{3},\mathbf{2})_{\frac{1}{6}}$
$\Phi^{\{AB\}}(\mathbf{15})$	$\phi_1^{\{ab\}} = (\mathbf{1},\mathbf{3})_1, \quad \phi_2^{\{\alpha\beta\}} = (\mathbf{6},\mathbf{1})_{-\frac{2}{3}}, \quad \phi_3^{a\alpha} = (\mathbf{3},\mathbf{2})_{\frac{1}{6}}$
$\Phi_B^A(\mathbf{24})$	$(\phi_1)_b^a = (\mathbf{1},\mathbf{3})_0, \quad (\phi_2)_\beta^\alpha = (\mathbf{8},\mathbf{1})_0, \quad \phi_3 = (\mathbf{1},\mathbf{1})_0,$ $(\phi_4)_a^\alpha = (\mathbf{3},\mathbf{2})_{-\frac{5}{6}}, \quad (\phi_5)_\alpha^a = (\overline{\mathbf{3}},\mathbf{2})_{\frac{5}{6}}$
$\Phi^{\{ABC\}}(\mathbf{35})$	$\phi_1^{\{abc\}} = (\mathbf{1},\mathbf{4})_{-\frac{3}{2}}, \quad \phi_2^{\{ab\}\alpha} = (\mathbf{3},\mathbf{3})_{\frac{2}{3}}$, $\phi_3^{\{\alpha\beta\}a} = (\mathbf{6},\mathbf{2})_{-\frac{1}{6}}, \quad \phi_4^{\{\alpha\beta\gamma\}} = (\mathbf{10},\mathbf{1})_{-1}$
$\Phi^{\{AB\}C}(\mathbf{40})$	$\phi_1^a = (\mathbf{1},\mathbf{2})_{\frac{3}{2}}, \quad \phi_2^\alpha = (\mathbf{3},\mathbf{1})_{\frac{2}{3}}, \quad \phi_3^{\{ab\}\alpha} = (\mathbf{3},\mathbf{3})_{\frac{2}{3}},$ $(\phi_4)_\alpha^a = (\overline{\mathbf{3}},\mathbf{2})_{-\frac{1}{6}}, \quad \phi_5^{\{\alpha\beta\}a} = (\mathbf{6},\mathbf{2})_{-\frac{1}{6}}, \quad (\phi_6)_\beta^\alpha = (\mathbf{8},\mathbf{1})_{-1}$
$\Phi_{[BC]}^A(\mathbf{45})$	$\phi_1^a = (\mathbf{1},\mathbf{2})_{-\frac{1}{2}}, \quad \phi_2^\alpha = (\mathbf{3},\mathbf{1})_{-\frac{4}{3}}, \quad \phi_3^{a\alpha} = (\mathbf{3},\mathbf{2})_{\frac{7}{6}}$, $(\phi_4)_\beta^{\alpha a} = (\mathbf{8},\mathbf{2})_{-\frac{1}{2}}, \quad (\phi_5)_\alpha = (\overline{\mathbf{3}},\mathbf{1})_{\frac{1}{3}},$ $(\phi_6)_{b\alpha}^a = (\overline{\mathbf{3}},\mathbf{3})_{\frac{1}{3}}, \quad \phi_7^{\{\alpha\beta\}} = (\mathbf{6},\mathbf{1})_{\frac{1}{3}}$
$\Phi_{[AB][CD]}(\mathbf{50})$	$\phi_1 = (\mathbf{1},\mathbf{1})_{-2}, \quad \phi_2^\alpha = (\mathbf{3},\mathbf{1})_{-\frac{1}{3}}, \quad (\phi_3)_\alpha^a = (\overline{\mathbf{3}},\mathbf{2})_{-\frac{7}{6}}$, $(\phi_4)^{[\alpha\beta]} = (\mathbf{6},\mathbf{1})_{\frac{4}{3}}, \quad (\phi_5)_{[\alpha\beta][ab]} = (\overline{\mathbf{6}},\mathbf{3})_{-\frac{1}{3}}, \quad (\phi_6)_{\alpha a}^\beta = (\mathbf{8},\mathbf{2})_{\frac{1}{2}},$
$\Phi_C^{\{AB\}}(\mathbf{70})$	$\phi_1^a = (\mathbf{1},\mathbf{2})_{\frac{1}{2}}, \quad \phi_2^\alpha = (\mathbf{3},\mathbf{1})_{-\frac{1}{3}}, \quad (\phi_3)_c^{\{ab\}} = (\mathbf{1},\mathbf{4})_{\frac{1}{2}}$, $(\phi_4)_b^{a\alpha} = (\mathbf{3},\mathbf{3})_{-\frac{1}{3}}, \quad (\phi_5)_\alpha^{\{ab\}} = (\overline{\mathbf{3}},\mathbf{3})_{\frac{4}{3}},$ $(\phi_6)_\beta^{\alpha a} = (\mathbf{8},\mathbf{2})_{\frac{1}{2}}, \quad (\phi_7)_a^{\{\alpha\beta\}} = (\mathbf{6},\mathbf{2})_{-\frac{7}{6}}, \quad (\phi_8)_\gamma^{\{\alpha\beta\}} = (\mathbf{15},\mathbf{1})_{-\frac{1}{3}}$
$\Phi_{[CD]}^{[AB]}(\mathbf{75})$	$(\phi_1)_\alpha = (\overline{\mathbf{3}},\mathbf{1})_{-\frac{5}{3}}, \quad \phi_2^\alpha = (\mathbf{3},\mathbf{1})_{\frac{5}{3}}, \quad \phi_3^{\alpha a} = (\mathbf{3},\mathbf{2})_{-\frac{5}{6}}$, $(\phi_4)_{\alpha a} = (\overline{\mathbf{3}},\mathbf{2})_{\frac{5}{6}}, \quad \phi_5^{\{\alpha\beta\}a} = (\overline{\mathbf{6}},\mathbf{2})_{-\frac{5}{6}}, \quad (\phi_6)_{\{\alpha\beta\}a} = (\mathbf{6},\mathbf{2})_{\frac{5}{6}},$ $\phi_7 = (\mathbf{1},\mathbf{1})_0, \quad (\phi_8)_\beta^\alpha = (\mathbf{8},\mathbf{1})_0, \quad (\phi_9)_{\beta b}^{\alpha a} = (\mathbf{8},\mathbf{3})_0$

Table D.2: Vacuum polarization coefficients b_{ij}. b_{ij} which are not listed here are zero.

Reps.	b_{ij}
$\mathbf{15}+\overline{\mathbf{15}}$	$b_{13}=b_{\overline{13}}=3$ $b_{23}=b_{\overline{23}}=4$
$\mathbf{35}+\overline{\mathbf{35}}$	$b_{12}=b_{\overline{12}}=6$ $b_{23}=b_{\overline{23}}=12$ $b_{34}=b_{\overline{34}}=10$
$\mathbf{40}+\overline{\mathbf{40}}$	$b_{12}=b_{13}=b_{\overline{12}}=b_{\overline{13}}=3/2$ $b_{25}=b_{35}=b_{\overline{25}}=b_{\overline{35}}=3$ $b_{24}=b_{\overline{24}}=1/2$ $b_{34}=b_{\overline{34}}=9/2$ $b_{46}=b_{56}=b_{\overline{46}}=b_{\overline{56}}=4$
$\mathbf{45}+\overline{\mathbf{45}}$	$b_{12}=b_{\overline{12}}=4/3$ $b_{15}=b_{\overline{15}}=1/3$ $b_{16}=b_{35}=b_{37}=b_{\overline{16}}=b_{\overline{35}}=b_{\overline{37}}=2$ $b_{24}=b_{45}=b_{\overline{24}}=b_{\overline{45}}=8/3$ $b_{36}=b_{\overline{36}}=3$ $b_{46}=b_{47}=b_{\overline{46}}=b_{\overline{47}}=4$
$\mathbf{50}+\overline{\mathbf{50}}$	$b_{13}=b_{\overline{13}}=2$ $b_{23}=b_{\overline{23}}=3$ $b_{26}=b_{\overline{26}}=4$ $b_{35}=b_{\overline{35}}=6$ $b_{46}=b_{\overline{46}}=8$ $b_{56}=b_{\overline{56}}=12$
$\mathbf{70}+\overline{\mathbf{70}}$	$b_{12}=b_{15}=b_{26}=b_{35}=b_{\overline{12}}=b_{\overline{15}}=b_{\overline{26}}=b_{\overline{35}}=2$ $b_{14}=b_{\overline{14}}=1$ $b_{27}=b_{\overline{27}}=3$ $b_{34}=b_{46}=b_{\overline{34}}=b_{\overline{46}}=4$ $b_{47}=b_{\overline{47}}=6$ $b_{56}=b_{\overline{56}}=8$ $b_{68}=b_{\overline{68}}=10$ $b_{78}=b_{\overline{78}}=5$

Appendix E

Radiative Corrections at One-loop

In this appendix, we give the explicit formulae of the loop integrals in terms of supergraphs. All the external momenta of the chiral (anti-chiral) superfields are set to be p, and the masses of the MSSM vector superfields are set to be μ_{IR} in order to regularize the IR divergence. For simplicity, we set all coupling constants to be 1 through this appendix. For the corrections to the three-point vertex functions and the box-like corrections, the loop integrals in text are the coefficients of Kähler potentials in the limit that the external momenta p^2 vanishes.

Radiative Corrections to Two-Point Functions for Matter Superfields

The correction to the self energy of the chiral and anti-chiral matter superfields in the first generation is induced by the gauge interactions. The one-loop contribution is given as

$$\begin{aligned} i\Gamma_\Phi &= i^2 \int d^4\theta_1 d^4\theta_2 \int \frac{d^D l}{(2\pi)^D} \frac{-i}{2(l^2 - M^2)} \frac{i}{(l+p)^2} \frac{1}{16} (\mathcal{D}_2^2 \delta_{21} \overleftarrow{\overline{\mathcal{D}}_1^2}) \delta_{12} \Phi(p, \theta_1) \Phi^\dagger(p, \theta_2) \\ &= -\frac{1}{2} \int \frac{d^D l}{(2\pi)^D} \frac{1}{l^2 - M^2} \frac{1}{(l+p)^2} \int d^4\theta \Phi^\dagger(p, \theta) \Phi(p, \theta) \,, \end{aligned} \tag{E.1}$$

where p is external momentum and M is the mass for the internal vector superfield. δ_{ij} denotes the δ-function for the Grassmann valuable, $\delta_{ij} \equiv (\delta_i - \delta_j)^2 (\overline{\delta}_i - \overline{\delta}_j)^2$. The renormalized one-loop two-point function of matter superfields in the $SU(5)$ GUTs are given as

$$\Gamma_\Phi = -\frac{g_5^2}{8\pi^2} \left[(c_5^\Phi - \sum_{n=1}^{3} c_n^\Phi) f(M_X^2) + \sum_{n=1}^{3} c_n^\Phi f(\mu_{\mathrm{IR}}^2) \right] \int d^4\theta \Phi^\dagger \Phi \,, \tag{E.2}$$

where function $f(M^2)$ is defined in Eq. (3.18). c_5^Φ and c_n^Φ ($n = 3, 2, 1$) are the quadratic Casimir defined in text. In the MSSM, we also obtain

$$\Gamma_\Phi^{\mathrm{EFT}} = -\frac{g_5^2}{8\pi^2} \sum_{n=1}^{3} c_n^\Phi f(\mu_{\mathrm{IR}}^2) \int d^4\theta \Phi^\dagger \Phi \,. \tag{E.3}$$

