
Quantum Mechanics

Jilin U. Center for Theoretical Physics
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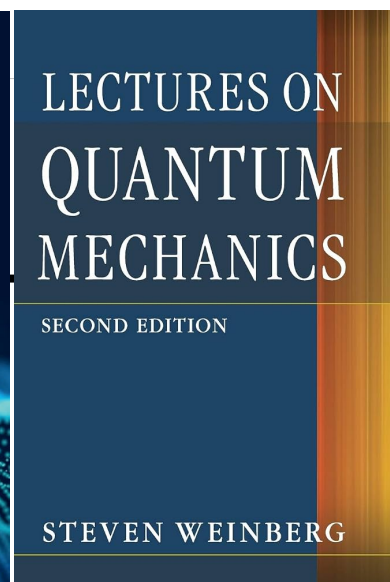
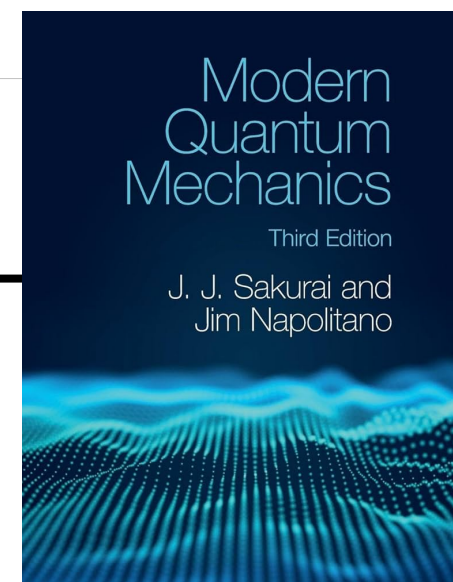
Preface

Textbooks

This lecture is based on

Modern Quantum Mechanics: **Jun J. Sakurai**

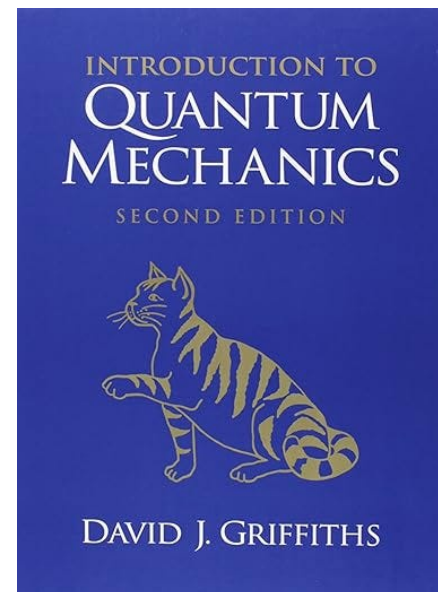
Lectures on Quantum Mechanics: **Steven Weinberg**