Radiative Corrections to Two-Point Function for Vector Superfield

Three diagrams in Fig. 3.2 contribute to the radiative corrections to two-point functions from the (massive) chiral superfields. The corrections from the diagram (a) in Fig. 3.2 are

$$i\Gamma^{(a)}_{XX} = -i^2 \int d^4\theta X^{\dagger\alpha}_r(-p,\theta) \int \frac{d^D l}{(2\pi)^D} \frac{l^2 - \frac{1}{2} l_\mu \sigma^\mu_{\alpha\dot{\alpha}} \mathcal{D}^\alpha \overline{\mathcal{D}}^{\dot{\alpha}} + \frac{1}{16}\mathcal{D}^2\overline{\mathcal{D}}^2}{(l^2 - M_1^2)[(l+p)^2 - M_2^2]} X^r_\alpha(p,\theta)\,, \tag{E.4}$$

where M_1 and M_2 are the masses of the chiral superfields in the loop diagram. After picking the transverse mode and regularizing the UV divergence, we obtain the finite correction to the two-point function as follows:

$$\Gamma^{(a)}_{XX} = \frac{1}{16\pi^2} B(p^2, M_1^2, M_2^2) \int d^4\theta X^{\dagger\alpha}_r(-p,\theta) P_T X^r_\alpha(p,\theta) + (\text{longitudinal mode})\,, \tag{E.5}$$

where the loop function is defined in Eq. (3.24). The massive chiral superfields also have the non-zero contribution from the diagram (b) in Fig. 3.2,

$$\Gamma^{(b)}_{XX} = -\frac{M^2}{16\pi^2}\left(1 - \ln\frac{M^2}{\mu^2}\right) \int d^4\theta X^{\dagger\alpha}_r(-p,\theta) P_T X^r_\alpha(p,\theta)\,, \tag{E.6}$$

where M is for the masses of chiral superfields running in the internal line. The third contribution (the diagram (c) in Fig. 3.2) comes from the vertex which includes the VEV of the adjoint Higgs superfield,

$$\Gamma^{(c)}_{XX} = \frac{1}{16\pi^2} A(p^2, M_1^2, M_2^2) \int d^4\theta X^{\dagger\alpha}_r(-p,\theta) P_T X^r_\alpha(p,\theta)\,. \tag{E.7}$$

Here, the loop function A is also defined in Eq. (3.24).

Radiative Corrections to Three-Point Vertices

The one-loop diagrams for the three-point vertex correction are shown in Fig. 3.4. In our momentum assignment, the momentum of the X boson is $q = 0$. The one-loop vertex correction induced by the diagram in Fig. 3.4 (a) is given as

$$\begin{aligned} i\Gamma^{(v)}_1(p;M) = i^3 \int d^4\theta_1 d^4\theta_2 d^4\theta_3 \int \frac{d^D l}{(2\pi)^D} \frac{i}{(l+p)^2} \frac{i}{(l+p)^2} \frac{-i}{2(l^2 - M^2)} \\ \times \frac{1}{16}(\overline{\mathcal{D}}_2^2 \delta_{23} \overleftarrow{\mathcal{D}}_3^2) \frac{1}{16}(\overline{\mathcal{D}}_3^2 \delta_{31} \overleftarrow{\mathcal{D}}_1^2) \delta_{12} \Phi(\theta_1)\Phi^\dagger(\theta_2) V(\theta_3)\,. \end{aligned} \tag{E.8}$$

By integrating by part and also using the $\mathcal{D}$ algebra, we always decompose the vertex correction into the effective Kähler terms $\mathcal{K}$ and the auxiliary terms which vanish as $\mathcal{D}_\alpha\Phi, \overline{\mathcal{D}}_{\dot{\alpha}}\Phi^\dagger = 0$. The effective Kähler term induced by the diagram Fig. 3.4 (a) has the following form:

$$i\mathcal{K}^{(v)}_1(p;M) = \frac{1}{2} \int \frac{d^D l}{(2\pi)^D} \frac{1}{[(l+p)^2]^2} \frac{1}{l^2 - M^2} (l+2p)^2 \Phi\Phi^\dagger V\,, \tag{E.9}$$

where we remove the Grassmann valuables in the effective Kähler term, for simplicity.

Next we show the effective Kähler term described in Fig. 3.4 (b) and (c). In our momentum assignment, the diagrams both of Fig. 3.4 (b) and (c) give the same expression, and we find the one-loop vertex correction and the effective Kähler term as

$$\begin{aligned} i\Gamma_2^{(\mathrm{v})}(p;M) &= i^2 \int d^4\theta_1 d^4\theta_2 \int \frac{d^D l}{(2\pi)^D} \frac{i}{(l+p)^2} \frac{-i}{2(l^2-M^2)} \\ &\quad \times \frac{1}{16} \overline{\mathcal{D}}_2^2 \delta_{21} \overleftarrow{\mathcal{D}_1^2} \delta_{12} \Phi(\theta_1) \Phi^\dagger(\theta_2) V(\theta_2), \\ i\mathcal{K}_2^{(\mathrm{v})}(p;M) &= -\frac{1}{2} \int \frac{d^D l}{(2\pi)^D} \frac{1}{(l+p)^2} \frac{1}{l^2-M^2} \Phi\Phi^\dagger V \,. \end{aligned} \tag{E.10}$$

The diagrams (d) and (e) in Fig. 3.4 include the three-point vertices of vector superfields. After carrying out the superspace integral, the vertex corrections from the diagrams Fig. 3.4(d) and (e) are obtained as

$$\begin{aligned} i\Gamma_3^{(\mathrm{v})}(p;M) &= -4 \int \frac{d^D l}{(2\pi)^D} \frac{1}{l^2-\mu_{\mathrm{IR}}^2} \frac{1}{(l+p)^2} \frac{(l+p)^2+p^2}{l^2-M^2} \int d^4\theta \; \Phi^\dagger \Phi V, \\ i\Gamma_4^{(\mathrm{v})}(p;M) &= 8 \int \frac{d^D l}{(2\pi)^D} \frac{1}{l^2-M^2} \frac{1}{l^2-\mu_{\mathrm{IR}}^2} \int d^4\theta \; \Phi^\dagger \Phi V \,. \end{aligned} \tag{E.11}$$

Since they do not include the auxiliary terms, $\Gamma_n^{(\mathrm{v})}(p;M)$ $(n=3,4)$ is just the Kähler term $\int d^4\theta \mathcal{K}_n^{(\mathrm{v})}(p;M)$ $(n=3,4)$.

The contribution from a diagram (f) in Fig. 3.4 is zero as mentioned in the text.

Box-like Corrections

Now we show the effective Kähler terms from the box-like diagrams presented in Fig. 3.5. These diagrams include one massless and one massive vector superfields. The correction from the box diagram (Fig. 3.5(a)) is given as:

$$i\Gamma_{\mathrm{box}}(p;M) \equiv p^2 \int \frac{d^D q}{(2\pi)^D} \frac{1}{q^2-M^2} \frac{1}{(q+p)^2} \frac{1}{q^2-\mu_{\mathrm{IR}}^2} \frac{1}{(q-p)^2} \int d^4\theta \Phi_1^\dagger \Phi_2 \Phi_3^\dagger \Phi_4 \,. \tag{E.12}$$

Here, we do not write the external momenta of external superfields for simplicity since we set them to be the same momentum p. As mentioned above, we set the mass of massless vector superfields to be μ_{IR} as IR regularization. $\Gamma_{\mathrm{box}}(p;M)$ vanishes at the point with $p^2 = 0$, as mentioned in the text.

The contribution of the crossing-box diagram (Fig. 3.5(b)) is given by

$$i\Gamma_{\rm cross}(p;M) \equiv \frac{1}{4}\int \frac{d^Dq}{(2\pi)^D}\frac{1}{q^2-M^2}\frac{1}{[(q-p)^2]^2}\frac{1}{q^2-\mu_{\rm IR}^2}\int d^4\theta \Big[(q-p)^2\Phi_1^\dagger\Phi_2\Phi_3^\dagger\Phi_4 + \frac{1}{2}(q-p)_\mu(\overline{\sigma}^\mu\overline{\mathcal{D}}\mathcal{D})\left(\Phi_1^\dagger\Phi_4\right)\Phi_2\Phi_3^\dagger + \frac{1}{16}\overline{\mathcal{D}}^2\mathcal{D}^2\left(\Phi_1^\dagger\Phi_4\right)\Phi_3^\dagger\Phi_2\Big]. \tag{E.13}$$

Here, we define the mnemonic symbol $(\overline{\sigma}^\mu\overline{\mathcal{D}}\mathcal{D}) \equiv (\overline{\sigma}^\mu)^{\dot{\alpha}\alpha}\overline{\mathcal{D}}_{\dot{\alpha}}\mathcal{D}_\alpha$. This correction has the auxiliary terms. The corresponding Kähler term is given by removing the auxiliary terms as

$$i\mathcal{K}_{\rm cross}(p;M) = \frac{1}{4}\int \frac{d^Dq}{(2\pi)^D}\frac{(q-2p)^2}{(q^2-M^2)[(q-p)^2]^2(q^2-\mu_{\rm IR}^2)}\Phi_1^\dagger\Phi_2\Phi_3^\dagger\Phi_4 . \tag{E.14}$$

Finally, we show the contribution from the triangle diagram in Fig. 3.5(c). The correction from the triangle diagram is obtained as follows:

$$i\Gamma_{\rm triangle}(p;M) \equiv -\frac{1}{4}\int \frac{d^Dq}{(2\pi)^D}\frac{1}{q^2-M^2}\frac{1}{(q+p)^2}\frac{1}{q^2-\mu_{\rm IR}^2}\int d^4\theta\Phi_1^\dagger\Phi_2\Phi_3^\dagger\Phi_4 . \tag{E.15}$$

Since auxiliary terms are not included in the radiative corrections $\Gamma_{\rm box}$ and $\Gamma_{\rm triangle}$, the corresponding Kähler terms are just written by these corrections as $\Gamma_n = \int d^4\theta\ \mathcal{K}_n$ ($n =$ box, triangle).

The diagram in Fig. 3.5(d) vanishes as mentioned in the text.