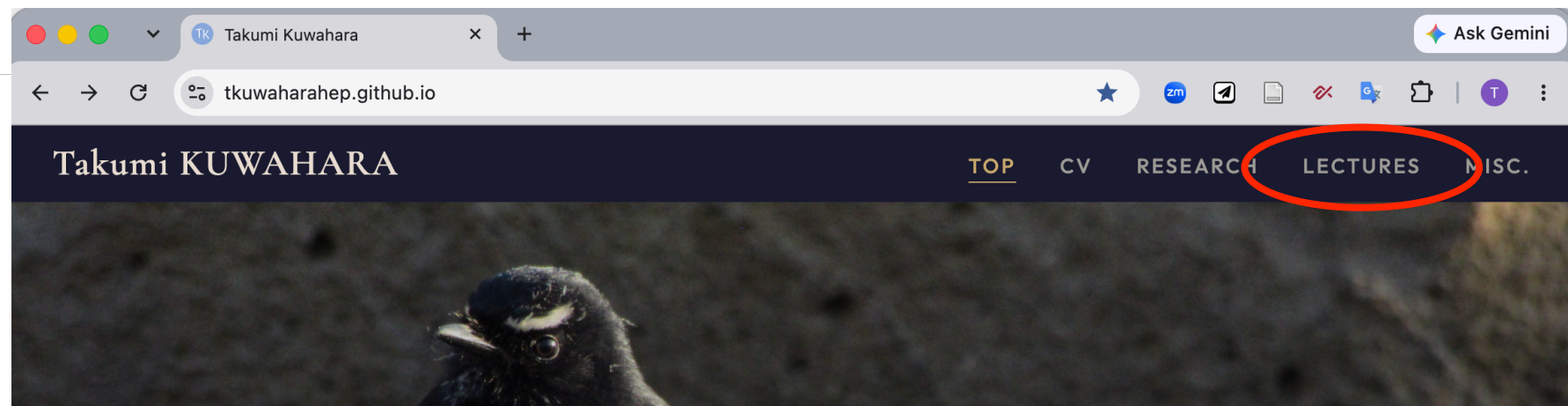


Further References

Introduction to Quantum Mechanics: **David J. Griffiths**

David Tong's Lecture [[URL](#)] (and Lecture note PDF)

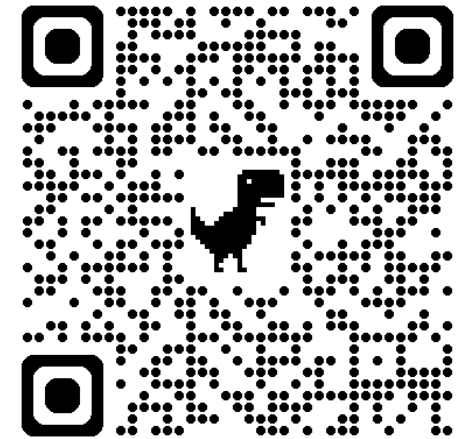




Courses Materials:

If necessary,
lecture slides will be uploaded on my webpage [[URL](#)]

webpage



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Takumi Kuwahara



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Beyond Classical Mechanics



Breakdown of Classical Mechanics

Many evidences for breakdown: have been discovered in the late 19th century

Classical Mechanics

$$ma = F$$

microscopic

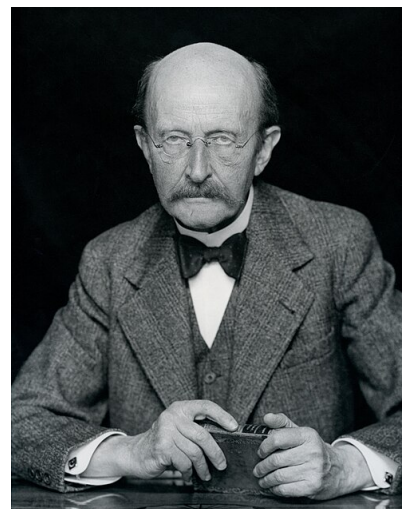
Quantum Mechanics

- wave-particle duality
- sometimes, discrete

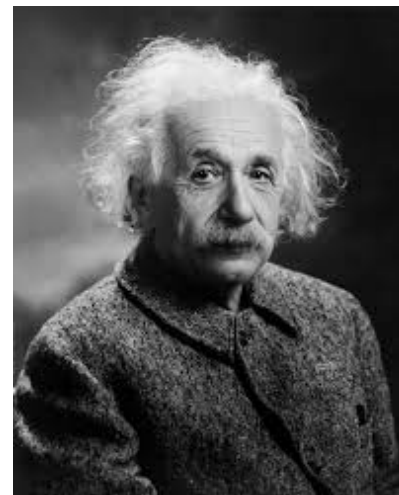
Founders of old-Quantum Mechanics (1900's - 1920's)



N. Bohr



Max Planck



A. Einstein



Louis de Broglie

Breakdown of Classical Mechanics

Many evidences for breakdown: have been discovered in the late 19th century

Classical Mechanics

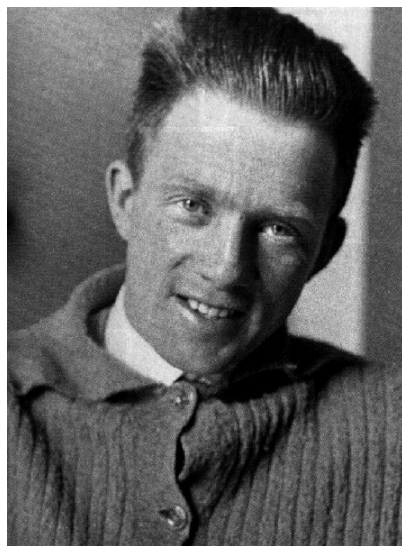
$$ma = F$$

microscopic

Quantum Mechanics

- wave-particle duality
- sometimes, discrete

Founders of Modern Quantum Mechanics (1925 - 1930)