One-loop Corrections in EFT

In the last of this appendix, we show the radiative corrections in EFT presented in Fig. 3.6. We obtain the one-loop effective vertex functions $\Gamma_1^{\rm EFT}, \Gamma_2^{\rm EFT}$, and $\Gamma_3^{\rm EFT}$ which correspond to the diagram Fig. 3.6 (b), (c), and (a), respectively, as follows:

$$\begin{aligned}
i\Gamma_1^{\rm EFT}(p;\mu_{\rm IR}) &= \frac{1}{2}\int \frac{d^Dq}{(2\pi)^D}\frac{1}{[(q+p)^2]^2}\frac{1}{q^2-\mu_{\rm IR}^2}\int d^4\theta\Big[(q+p)^2\Phi_1^\dagger\Phi_2\Phi_3^\dagger\Phi_4 \\
&\quad + \frac{1}{2}(q+p)_\mu(\overline{\sigma}^\mu\overline{\mathcal{D}}\mathcal{D})\left(\Phi_1^\dagger\Phi_2\right)\Phi_3^\dagger\Phi_4 + \frac{1}{16}\overline{\mathcal{D}}^2\mathcal{D}^2\left(\Phi_1^\dagger\Phi_2\right)\Phi_3^\dagger\Phi_4\Big], \\
i\Gamma_2^{\rm EFT}(p;\mu_{\rm IR}) &= -\frac{1}{2}\int \frac{d^Dq}{(2\pi)^D}\frac{1}{q^2-\mu_{\rm IR}^2}\frac{1}{(q+p)^2}\int d^4\theta\Phi_1^\dagger\Phi_2\Phi_3^\dagger\Phi_4 , \\
i\Gamma_3^{\rm EFT}(p;\mu_{\rm IR}) &= \frac{1}{2}\int \frac{d^Dq}{(2\pi)^D}\frac{1}{[(q+p)^2]^2}\frac{1}{q^2-\mu_{\rm IR}^2}(2p)^2\int d^4\theta\Phi_1^\dagger\Phi_2\Phi_3^\dagger\Phi_4 .
\end{aligned} \tag{E.16}$$

The momentum assignment is the same as in calculation of the box-like diagrams. The corresponding Kähler terms are given by removing the auxiliary terms as

$$
\begin{aligned}
i\mathcal{K}_1^{\rm EFT}(p;\mu_{\rm IR}) &= \frac{1}{2}\int\frac{d^Dq}{(2\pi)^D}\frac{(q+2p)^2}{[(q+p)^2]^2(q^2-\mu_{\rm IR}^2)}\Phi_1^\dagger\Phi_2\Phi_3^\dagger\Phi_4,\\
i\mathcal{K}_2^{\rm EFT}(p;\mu_{\rm IR}) &= -\frac{1}{2}\int\frac{d^Dq}{(2\pi)^D}\frac{1}{q^2-\mu_{\rm IR}^2}\frac{1}{(q+p)^2}\Phi_1^\dagger\Phi_2\Phi_3^\dagger\Phi_4\,,\\
i\mathcal{K}_3^{\rm EFT}(p;\mu_{\rm IR}) &= 2p^2\int\frac{d^Dq}{(2\pi)^D}\frac{1}{[(q+p)^2]^2}\frac{1}{q^2-\mu_{\rm IR}^2}\Phi_1^\dagger\Phi_2\Phi_3^\dagger\Phi_4\,.
\end{aligned}
\tag{E.17}
$$

Here, we skip over the ways in which we obtain the effective vertex functions from the diagrams since these structures are similar as mentioned above. $i\mathcal{K}_3^{\rm EFT}(p;\mu_{\rm IR})$ vanishes when $p^2=0$ is set, as mentioned in the text.

Appendix F

Anomalous Dimensions via Effective Kähler Potential

In this chapter, we discuss anomalous dimensions for higher-dimensional operators in the supersymmetric effective theories. Let us consider the non-renormalizable Kähler potential.

$$\mathcal{K} = \mathcal{K}_0 + \Delta\mathcal{K}\,, \tag{F.1}$$

with the canonical part $\mathcal{K}_0$ and the non-renormalizable part $\Delta\mathcal{K}$.

$$\begin{aligned} \mathcal{K}_0 &= \sum_{\Phi} \overline{\Phi}_a \left(e^{2gV_G^I t^I}\right)^a_{\ b} \Phi^b\,, \\ \Delta\mathcal{K} &= (\lambda_A^{a_1\cdots a_m}\overline{\Phi}_{a_1}\cdots\overline{\Phi}_{a_m}) \left(e^{2gV_G^I T^I}\right)^A_{\ B} (\lambda^B_{b_1\cdots b_n}\Phi^{b_1}\cdots\Phi^{b_n})\,. \end{aligned} \tag{F.2}$$

Here, $\{a_1, \cdots, a_m\}$ and $\{b_1, \cdots, b_n\}$ denote gauge indices, but we do not restrict them to fundamental ones. $I = 1, \cdots, \dim G$ represents the indices of gauge generators. The generic two-loop effective Kähler potential is calculated by using the background field method. In this chapter, we write anti-chiral superfield as a superfield with bar. In the previous work [137], the chiral and anti-chiral superfields are expanded around background superfields ϕ and $\overline{\phi}$, respectively. $\Phi, \overline{\Phi}$, and V denote quantum superfields. We can include also a generic superpotential W and a generic gauge kinetic function f_{IJ} if needed.

It is convenient to use the geometric structure in the generic Kähler potential. A Kähler metric is defined as the second-derivative of the Kähler potential,

$$G^a_{\ b} \equiv \mathcal{K}^a_{\ b} = \frac{\partial^2}{\partial\overline{\phi}_a \partial\phi^b}\mathcal{K}\,. \tag{F.3}$$

G^{-1} indicates the inverse of Kähler metric. Given the metric, we construct the geometric quantities such as the connection

$$\Gamma_{ab}^{\ \ c} = \mathcal{K}_{ab}^{\ \ d}(G^{-1})_d^c\,, \qquad \overline{\Gamma}^{ab}_{\ \ c} = \mathcal{K}^{ab}_{\ \ d}(G^{-1})^d_c\,, \tag{F.4}$$

and the curvature,

$$R^{a\ b}_{\ c\ d} \equiv \mathcal{K}^{ab}_{cd} - \mathcal{K}^{ab}_{\ \ e}(G^{-1})^e_f \mathcal{K}_{cd}^{\ \ f}\,. \tag{F.5}$$

In the background field method, gauge symmetries are spontaneously broken. All superfields obtain masses as function of background superfields. For vector superfields, the mass matrix $(M_V^2)_{IJ}$ consists of the mass matrix for ghost superfield M_C^2 and its transpose.

$$(M_C^2)_{IJ} \equiv 2g_I g_J \overline{\phi}_a (t_I)^a_b G^b_c (t_J)^c_d \phi^d\,, \qquad M_V^2 = \frac{1}{2}\left(M_C^2 + M_C^{2\,T}\right)\,. \tag{F.6}$$

Here, the summation symbols for the constituent superfields ϕ and gauge groups are implicit. The mass matrix for chiral and anti-chiral superfields arises from the superpotential contribution M_W^2 and the Kähler potential contribution M_G^2.

$$M^2 = M_G^2 + M_W^2\,, \tag{F.7}$$

with

$$\begin{aligned}(M_G^2)^b_a &= 2g_I g_J (t_I\phi)^b (h^{-1})^{IJ} (\overline{\phi} t_J G)_a\,,\\ (M_W^2)^b_a &= (G^{-1})^b_c \overline{W}^{cd} (G^{-1})^e_d W_{ea}\,.\end{aligned} \tag{F.8}$$

Here, h_{IJ} is the sum of gauge kinetic functions, which is Kronecker delta in the limit of renormalizable theories.

$$h_{IJ} = \frac{1}{2}(f_{IJ} + \overline{f}_{IJ}) \sim \frac{1}{g_I^2}\delta_{IJ} \tag{F.9}$$

In this chapter, we focus only on non-renormalized Kähler terms, so that $h_{IJ} \sim \delta_{IJ}$.

One- and Two-loop Effective Kähler Potential

One- and two-loop effective Kähler potentials have been already given in Ref. [137]. The one-loop effective Kähler potential is

$$\mathcal{K}_{1L} = -\frac{1}{16\pi^2}\mathrm{Tr}\, h^{-1}M_C^2\left(2 - \ln\frac{h^{-1}M_C^2}{\overline{\mu}^2}\right) + \frac{1}{32\pi^2}\mathrm{tr}\, M_W^2\left(2 - \ln\frac{M_W^2}{\overline{\mu}^2}\right)\,. \tag{F.10}$$

Here, "Tr" denotes trace of gauge adjoint indices $I, J, \cdots$, and "tr" denotes trace of gauge indices $a, b, \cdots$ associated with the chiral and anti-chiral superfields. We define $\overline{\mu} \equiv \mu e^{-4\pi\gamma}$ with the renormalization scale μ and the Euler's constant γ.

Two-loop effective Kähler potential is *

$$
\begin{aligned}
\mathcal{K}_{2L} =& \frac{1}{2} R^{a\ b}_{\ c\ d} \overline{J}^{c\ d}_{\ a\ b}(M^2) + \frac{1}{6} \overline{W}^{;abc} W_{def} \overline{I}^{d\ e\ f}_{\ a\ b\ c}(M^2, M^2, M^2) \\
&- \frac{1}{2} h_{LP} f^{P}_{IN} h_{JQ} f^{Q}_{KM} \left[\overline{I}^{IJKLMN}(M_C^2, M_C^2, M_V^2) - \overline{I}^{IJKLMN}(M_C^2, (M_C^2)^T, M_V^2) \right] \\
&- (G t_I \phi)^a{}_{;b} (\overline{\phi} t_J G)_d^{;c} \overline{I}^{d\ b\ IJ}_{\ a\ c}(M^2, M^2, M_V^2) \\
&- h_{LP} f^{P}_{IN} h_{JQ} f^{Q}_{KM} \overline{I}^{IJKLMN}(M_V^2, M_V^2, M_V^2) \,.
\end{aligned}
\tag{F.11}
$$

Here, J and I denotes the loop integrals which are given in Ref. [137].

Two-loop Effective Kähler Potential for Non-renormalizable Operators

We reduce the formulae for the two-loop effective Kähler potential to derive the two-loop RGEs. We only consider two-loop RGEs via gauge interactions, and hence the superpotential is zero ($W = 0$). We divide the tree-level Kähler potential as follows,

$$
\mathcal{K} = \overline{\phi}_a \phi^a + \Delta\mathcal{K} \,. \tag{F.12}
$$

$\Delta\mathcal{K}$ denotes the higher dimensional operators. We neglect the higher order terms of $\Delta\mathcal{K}$ since it represents much higher dimensional operators. The Kähler metric and its inverse are easily found as

$$
G^a_b = \delta^a_b + (\Delta\mathcal{K})^a_b \,, \qquad (G^{-1})^a_b = \delta^a_b - (\Delta\mathcal{K})^a_b \,. \tag{F.13}
$$

Moreover, the Kähler curvature is also just the fourth derivative of $\Delta\mathcal{K}$.

$$
R^{a\ b}_{\ c\ d} = (\Delta\mathcal{K})^{ab}_{cd} \,. \tag{F.14}
$$

We divide all mass matrices into the canonical and $\Delta\mathcal{K}$ contributions. Firstly, for the ghost mass matrices, we have

$$
M_C^2 \equiv M_{C0}^2 + \Delta M_C^2 \,, \tag{F.15}
$$

with

$$
(M_{C0}^2)_{IJ} = 2 g_I g_J \overline{\phi}_a (t_I)^a_b (t_J)^b_c \phi^c \,, \qquad (\Delta M_C^2)_{IJ} = g_I g_J \overline{\phi}_a (t_I)^a_b (\Delta\mathcal{K})^b_c (t_J)^c_d \phi^d \,. \tag{F.16}
$$

*If one consider the non-trivial gauge kinetic function, there is an additional term with derivatives of gauge kinetic functions with respect to ϕ and $\overline{\phi}$.

Secondly, for the mass matrices of (anti-)chiral superfields, we get

$$M^2 = M_0^2 + \Delta M^2 \,, \tag{F.17}$$

with

$$(M_0^2)_a^b = 2g_I^2 (t_I\phi)^b (\overline{\phi} t_I)_a \,, \quad (\Delta M^2)_a^b = 2g_I^2 (t_I\phi)^b (\overline{\phi} t_I \Delta\mathcal{K})_a \,. \tag{F.18}$$

Here, $M^2 = M_G^2$ since we assume the superpotential is vanished.

In order to compute the two-loop RGEs, it is sufficient to consider only $\ln\overline{\mu}$ term. We obtain the first line in Eq. (F.11) as follows.

$$\frac{1}{2} R^{a\ b}_{\ c\ d} \overline{J}^{c\ d}_{\ a\ b}(M^2)\Big|_{\ln\overline{\mu}^2} = \frac{4g_I^2 g_J^2}{(16\pi^2)^2} (\ln\overline{\mu}^2) (\Delta\mathcal{K})^{ab}_{cd} (\overline{\phi} t_I)_a (\overline{\phi} t_J)_b (t_I\phi)^c (t_J\phi)^d \,. \tag{F.19}$$

Moreover, it is easy to find that the second line in Eq. (F.11) is vanished and the fourth line is

$$\begin{aligned} &g_L^{-2} g_J^{-2} f_{LIN} f_{JKM}\, \overline{I}^{IJKLMN}(M_V^2, M_V^2, M_V^2)\Big|_{\ln\overline{\mu}^2} \\ &= \frac{6g_I^4}{(16\pi^2)^2} (\ln\overline{\mu}^2) C(I) \left(2C_I(\phi) \overline{\phi}_a \phi^a + C_I^{\text{comp.}}(\Delta\mathcal{K})\Delta\mathcal{K} \right) \,. \end{aligned} \tag{F.20}$$

Here, we use

$$\begin{aligned} &(\overline{\phi} t_I)_a (\Delta\mathcal{K})^a_b (t_I\phi)^b \\ &= \left(\sum_a \lambda_A^{a_1\cdots a\cdots a_m} (t_I)_a^{a'} \right) \left(\sum_b \lambda^A_{b_1\cdots b\cdots b_n} (t_I)^b_{b'} \right) (\overline{\phi}_{a_1} \cdots \overline{\phi}_{a'} \cdots \overline{\phi}_{a_m})(\phi^{b_1} \cdots \phi^{b'} \cdots \phi^{b_n}) \,, \end{aligned} \tag{F.21}$$

and group theoretical arguments

$$\begin{aligned} (T_I)^B_A \lambda_B^{a_1\cdots a'\cdots a_m} &= \sum_a \lambda_A^{a_1\cdots a\cdots a_m} (t_I)_a^{a'} \,, \\ (T_I)^B_A \lambda_B^{b_1\cdots b'\cdots b_m} &= \sum_b \lambda^A_{b_1\cdots b\cdots b_n} (t_I)^b_{b'} \,, \\ (T_I)^B_A (T_I)^A_C &= C_I^{\text{comp.}} \delta^B_C \,. \end{aligned} \tag{F.22}$$

$C_I^{\text{comp.}}$ represents the quadratic Casimir invariant of composite operator $\lambda^B_{b_1\cdots b_n}\phi^{b_1}\cdots\phi^{b_n}$. In particular, the following relation is satisfied for any composite operators.

$$C_I^{\text{comp.}} \Delta\mathcal{K} = (\overline{\phi} t_I)_a (\Delta\mathcal{K})^a_b (t_I\phi)^b \,. \tag{F.23}$$

At last of this section, we subtract the $\ln \overline{\mu}^2$ terms from the third term in Eq. (F.11). The situation for this part is more complicated than that for other part. M_G^2 is not an Hermitian operator, so that we redefine the superpropagator along the following procedure. E and $\overline{E}$ represent the square root of Kähler metric defined as

$$G^a_b = \overline{E}^a_c E^c_b \,. \tag{F.24}$$

The quadratic Kähler term for Φ is written under this field re-definition as follows.

$$S^2_\Phi = \int d^8 z\, \overline{\Phi}_a \overline{E}^a_x \left[\delta^x_y - (M'^2_G)^x_y \frac{1}{\Box} \right] E^y_b \Phi^b \,, \tag{F.25}$$

The mass matrix is also re-defined with vielbein of Kähler metric as follows.

$$(M'^2_G)^x_y \equiv 2(Et_I\phi)^x (h^{-1})_{IJ} (\overline{\phi} t_J \overline{E})_y \,. \tag{F.26}$$

Due to the re-definition of the superpropagator, the third line of Eq. (F.11) changes as follows.

$$-(Gt_I\phi)^a{}_{;b} (\overline{\phi} t_J G)_d^{;c} (E^{-1})^b_x (E^{-1})^d_y \overline{I}'^{y\ x\ IJ}_{a\ c}(M'^2, M'^2, M_V^2) \,. \tag{F.27}$$

Here, the $\ln \overline{\mu}$ term of loop integral $\overline{I}'$ is given as

$$\begin{aligned}
&\overline{I}'^{y\ x\ IJ}_{a\ c}(M'^2, M'^2, M_V^2)\Big|_{\ln \overline{\mu}^2} \\
&= \frac{1}{2} \frac{g_I g_J}{(16\pi^2)^2} \ln \overline{\mu}^2 \Big\{ (-4M'^2 \overline{E}^{-1} + 2M'^2 \ln M'^2 \overline{E}^{-1})^y_a (\overline{E}^{-1})^x_c (h^{-1})^{IJ} \\
&+ (\overline{E}^{-1})^y_a (-4M'^2 \overline{E}^{-1} + 2M'^2 \ln M'^2 \overline{E}^{-1})^x_c (h^{-1})^{IJ} \\
&+ (\overline{E}^{-1})^y_a (\overline{E}^{-1})^x_c (-4M_V^2 h^{-1} + 2M_V^2 \ln M_V^2 h^{-1})^{IJ} \Big\} \,.
\end{aligned} \tag{F.28}$$

Using the following relation,

$$\begin{aligned}
(\overline{E}^{-1})^w_a (M'^2)^x_z (E^{-1})^b_z &= 2(t_I\phi)^b (h^{-1})_{IJ} (\overline{\phi} t_J)_a \,, \\
(\overline{E}^{-1})^w_a (\Delta M'^2)^x_z (E^{-1})^b_z &= (\Delta\mathcal{K})^b_c (t_I\phi)^c (h^{-1})_{IJ} (\overline{\phi} t_J)_a + (t_I\phi)^b (h^{-1})_{IJ} (\overline{\phi} t_J)_c (\Delta\mathcal{K})^c_a,
\end{aligned} \tag{F.29}$$

we obtain

$$\begin{aligned}
&-(Gt_I\phi)^a{}_{;b} (\overline{\phi} t_J G)_d^{;c} (E^{-1})^b_x (E^{-1})^d_y\, \overline{I}'^{y\ x\ IJ}_{a\ c}(M'^2, M'^2, M_V^2)\Big|_{\ln \overline{\mu}^2} \\
&= \frac{1}{2} \frac{\ln \overline{\mu}^2}{(16\pi^2)^2} \Big\{ 16 g_I^2 g_J^2 C_I(\phi) C_J(\phi) \overline{\phi}_a \phi^a + 8 g_I^4 S_I C_J(\phi) \overline{\phi}_a \phi^a + 4 g_I^4 S_I C_I^{\text{comp.}}(\Delta\mathcal{K}) \Delta\mathcal{K} \\
&+ 4 g_I^2 g_J^2 \left[(\overline{\phi} t_J t_I t_I)_a (\Delta\mathcal{K})^a_b (t_J\phi)^b + (\overline{\phi} t_J)_a (\Delta\mathcal{K})^a_b (t_I t_I t_J \phi)^b \right] \\
&+ 4 g_I^2 g_J^2 \overline{\phi} (t_I t_J + t_J t_I) \phi \ \mathrm{tr} \left[t_J (\Delta\mathcal{K} t_I \phi) + t_I (\Delta\mathcal{K} \overline{\phi} t_J) \right] \\
&+ 8 g_I^2 g_J^2 (\overline{\phi} t_J)_a \left[\Delta\mathcal{K} t_I \phi + \Delta\mathcal{K} \overline{\phi} t_I \right]^a_b (t_I t_J \phi)^b \\
&+ 8 g_I^2 g_J^2 (\overline{\phi} t_J t_I)_a \left[\Delta\mathcal{K} t_I \phi + \Delta\mathcal{K} \overline{\phi} t_I \right]^a_b (t_J \phi)^b \Big\} \,.
\end{aligned} \tag{F.30}$$

Here, we define $S_I \equiv \sum_{\phi'} I_I(\phi')$. We also define the short-hand notation $(\Delta\mathcal{K}T_K\phi)^a_b$ and $(\Delta\mathcal{K}\overline{\phi}T_K)^a_b$ as follows.

$$(\Delta\mathcal{K}t_I\phi)^a_b = (\Delta\mathcal{K})^a_{bc}(t_I\phi)^c\,, \qquad (\Delta\mathcal{K}\overline{\phi}t_I)^a_b = (\Delta\mathcal{K})^{ac}_b(\overline{\phi}t_I)_c\,. \tag{F.31}$$

Finally, we obtain $\ln\mu$ derivative of the two-loop effective Kähler potential as follows.

$$\begin{aligned}
&(16\pi^2)^2\mu\frac{\mathrm{d}\mathcal{K}_{2L}}{\mathrm{d}\mu}\\
&= 8\left[(S_I - 3C_2(I))C_I(\phi)g_I^4 + 2C_I(\phi)C_J(\phi)g_I^2g_J^2\right]\overline{\phi}_a\phi^a\\
&+ 4(S_I - 3C_2(I))C_I^{\text{comp.}}(\Delta\mathcal{K})g_I^4\,\Delta\mathcal{K}\\
&+ 4g_I^2g_J^2\left[(\overline{\phi}t_Jt_It_I)_a(\Delta\mathcal{K})^a_b(t_J\phi)^b + (\overline{\phi}t_J)_a(\Delta\mathcal{K})^a_b(t_It_It_J\phi)^b\right]\\
&+ 4g_I^2g_J^2\overline{\phi}(t_It_J + t_Jt_I)\phi\ \mathrm{tr}\left[t_J(\Delta\mathcal{K}t_I\phi) + t_I(\Delta\mathcal{K}\overline{\phi}t_J)\right]\\
&+ 8g_I^2g_J^2(\overline{\phi}t_J)_a\left[\Delta\mathcal{K}t_I\phi + \Delta\mathcal{K}\overline{\phi}t_I\right]^a_b(t_It_J\phi)^b\\
&+ 8g_I^2g_J^2(\overline{\phi}t_Jt_I)_a\left[\Delta\mathcal{K}t_I\phi + \Delta\mathcal{K}\overline{\phi}t_I\right]^a_b(t_J\phi)^b\\
&+ 8g_I^2g_J^2(\Delta\mathcal{K})^{ab}_{cd}(\overline{\phi}t_I)_a(\overline{\phi}t_J)_b(t_I\phi)^c(t_J\phi)^d\,.
\end{aligned} \tag{F.32}$$

F.1 Two-Loop Anomalous Dimension

Now, we derive a general form of two-loop anomalous dimensions for higher-dimensional operators in the SUSY SM. The Callan-Symanzik equation at two-loop level is given by the following form.

$$\begin{aligned}
&\mu\frac{\partial\Gamma^{(2)}_{\Delta\mathcal{K}}}{\partial\mu} + \beta^{(1)}_I\frac{\partial\Gamma^{(1)}_{\Delta\mathcal{K}}}{\partial g_I} - \sum_\phi\left[\gamma^{(1)}_\phi\phi\frac{\partial}{\partial\phi} + \gamma^{(1)}_{\overline{\phi}}\overline{\phi}\frac{\partial}{\partial\overline{\phi}}\right]\Gamma^{(1)}_{\Delta\mathcal{K}}\\
&-\sum_\phi\left[\gamma^{(2)}_\phi\phi\frac{\partial}{\partial\phi} + \gamma^{(2)}_{\overline{\phi}}\overline{\phi}\frac{\partial}{\partial\overline{\phi}}\right]\Gamma^{(0)}_{\Delta\mathcal{K}} + \gamma^{(1)}_{\Delta\mathcal{K}}\Gamma^{(1)}_{\Delta\mathcal{K}} + \gamma^{(2)}_{\Delta\mathcal{K}}\Gamma^{(0)}_{\Delta\mathcal{K}} = 0\,.
\end{aligned} \tag{F.33}$$

Here, $\Gamma^{(i)}_{\Delta\mathcal{K}}$ represents the $\Delta\mathcal{K}$ part of the i-th loop order effective Kähler potential. Since the corresponding part of the one-loop effective Kähler potential is given by

$$\Gamma^{(1)}_{\Delta\mathcal{K}} = -\frac{1}{16\pi^2}\mathrm{Tr}\Delta M_C^2 = -\frac{1}{16\pi^2}\cdot 2g_I^2C_I^{\text{comp.}}\Delta\mathcal{K}\,. \tag{F.34}$$

The anomalous dimension for a superfield ϕ is given by

$$\begin{aligned}
\gamma^{(1)}_\phi &= -2\sum_I\frac{g_I^2}{16\pi^2}C_I(\phi)\,,\\
\gamma^{(2)}_\phi &= 2\sum_{I,J}\frac{g_I^2g_J^2}{(16\pi^2)^2}C_I(\phi)\left[b_I\delta_{IJ} + 2C_J(\phi)\right]\,,
\end{aligned} \tag{F.35}$$

where $b_I = S_I - 3C_2(I)$ is the coefficient of the one-loop beta function. Then, we obtain each parts of Eq. (F.33) combining above-listed pieces. We find the second term of Eq. (F.33) as follows,

$$\beta_I^{(1)} \frac{\partial \Gamma^{(1)}_{\Delta\mathcal{K}}}{\partial g_I} = -\frac{1}{(16\pi^2)^2} \cdot 4 b_I g_I^4 C_I^{\text{comp.}} \Delta\mathcal{K}, \tag{F.36}$$

and the third and fourth term of Eq. (F.33) as

$$\begin{aligned}
\gamma_\phi^{(1)} \phi \frac{\partial}{\partial \phi} \Gamma^{(1)}_{\Delta\mathcal{K}} &= \frac{4 n_\phi}{(16\pi^2)^2} g_I^2 g_J^2 C_J(\phi) C_I^{\text{comp.}} \Delta\mathcal{K}\,, \\
\gamma_{\overline{\phi}}^{(1)} \overline{\phi} \frac{\partial}{\partial \overline{\phi}} \Gamma^{(1)}_{\Delta\mathcal{K}} &= \frac{4 n_{\overline{\phi}}}{(16\pi^2)^2} g_I^2 g_J^2 C_J(\overline{\phi}) C_I^{\text{comp.}} \Delta\mathcal{K}\,, \\
\gamma_\phi^{(2)} \phi \frac{\partial}{\partial \phi} \Gamma^{(0)}_{\Delta\mathcal{K}} &= \frac{n_\phi}{(16\pi^2)^2} \left[2 b_I C_I(\phi) g_I^4 + 4 g_I^2 g_J^2 C_I(\phi) C_J(\phi) \right] \Delta\mathcal{K}\,, \\
\gamma_{\overline{\phi}}^{(2)} \overline{\phi} \frac{\partial}{\partial \overline{\phi}} \Gamma^{(0)}_{\Delta\mathcal{K}} &= \frac{n_{\overline{\phi}}}{(16\pi^2)^2} \left[2 b_I C_I(\overline{\phi}) g_I^4 + 4 g_I^2 g_J^2 C_I(\overline{\phi}) C_J(\overline{\phi}) \right] \Delta\mathcal{K}\,.
\end{aligned} \tag{F.37}$$

Here, n_ϕ denotes the number of superfield ϕ which is contained in $\Delta\mathcal{K}$.

Finally, $\gamma^{(i)}_{\Delta\mathcal{K}}$ denotes the i-th loop anomalous dimension for the composite operator $\Delta\mathcal{K}$. The one-loop anomalous dimension is calculated in Ref. [47] for the generic composite operator $\Delta\mathcal{K}$.

$$\gamma^{(1)}_{\Delta\mathcal{K}} = \frac{g_I^2}{16\pi^2} [4 C_I^{\text{comp.}} - 2 \sum_\phi (C_I(\overline{\phi}) n_{\overline{\phi}} + C_I(\phi) n_\phi)]\,. \tag{F.38}$$

Combining the above results and Eq. (F.32), we have

$$\begin{aligned}
&(16\pi^2)^2 \gamma^{(2)}_{\Delta\mathcal{K}} \Delta\mathcal{K} \\
&= 2 \Bigg[b_I \sum_\phi (C_I(\overline{\phi}) n(\overline{\phi}) + C_I(\phi) n_\phi) \delta_{IJ} + 2 \sum_\phi (C_I(\phi) C_J(\phi) n_\phi + C_I(\overline{\phi}) C_J(\overline{\phi}) n_{\overline{\phi}}) \\
&+ 4 C_I^{\text{comp.}} C_J^{\text{comp.}} \Bigg] g_I^2 g_J^2 \Delta\mathcal{K} \\
&- 4 g_I^2 g_J^2 \left[(\overline{\phi} t_J t_I t_I)_a (\Delta\mathcal{K})^a_b (t_J \phi)^b + (\overline{\phi} t_J)_a (\Delta\mathcal{K})^a_b (t_I t_I t_J \phi)^b \right] \\
&- 4 g_I^2 g_J^2 \overline{\phi} (t_I t_J + t_J t_I) \phi \ \text{tr} \left[t_J (\Delta\mathcal{K} t_I \phi) + t_I (\Delta\mathcal{K} \overline{\phi} t_J) \right] \\
&- 8 g_I^2 g_J^2 (\overline{\phi} t_J)_a \left[\Delta\mathcal{K} t_I \phi + \Delta\mathcal{K} \overline{\phi} t_I \right]^a_b (t_I t_J \phi)^b \\
&- 8 g_I^2 g_J^2 (\overline{\phi} t_J t_I)_a \left[\Delta\mathcal{K} t_I \phi + \Delta\mathcal{K} \overline{\phi} t_I \right]^a_b (t_J \phi)^b \\
&- 8 g_I^2 g_J^2 (\Delta\mathcal{K})^{ab}_{cd} (\overline{\phi} t_I)_a (\overline{\phi} t_J)_b (t_I \phi)^c (t_J \phi)^d\,.
\end{aligned} \tag{F.39}$$

We show an identity for the quadratic Casimir invariant for the composite operator $\Delta\mathcal{K}$ in Eq. (F.23). By multiplying $(\overline{\phi}t_J)_b(t_J\phi)^d$ and the second derivative of the above relation, we obtain the following relation about the fourth derivative of $\Delta\mathcal{K}$.

$$\begin{aligned}&(\Delta\mathcal{K})^{ab}_{cd}(\overline{\phi}t_I)_a(\overline{\phi}t_J)_b(t_I\phi)^c(t_J\phi)^d\\&=C^{\text{comp.}}_I C^{\text{comp.}}_J\Delta\mathcal{K}\\&-(\overline{\phi}t_J)_a(\Delta\mathcal{K}\overline{\phi}t_I)^a_b(t_It_J\phi)^b-(\overline{\phi}t_Jt_I)_a(\Delta\mathcal{K}t_I\phi)^a_b(t_J\phi)^b-(\overline{\phi}t_Jt_I)_a(\Delta\mathcal{K})^a_b(t_It_J\phi)^b\,.\end{aligned}\tag{F.40}$$

We eliminate the fourth derivative term by using this identity.

As a result, the two-loop anomalous dimension for higher-dimensional operator $\Delta\mathcal{K}$ is given by the following form.

$$\begin{aligned}&(16\pi^2)^2\gamma_{\Delta\mathcal{K}^{(2)}}\Delta\mathcal{K}\\&=2\left[b_I\sum_\phi(C_I(\overline{\phi})n(\overline{\phi})+C_I(\phi)n_\phi)\delta_{IJ}+2\sum_\phi(C_I(\phi)C_J(\phi)n_\phi+C_I(\overline{\phi})C_J(\overline{\phi})n_{\overline{\phi}})\right]g_I^2g_J^2\Delta\mathcal{K}\\&-4g_I^2g_J^2\left[(\overline{\phi}t_Jt_It_I)_a(\Delta\mathcal{K})^a_b(t_J\phi)^b+(\overline{\phi}t_J)_a(\Delta\mathcal{K})^a_b(t_It_It_J\phi)^b\right]\\&-4g_I^2g_J^2\overline{\phi}(t_It_J+t_Jt_I)\phi\ \text{tr}\left[t_J(\Delta\mathcal{K}t_I\phi)+t_I(\Delta\mathcal{K}\overline{\phi}t_J)\right]\\&-8g_I^2g_J^2(\overline{\phi}t_J)_a(\Delta\mathcal{K}t_I\phi)^a_b(t_It_J\phi)^b-8g_I^2g_J^2(\overline{\phi}t_Jt_I)_a(\Delta\mathcal{K}\overline{\phi}t_I)^a_b(t_J\phi)^b\\&+8g_I^2g_J^2(\Delta\mathcal{K})^a_b(\overline{\phi}t_It_J)_a(t_Jt_I\phi)^b\,.\end{aligned}\tag{F.41}$$

This is the generic formula of two-loop anomalous dimension for arbitrary higher-dimensional Kähler potential. However, this result is still complicated except the first term.

Now, we apply the above formula to the dimension-six operators under some assumptions. It is required that the generic Kähler potential is invariant under the gauge transformation, that is

$$\mathcal{K}(\overline{\phi}\,e^{-i\alpha},e^{i\alpha}\phi)-\mathcal{K}(\overline{\phi},\phi)=0\,,\tag{F.42}$$

where $\alpha=\alpha_It_I$ denotes the gauge transformation parameter. We expand the above identity in α, each order of α should be vanished for the gauge invariance. We especially obtain a relation on the Kähler potential from the α^2 term as follows.

$$(\overline{\phi}t_It_K)_a\mathcal{K}^a+(t_It_K\phi)^a\mathcal{K}_a+(\overline{\phi}t_I)_a(\overline{\phi}t_K)_b\mathcal{K}^{ab}+(t_I\phi)^a(t_K\phi)^b\mathcal{K}_{ab}=2(\overline{\phi}t_I)_a\mathcal{K}^a_b(t_K\phi)^b\,,\tag{F.43}$$

We take the gauge adjoint indices $I=K$ and sum up them in both sides. Since the canonical part of $\mathcal{K}$ itself is trivially satisfied the above identity, we concentrate only on the higher-dimensional part. Besides, we operate $(t_J\phi)^c(\overline{\phi}t_J)_d\partial^2/\partial\phi^c\partial\overline{\phi}_d$ on both sides, and thus we obtain,

$$\begin{aligned}
&\sum_{\phi}\left[n_{\phi}C_I(\phi)+n_{\overline{\phi}}C_I(\overline{\phi})\right]C_J^{\text{comp.}}\Delta\mathcal{K} \\
&+2(\overline{\phi}t_Jt_I)_a(\Delta\mathcal{K})_d^{ab}(\overline{\phi}t_I)_b(t_J\phi)^d+2(\overline{\phi}t_J)_a(\Delta\mathcal{K})_{bd}^{a}(t_I\phi)^b(t_It_J\phi)^d \\
&+(\overline{\phi}t_I)_a(\overline{\phi}t_I)_b(\overline{\phi}t_J)_c(\Delta\mathcal{K})_d^{abc}(t_J\phi)^d+(\overline{\phi}t_J)_c(\Delta\mathcal{K})_{abd}^{c}(t_I\phi)^a(t_I\phi)^b(t_J\phi)^d \\
&=2C_I^{\text{comp.}}C_J^{\text{comp.}}\Delta\mathcal{K}\,.
\end{aligned} \tag{F.44}$$

Here, we use $(\overline{\phi}t_I)_a(\Delta\mathcal{K})_b^a(t_I\phi)^b = C_I^{\text{comp.}}\Delta\mathcal{K}$, again. If we assume that the dimension-six operators contain two chiral and two anti-chiral superfields, the third line is vanished. By using this relation, we remove the third derivative terms in Eq. (F.41).

For dimension-six Kähler potentials, such as $\Delta\mathcal{K}=(\lambda_A^{a_1a_2}\overline{\phi}_{a_1}\overline{\phi}_{a_2})(\lambda^A_{b_1b_2}\phi^{b_1}\phi^{b_2})$, the generic two-loop anomalous dimension is reduced the following form.

$$\begin{aligned}
&(16\pi^2)^2\gamma_{\Delta\mathcal{K}^{(2)}}\Delta\mathcal{K} \\
&=2\left[b_I\sum_{\phi}(C_I(\overline{\phi})n(\overline{\phi})+C_I(\phi)n_{\phi})\delta_{IJ}+2\sum_{\phi}(C_I(\phi)C_J(\phi)n_{\phi}+C_I(\overline{\phi})C_J(\overline{\phi})n_{\overline{\phi}})\right. \\
&\left.-\left(4C_I^{\text{comp.}}-2\sum_{\phi}(C_I(\overline{\phi})n_{\overline{\phi}}+C_I(\phi)n_{\phi})\right)C_J^{\text{comp.}}\right]g_I^2g_J^2\Delta\mathcal{K} \\
&-4g_I^2g_J^2\left[(\overline{\phi}t_Jt_It_I)_a(\Delta\mathcal{K})_b^a(t_J\phi)^b+(\overline{\phi}t_J)_a(\Delta\mathcal{K})_b^a(t_It_It_J\phi)^b\right] \\
&-4g_I^2g_J^2\overline{\phi}(t_It_J+t_Jt_I)\phi\ \mathrm{tr}\left[t_J(\Delta\mathcal{K}t_I\phi)+t_I(\Delta\mathcal{K}\overline{\phi}t_J)\right] \\
&+8g_I^2g_J^2(\overline{\phi}t_It_J)_a(\Delta\mathcal{K})_b^a(t_Jt_I\phi)^b\,.
\end{aligned} \tag{F.45}$$

F.2 Application: Proton Decay Operators

We consider the following higher-dimensional Kähler potential causing the baryon number violation.

$$\Delta\mathcal{K}=\sum_{i=1}^{2}C^{(i)}\mathcal{O}^{(i)}+\text{h.c.}\,, \tag{F.46}$$

with operators

$$\begin{aligned}
\mathcal{O}^{(1)}&=\epsilon_{\alpha\beta\gamma}\epsilon_{rs}(\overline{U}^{\dagger})^{\alpha}(\overline{D}^{\dagger})^{\beta}Q^{r\gamma}L^{s}\,, \\
\mathcal{O}^{(2)}&=\epsilon_{\alpha\beta\gamma}\epsilon_{rs}\overline{E}^{\dagger}(\overline{U}^{\dagger})^{\alpha}Q^{r\beta}Q^{s\gamma}\,,
\end{aligned} \tag{F.47}$$

and $C^{(i)}$ $(i=1,2)$ corresponding to the Wilson coefficients.

Diagonal term

We consider the case of t_I and t_J are generators of the same non-Abelian group. Since all superfields are in the (anti-)fundamental or singlet, we can factor out C_I for fundamental representations,

$$(\overline{\phi}t_J t_I t_I)_a(\mathcal{O}^{(i)})^a_b(t_J\phi)^b + (\overline{\phi}t_J)_a(\mathcal{O}^{(i)})^a_b(t_I t_I t_J\phi)^b = (C_{I:\overline{\phi}} + C_{I:\phi})C_J^{\text{comp.}}\mathcal{O}^{(i)}\,. \tag{F.48}$$

Here, $C_{I:\overline{\phi}}$ and $C_{I:\phi}$ come from the first and second term of LHS, respectively. In other words, if the (anti-)chiral part of $\mathcal{O}^{(i)}$ includes only the singlet superfields, $C_{I:\phi} = 0$ ($C_{I:\overline{\phi}} = 0$). This expression is applicable to the case of non-Abelian mixing terms ($SU(2)$-$SU(3)$ term). We also obtain the following after carrying out the brute-force calculation

$$(\overline{\phi}t_J t_I)_a(\mathcal{O}^{(i)})^a_b(t_I t_J\phi)^b = \frac{8}{9}\mathcal{O}^{(i)}\,, \qquad (i = 1, 2)\,, \tag{F.49}$$

for $SU(3)$ generators. For $SU(2)$ generators, since the all anti-chiral superfields are weak singlets, the corresponding terms are zero. For $U(1)_Y$ generators, we find that

$$(\overline{\phi}t_J t_I t_I)_a(\mathcal{O}^{(i)})^a_b(t_J\phi)^b + (\overline{\phi}t_J)_a(\mathcal{O}^{(i)})^a_b(t_I t_I t_J\phi)^b = (S^3_{\overline{\phi}}S^1_{\phi} + S^1_{\overline{\phi}}S^3_{\phi})\mathcal{O}^{(i)}\,, \tag{F.50}$$

and

$$(\overline{\phi}t_J t_I)_a(\mathcal{O}^{(i)})^a_b(t_I t_J\phi)^b = S^2_{\overline{\phi}}S^2_{\phi}\mathcal{O}^{(i)}\,. \tag{F.51}$$

Here, we define the polynomial of $U(1)$ charge as follows.

$$S^n_{\overline{\phi}} = \sum_{\overline{\phi}} q^n_{\overline{\phi}}\,, \qquad S^n_{\phi} = \sum_{\phi} q^n_{\phi}\,, \tag{F.52}$$

where q_ϕ and $q_{\overline{\phi}}$ are $U(1)_Y$ charges for the chiral superfield ϕ and the anti-chiral superfield $\overline{\phi}$, respectively.

Mixed terms

By brute-force calculations, we obtain the $SU(3)$-$U(1)$ terms as follows.

$$(\overline{\phi}t_J t_I)_a(\mathcal{O}^{(i)})^a_b(t_I t_J\phi)^b = \frac{2}{3}S^1_{C\overline{\phi}}S^1_{C\phi}\mathcal{O}^{(i)}\,, \tag{F.53}$$

where $S^n_{C\phi}$ ($S^n_{C\overline{\phi}}$) is defined as the sum of n-th power of $U(1)$ charges of ϕ ($\overline{\phi}$) with color charge.

$$S^n_{C\overline{\phi}} = \sum_{\overline{\phi}\in SU(3)} q^n_{\overline{\phi}}\,, \qquad S^n_{C\phi} = \sum_{\phi\in SU(3)} q^n_{\phi}\,, \tag{F.54}$$

Since there is no anti-chiral superfield with non-zero $SU(2)$ charge, $SU(2)$-$U(1)$ and $SU(3)$-$SU(2)$ terms do not appear from the above term.

Finally, we consider the mixing terms from

$$(\overline{\phi}t_J t_I t_I)_a(\mathcal{O}^{(i)})^a_b(t_J\phi)^b + (\overline{\phi}t_J)_a(\mathcal{O}^{(i)})^a_b(t_I t_I t_J\phi)^b \,. \tag{F.55}$$

If the subscript I corresponds to the non-Abelian group and J denotes the $U(1)$ group, we can factor out the quadratic Casimir for the non-Abelian group, and then we obtain,

$$\begin{aligned}&(\overline{\phi}t_J t_I t_I)_a(\mathcal{O}^{(i)})^a_b(t_J\phi)^b + (\overline{\phi}t_J)_a(\mathcal{O}^{(i)})^a_b(t_I t_I t_J\phi)^b\\ &= (C_{I:\overline{\phi}}S^1_{I\overline{\phi}}S^1_{\phi} + C_{I:\phi}S^1_{\overline{\phi}}S^1_{I\phi})\mathcal{O}^{(i)}\,,\end{aligned} \tag{F.56}$$

where $S^n_{I\overline{\phi}}$ and $S^n_{I\phi}$ denote the sum of n-th power of $U(1)$ charges restricted on the fields with non-trivial non-Abelian charges. If I denotes an index of $U(1)$ group, we can not factor out the square of $U(1)$ charges. In such case, there is non-zero contribution if $J = SU(3)$.

$$\begin{aligned}&(\overline{\phi}t_J t_I t_I)_a(\mathcal{O}^{(i)})^a_b(t_J\phi)^b + (\overline{\phi}t_J)_a(\mathcal{O}^{(i)})^a_b(t_I t_I t_J\phi)^b\\ &= \frac{2}{3}(n^C_{\mathcal{O}}S^2_{C\overline{\phi}} + \overline{n}^C_{\mathcal{O}}S^2_{C\phi})\mathcal{O}^{(i)}\,.\end{aligned} \tag{F.57}$$

Here, $n^C_{\mathcal{O}}$ ($\overline{n}^C$) denotes the number of chiral superfields (anti-chiral superfields) with color charge.

Anomalous Dimension

We divide the two-loop anomalous dimension into the following parts.

$$\gamma^{(2)}_{\mathcal{O}^{(i)}} = \sum_{I\geq J}\frac{g_I^2 g_J^2}{(16\pi^2)^2}[\gamma^{(2)}_{\mathcal{O}^{(i)}}]_{IJ}\,. \tag{F.58}$$

From the above computation, we obtain $[\gamma^{(2)}_{\mathcal{O}^{(i)}}]_{IJ}$ symbolically as follows.

$$\begin{aligned}
[\gamma^{(2)}_{\mathcal{O}^{(i)}}]_{33} &= 6C_3(b_3+2C_3)+\frac{64}{9}-4C_3^{\text{comp.}}(2C_3^{\text{comp.}}-C_3)\,,\\
[\gamma^{(2)}_{\mathcal{O}^{(i)}}]_{22} &= 4C_2(b_2+2C_2)\,,\\
[\gamma^{(2)}_{\mathcal{O}^{(i)}}]_{11} &= 2b_1(S^2_{\overline{\phi}}+S^2_{\phi})+4(S^4_{\overline{\phi}}+S^4_{\phi})-2q^2_{\text{comp.}}(4q^2_{\text{comp.}}-2(S^2_{\overline{\phi}}+S^2_{\phi}))\\
&\quad -4(S^3_{\overline{\phi}}S^1_{\phi}+S^1_{\overline{\phi}}S^3_{\phi}-2S^2_{\overline{\phi}}S^2_{\phi})\,,\\
[\gamma^{(2)}_{\mathcal{O}^{(i)}}]_{32} &= 8C_3C_2n_{(3,2)}+4C_2C_3^{\text{comp.}}\,,\\
[\gamma^{(2)}_{\mathcal{O}^{(i)}}]_{21} &= 8C_2(S^2_{W\overline{\phi}}+S^2_{W\phi})+8C_2q^2_{\text{comp.}}-4(C_{2:\overline{\phi}}S^1_{W\overline{\phi}}S^1_{\phi}+C_{2:\phi}S^1_{\overline{\phi}}S^1_{W\phi})\,,\\
[\gamma^{(2)}_{\mathcal{O}^{(i)}}]_{31} &= 8C_3(S^2_{C\overline{\phi}}+S^2_{C\phi})-16q^2_{\text{comp.}}C_3^{\text{comp.}}+12q^2_{\text{comp.}}C_3+4C_3^{\text{comp.}}(S^2_{\overline{\phi}}+S^2_{\phi})\\
&\quad -4(C_{3:\overline{\phi}}S^1_{C\overline{\phi}}S^1_{\phi}+C_{3:\phi}S^1_{\overline{\phi}}S^1_{C\phi})-\frac{8}{3}(n^C_{\mathcal{O}}S^2_{C\overline{\phi}}+\overline{n}^C_{\mathcal{O}}S^2_{C\phi})\,.
\end{aligned} \tag{F.59}$$

Here, $C_N=(N^2-1)/2N$ is the quadratic Casimir for fundamental representation of $SU(N)$. $S^n_{W\phi}$ ($S^n_{W\overline{\phi}}$) is defined as the sum of n-th power of $U(1)$ charges of chiral superfields (anti-chiral superfields) with weak charge as similar as $S^n_{C\phi}$ ($S^n_{C\overline{\phi}}$). $n_{(3,2)}$ in $[\gamma^{(2)}_{\mathcal{O}^{(i)}}]_{32}$ denotes the number of fields with both color and weak charges.

$$\begin{aligned}
&[\gamma^{(2)}_{\mathcal{O}^{(1)}}]_{33}=[\gamma^{(2)}_{\mathcal{O}^{(2)}}]_{33}=\frac{64}{3}+8b_3\,,\\
&[\gamma^{(2)}_{\mathcal{O}^{(1)}}]_{22}=[\gamma^{(2)}_{\mathcal{O}^{(2)}}]_{22}=\frac{9}{2}+3b_2\,,\\
&[\gamma^{(2)}_{\mathcal{O}^{(1)}}]_{11}=\frac{113}{150}+b_1\,,\quad [\gamma^{(2)}_{\mathcal{O}^{(2)}}]_{11}=\frac{91}{50}+\frac{9}{5}b_1\,,\\
&[\gamma^{(2)}_{\mathcal{O}^{(1)}}]_{32}=12\,,\quad [\gamma^{(2)}_{\mathcal{O}^{(2)}}]_{32}=20\,,\\
&[\gamma^{(2)}_{\mathcal{O}^{(1)}}]_{21}=\frac{6}{5}\,,\quad [\gamma^{(2)}_{\mathcal{O}^{(2)}}]_{21}=\frac{2}{5}\,,\\
&[\gamma^{(2)}_{\mathcal{O}^{(1)}}]_{31}=\frac{68}{15}\,,\quad [\gamma^{(2)}_{\mathcal{O}^{(2)}}]_{31}=\frac{76}{15}\,.
\end{aligned} \tag{F.60}$$

This result is consistent with the previous one [47].

Appendix G

Two-loop RGEs

G.1 Dimensionless Couplings

Gauge Couplings and Yukawa Couplings

In our analysis, we have used the RGEs at the two-loop level. The RGEs for the gauge coupling constants are given as follows [138, 139].

$$\frac{\mathrm{d}g_i}{\mathrm{d}\ln\mu} = \frac{g_i}{16\pi^2}\left[b_i g_i^2 + \frac{1}{16\pi^2}\left(\sum_j b_{ij} g_i^2 g_j^2 - \sum_j a_{ij} g_i^2 \mathrm{tr}[Y_j Y_j^\dagger]\right)\right] . \tag{G.1}$$

Here, $j = 1, 2$, and 3 in the last term respectively correspond to Y_u, Y_d, and Y_e in the SM, but to Y_U, Y_D, and Y_E in the MSSM. The coefficients in the SM are

$$b_{ij} = \begin{pmatrix} \frac{199}{50} & \frac{27}{10} & \frac{44}{5} \\ \frac{9}{10} & \frac{35}{6} & 12 \\ \frac{11}{10} & \frac{9}{2} & -26 \end{pmatrix} , \quad b_i = \left(\frac{41}{10}, -\frac{19}{6}, -7\right) , \quad a_{ij} = \begin{pmatrix} \frac{17}{10} & \frac{1}{2} & \frac{3}{2} \\ \frac{3}{2} & \frac{3}{2} & \frac{1}{2} \\ 2 & 2 & 0 \end{pmatrix} . \tag{G.2}$$

The one-loop RGEs for the Yukawa coupling matrices are given as*

$$\begin{aligned} \frac{\mathrm{d}Y_u}{\mathrm{d}\ln\mu} &= \frac{1}{16\pi^2}\left[-\sum_i c_i^{\mathrm{SM}} g_i^2 + \frac{3}{2}Y_u Y_u^\dagger - \frac{3}{2}Y_d Y_d^\dagger + Y_2(S)\right] Y_u , \\ \frac{\mathrm{d}Y_d}{\mathrm{d}\ln\mu} &= \frac{1}{16\pi^2}\left[-\sum_i c_i'^{\mathrm{SM}} g_i^2 + \frac{3}{2}Y_d Y_d^\dagger - \frac{3}{2}Y_u Y_u^\dagger + Y_2(S)\right] Y_d , \\ \frac{\mathrm{d}Y_e}{\mathrm{d}\ln\mu} &= \frac{1}{16\pi^2}\left[-\sum_i c_i''^{\mathrm{SM}} g_i^2 + \frac{3}{2}Y_e Y_e^\dagger + Y_2(S)\right] Y_e , \end{aligned} \tag{G.3}$$

*In our calculation, we need the RGEs for the gauge couplings at the two-loop level. It is sufficient to take into account the RGEs for the Yukawa couplings at the one-loop level since the Yukawa couplings appear in the two-loop-level RGEs for the gauge couplings.

where

$$c_i^{\rm SM} = \left(\frac{17}{20}, \frac{9}{4}, 8\right) , \quad c_i'^{\rm SM} = \left(\frac{1}{4}, \frac{9}{4}, 8\right) , \quad c_i''^{\rm SM} = \left(\frac{9}{4}, \frac{9}{4}, 0\right) , \tag{G.4}$$

and

$$Y_2(S) = {\rm tr}\left[3Y_u Y_u^\dagger + 3Y_d Y_d^\dagger + Y_e Y_e^\dagger\right] . \tag{G.5}$$

The coefficients of the RGEs for the gauge coupling constants in the MSSM are obtained as

$$b_{ij} = \begin{pmatrix} \frac{199}{25} & \frac{27}{5} & \frac{88}{5} \\ \frac{9}{5} & 25 & 24 \\ \frac{11}{5} & 9 & 14 \end{pmatrix} , \quad b_i = \left(\frac{33}{5}, 1, -3\right) , \quad a_{ij} = \begin{pmatrix} \frac{26}{5} & \frac{14}{5} & \frac{18}{5} \\ 6 & 6 & 2 \\ 4 & 4 & 0 \end{pmatrix} . \tag{G.6}$$

The one-loop RGEs for the Yukawa matrices in the MSSM are given as

$$\begin{aligned}
\frac{{\rm d}Y_U}{{\rm d}\ln\mu} &= \frac{1}{16\pi^2}\left[-\sum_i c_i^{\rm MSSM} g_i^2 + 3Y_U Y_U^\dagger + Y_D Y_D^\dagger + {\rm tr}(3Y_U Y_U^\dagger)\right] Y_U , \\
\frac{{\rm d}Y_D}{{\rm d}\ln\mu} &= \frac{1}{16\pi^2}\left[-\sum_i c_i'^{\rm MSSM} g_i^2 + 3Y_D Y_D^\dagger + Y_U Y_U^\dagger + {\rm tr}(3Y_D Y_D^\dagger + Y_E Y_E^\dagger)\right] Y_D , \\
\frac{{\rm d}Y_E}{{\rm d}\ln\mu} &= \frac{1}{16\pi^2}\left[-\sum_i c_i''^{\rm MSSM} g_i^2 + 3Y_E Y_E^\dagger + {\rm tr}(3Y_D Y_D^\dagger + Y_E Y_E^\dagger)\right] Y_E ,
\end{aligned} \tag{G.7}$$

where

$$c_i^{\rm MSSM} = \left(\frac{13}{15}, 3, \frac{16}{3}\right) , \quad c_i'^{\rm MSSM} = \left(\frac{7}{15}, 3, \frac{16}{3}\right) , \quad c_i''^{\rm MSSM} = \left(\frac{9}{5}, 3, 0\right) . \tag{G.8}$$

The boundary conditions for the Yukawa coupling constants at the SUSY breaking scale (M_S) are

$$Y_U(M_S) = \frac{1}{\sin\beta} Y_u(M_S) , \quad Y_D(M_S) = \frac{1}{\cos\beta} Y_d(M_S) , \quad Y_E(M_S) = \frac{1}{\cos\beta} Y_e(M_S) , \tag{G.9}$$

where $\tan\beta$ is the ratio of vacuum expectation values in the MSSM.

When the vector-like matters are introduced in the MSSM, the RGEs for the gauge coupling constants are modified as

$$b_i \to b_i + \delta b_i , \qquad b_{ij} \to b_{ij} + \delta b_{ij} , \tag{G.10}$$

where b_i and b_{ij} are the coefficients of the one-loop and two-loop RGEs in the MSSM, respectively. δb_i and δb_{ij} are given by Ref. [140]:

$$\delta b_i = (n_{\mathbf{5}} + 3n_{\mathbf{10}}, n_{\mathbf{5}} + 3n_{\mathbf{10}}, n_{\mathbf{5}} + 3n_{\mathbf{10}}) \ , \\ \delta b_{ij} = \begin{pmatrix} \frac{7}{15}n_{\mathbf{5}} + \frac{23}{5}n_{\mathbf{10}} & \frac{9}{5}n_{\mathbf{5}} + \frac{3}{5}n_{\mathbf{10}} & \frac{32}{15}n_{\mathbf{5}} + \frac{48}{5}n_{\mathbf{10}} \\ \frac{3}{5}n_{\mathbf{5}} + \frac{1}{5}n_{\mathbf{10}} & 7n_{\mathbf{5}} + 21n_{\mathbf{10}} & 16n_{\mathbf{10}} \\ \frac{4}{15}n_{\mathbf{5}} + \frac{6}{5}n_{\mathbf{10}} & 6n_{\mathbf{10}} & \frac{34}{3}n_{\mathbf{5}} + 34n_{\mathbf{10}} \end{pmatrix} , \tag{G.11}$$

where $n_{\mathbf{5}}$ and $n_{\mathbf{10}}$ denote the number of $\mathbf{5} + \overline{\mathbf{5}}$ and $\mathbf{10} + \overline{\mathbf{10}}$ vector-like matter superfields, respectively.

QCD Coupling

At the low-energy scale, the QCD coupling becomes strong, and thus the radiative corrections via the QCD coupling would be important. The RGE for the QCD coupling is given by

$$\mu \frac{\mathrm{d}g_S}{\mathrm{d}\mu} = \frac{b_1 g_S^3}{16\pi^2} + \frac{b_2 g_S^5}{(16\pi^2)^2} \, . \tag{G.12}$$

The coefficients of the beta function are

$$b_1 = -\left(\frac{11}{3}C_2 - \frac{2}{3}N_F\right) , \\ b_2 = -\left(\frac{34}{3}(C_2)^2 - \frac{8}{3}N_F - \frac{10}{3}C_2 N_F\right) , \tag{G.13}$$

with C_2 and N_F are the number of colors and quark flavors.

G.2 Wilson Coefficients

In this section, we show show anomalous dimensions for the baryon-number violating operators up to two-loop level. Then, we give numerical factors for the two-loop level estimations.

Short-Distance Effects –SM–

The effective Lagrangian below the EW scale is given by

$$\mathcal{L} = \sum_m C_m \mathcal{O}_m + \text{h.c.} \, , \tag{G.14}$$

with operators

$$\mathcal{O}_{LR} = \epsilon_{\alpha\beta\gamma}\epsilon_{rs}(\overline{u}_L^{C\gamma}\gamma^\mu q_L^{\beta r})(\overline{l}_R^{Cs}\gamma_\mu d_R^\alpha)\,,$$
$$\mathcal{O}_{LL} = \epsilon_{\alpha\beta\gamma}\epsilon_{rs}(\overline{u}_L^{C\gamma}\gamma^\mu q_L^{\beta r})(\overline{e}_L^{C}\gamma_\mu q_L^{\alpha s})\,. \tag{G.15}$$

Here, $\mathcal{O}_{LR}$ and $\mathcal{O}_{LL}$ respectively correspond to $\mathcal{O}_{4F}^{(1)}$ and $\mathcal{O}_{4F}^{(2)}$ in Eq. (3.9) in the text.

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}C_m(\mu) = \gamma_m C_m(\mu)\,, \tag{G.16}$$

$m = LL, LR$

$$\gamma_m = \sum_i \gamma_{i,m}^{(1)}\widetilde{\alpha}_i + \sum_{i\geq j}\gamma_{ij,m}^{(2)}\widetilde{\alpha}_i\widetilde{\alpha}_j + \cdots\,, \tag{G.17}$$

where $\widetilde{\alpha}_i = g_i^2/16\pi^2$ $(i = 1,2,3)$. Here, g_1 is the GUT-normalized $U(1)_Y$ gauge coupling, again. The one-loop anomalous dimensions are given by

$$\gamma_{3,m}^{(1)} = -4\,,\quad \gamma_{2,m}^{(1)} = -\frac{9}{2}\,,\quad \gamma_{1,m}^{(1)} = \begin{cases} -\dfrac{11}{10} & (m = LL) \\ -\dfrac{23}{10} & (m = LR) \end{cases}\,. \tag{G.18}$$

The above results are well-known by Ref. [42]. The two-loop anomalous dimensions were derived by authors of Ref. [45] in the context of the non-supersymmetric GUTs.

$$\gamma_{33,m}^{(2)} = -\frac{86}{3} + \frac{8}{9}n_g\,,\quad \gamma_{22,m}^{(2)} = -\frac{579}{16} + 3n_g\,,\quad \gamma_{11,m}^{(2)} = \begin{cases} \dfrac{479}{1200} - \dfrac{2}{15}n_g & (m = LL) \\ \dfrac{3143}{1200} - \dfrac{14}{15}n_g & (m = LR) \end{cases}\,,$$
$$\gamma_{21,m}^{(2)} = \begin{cases} -\dfrac{3}{40} & (m = LL) \\ \dfrac{17}{40} & (m = LR) \end{cases}\,,\quad \gamma_{31,m}^{(2)} = \begin{cases} \dfrac{59}{45} & (m = LL) \\ \dfrac{46}{9} & (m = LR) \end{cases}\,,\quad \gamma_{32,m}^{(2)} = \begin{cases} 9 & (m = LL) \\ 10 & (m = LR) \end{cases}\,. \tag{G.19}$$

Here, n_g denotes the number of generations.

The numerical factors for C_m are

$$\frac{C_{LL}(m_Z)}{C_{LL}(M_S)} = 1.12\,,\quad \frac{C_{LR}(m_Z)}{C_{LR}(M_S)} = 1.13\,. \tag{G.20}$$

Here, M_S is the SUSY scale and set to be 1 TeV.

Short-Distance Effects –SUSY SM–

As we have shown in the previous chapter, the two-loop anomalous dimensions for the baryon number violating operators are derived using effective Kähler potential. In this subsection, we summarize the anomalous dimensions up to two-loop level. The effective Kähler potential is defined in Eq. (F.46). The RGE running of the Wilson coefficients is

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} C^{(i)}(\mu) = \gamma_{\mathcal{O}^{(i)}} C^{(i)}(\mu)\,, \quad (i = 1, 2)\,. \tag{G.21}$$

The anomalous dimensions are decomposed as

$$\gamma_{\mathcal{O}^{(I)}} = \sum_i [\gamma^{(1)}_{\mathcal{O}^{(I)}}]_i \widetilde{\alpha}_i + \sum_{i \geq j} [\gamma^{(2)}_{\mathcal{O}^{(I)}}]_{ij} \widetilde{\alpha}_i \widetilde{\alpha}_j + \cdots\,. \tag{G.22}$$

The two-loop anomalous dimensions $[\gamma^{(2)}_{\mathcal{O}^{(I)}}]_{ij}$ are given in Eq. (F.60). The one-loop coefficients were derived in [43].

$$\begin{aligned}
&[\gamma^{(1)}_{\mathcal{O}^{(1)}}]_3 = [\gamma^{(1)}_{\mathcal{O}^{(2)}}]_3 = -\frac{8}{3}\,, \\
&[\gamma^{(1)}_{\mathcal{O}^{(1)}}]_2 = [\gamma^{(1)}_{\mathcal{O}^{(2)}}]_2 = -3\,, \\
&[\gamma^{(2)}_{\mathcal{O}^{(1)}}]_1 = -\frac{11}{15}\,, \quad [\gamma^{(2)}_{\mathcal{O}^{(2)}}]_1 = -\frac{23}{15}\,.
\end{aligned} \tag{G.23}$$

The numerical factors for $C^{(i)}$ $(i = 1, 2)$ are

$$\frac{C^{(1)}(M_S)}{C^{(1)}(M_{\mathrm{GUT}})} = 1.97\,, \quad \frac{C^{(2)}(M_S)}{C^{(2)}(M_{\mathrm{GUT}})} = 2.07\,. \tag{G.24}$$

Here, M_S is the SUSY scale and set to be 1 TeV, and $M_{\mathrm{GUT}} = 2 \times 10^{16}$ GeV.

Long-Distance Effect

Below the EW scale, the QCD correction to the Wilson coefficients would be important due to the strong coupling. The evolution of the Wilson coefficients for the proton decay operators has been evaluated by the authors of Ref. [46] up to two-loop level. The RGE for the Wilson coefficients is given by

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} C = \left[\gamma^{(1)}_C \frac{\alpha_S}{4\pi} + \gamma^{(2)}_C \left(\frac{\alpha_S}{4\pi} \right)^2 \right] C\,, \tag{G.25}$$

with anomalous dimensions

$$\begin{aligned}
\gamma^{(1)}_C &= -4\,, \\
\gamma^{(2)}_C &= -\left(\frac{14}{3} + \frac{4}{9} N_F + \Delta \right)\,.
\end{aligned} \tag{G.26}$$

Here, N_F is the number of quark flavors. The value of Δ changes depending on the proton decay operators: $\Delta = 0$ for left-handed (right-handed) quarks and lepton operators while $\Delta = -10/3$ for the operators including different chiralities. Thus, in our simulations, we use $\Delta = -10/3$ since the dimension-six operators via X-boson exchange include both of chiralities. The analytic solution of the RGE is easily found as follows.

$$\frac{C(\mu)}{C(\mu_0)} = \left(\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}\right)^{\frac{\gamma_C^{(1)}}{2b_1}} \left(\frac{4\pi b_1 + b_2\alpha_S(\mu)}{4\pi b_1 + b_2\alpha_S(\mu_0)}\right)^{\frac{\gamma_C^{(2)}}{2b_2} - \frac{\gamma_C^{(1)}}{2b_1}}, \tag{G.27}$$

with the running of QCD coupling given in Eq. (G.12).

We now give the numerical values for the long-distance effect. The long-distance effect A_L is defined as the ratio of the Wilson coefficients at the hadronic scale μ_H and the EW scale m_Z.

$$A_L = \frac{C(\mu_H)}{C(m_Z)}. \tag{G.28}$$

The hadronic scale is $\mu_H = 2$ GeV, where the hadron matrix elements are evaluated. Thus, the analytic expression for A_L is given by

$$A_L = \left(\frac{\alpha_S(\mu_H)}{\alpha_S(m_b)}\right)^{\frac{6}{25}} \left(\frac{\alpha_S(m_b)}{\alpha_S(m_Z)}\right)^{\frac{6}{23}} \left(\frac{\alpha_S(\mu_H) + \frac{50\pi}{77}}{\alpha_S(m_b) + \frac{50\pi}{77}}\right)^{-\frac{173}{825}} \left(\frac{\alpha_S(m_b) + \frac{23\pi}{29}}{\alpha_S(m_Z) + \frac{23\pi}{29}}\right)^{-\frac{430}{2001}}. \tag{G.29}$$

Then, we have the numerical value for A_L as

$$A_L = 1.25\,. \tag{G.30}$$

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