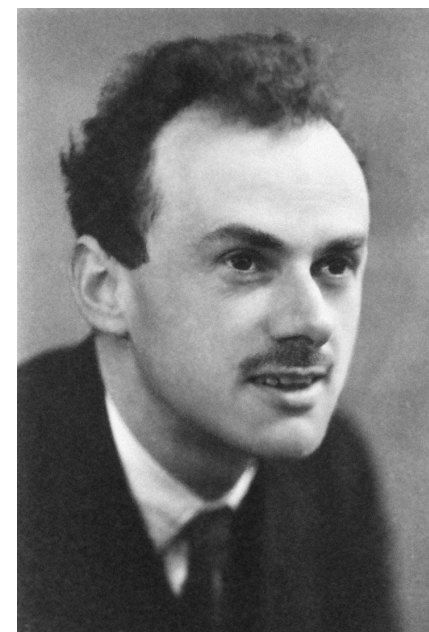
Werner Heisenberg



Erwin Schrödinger



Max Born



Paul Dirac

Breakdown of Classical Mechanics

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Classical Mechanics

$$ma = F$$

microscopic

Quantum Mechanics

- wave-particle duality
- sometimes, discrete

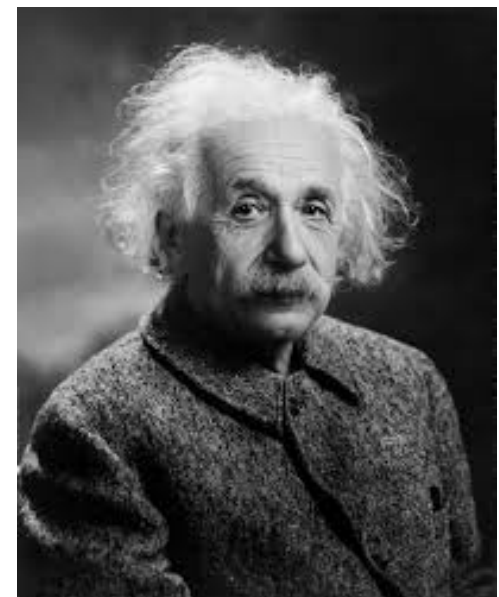
high velocity/acceleration

Special/General Relativity

- constancy of v_{light}
- equivalence principle

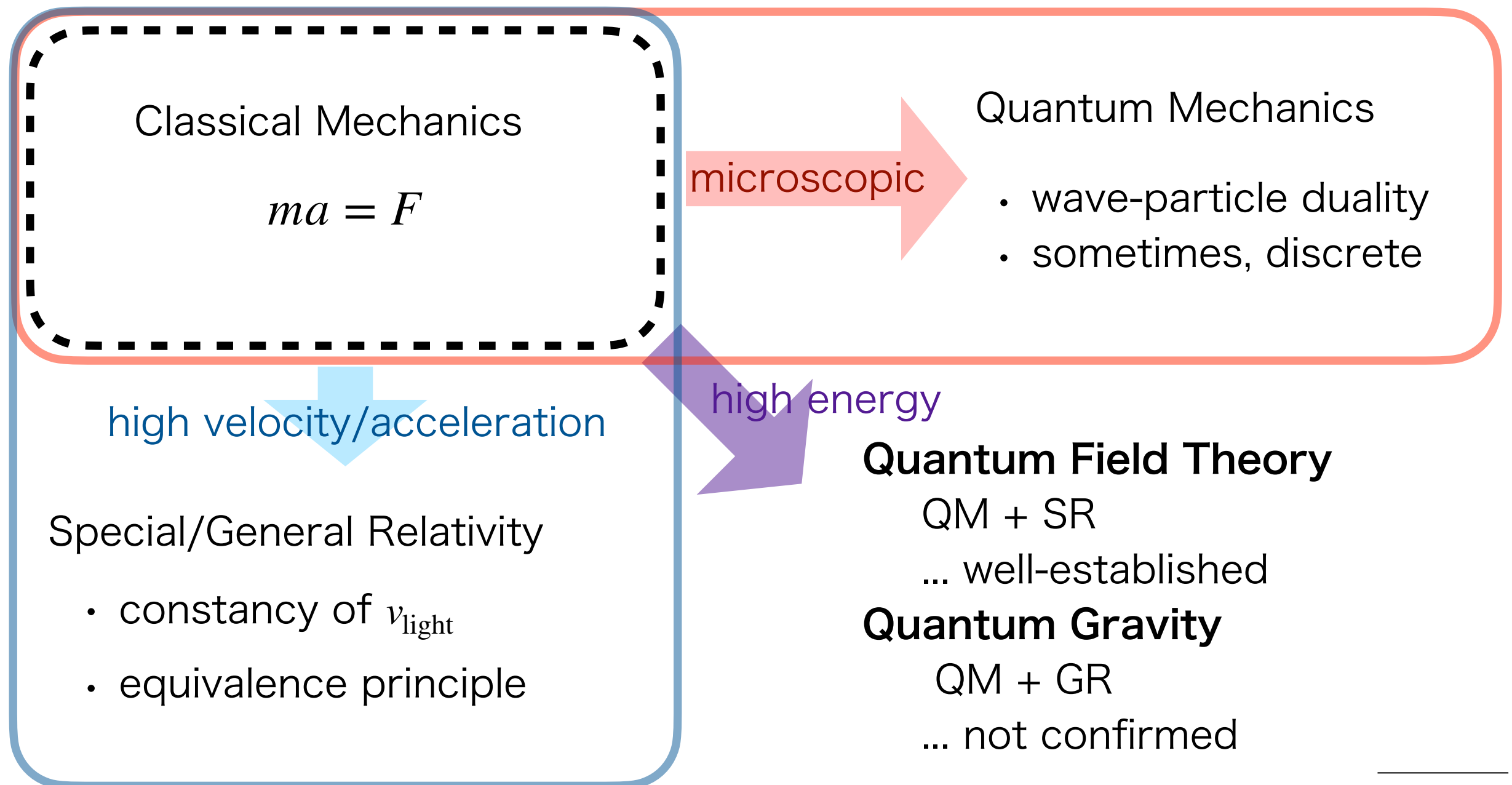
The same time:

A. Einstein developed another direction



Breakdown of Classical Mechanics

Many evidences for breakdown: have been discovered in the late 19th century



Old Quantum Mechanics



Wave to Particle

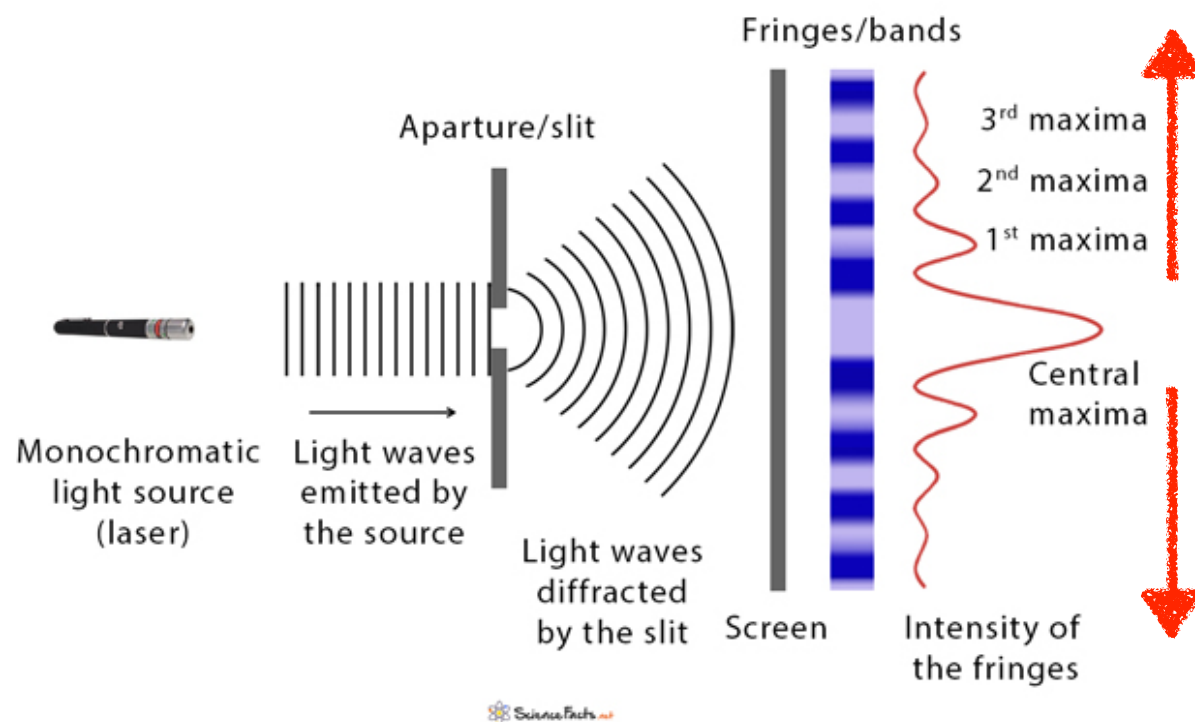
Wave and Particle

What is wave?

Characters of wave are

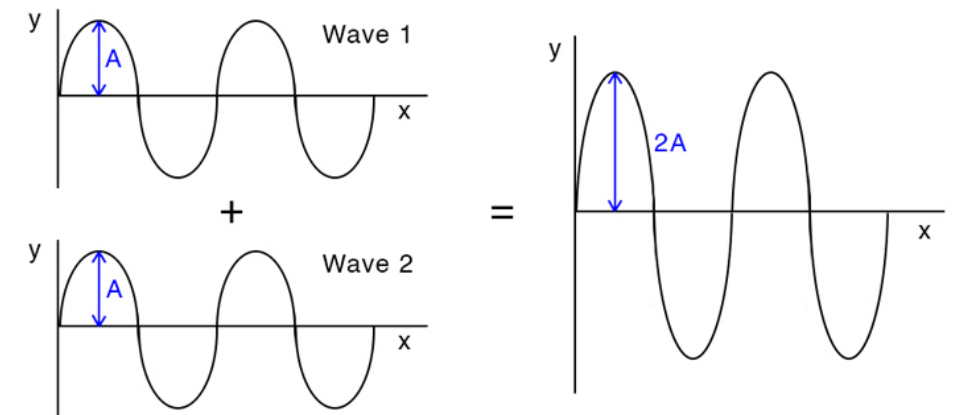
- Superposition (叠加)
- Interference (干涉)
- Diffraction (衍射)

Diffraction

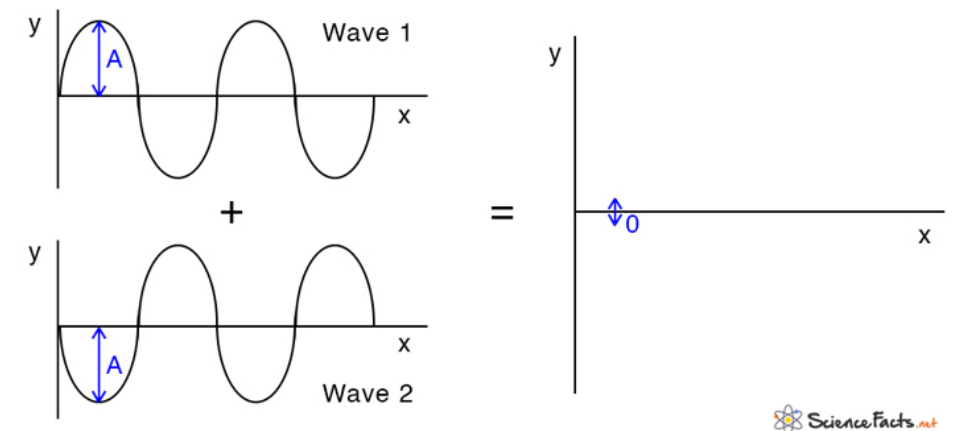


Principle of Superposition

Constructive Interference



Destructive Interference



Not allowed for straight-line propagation

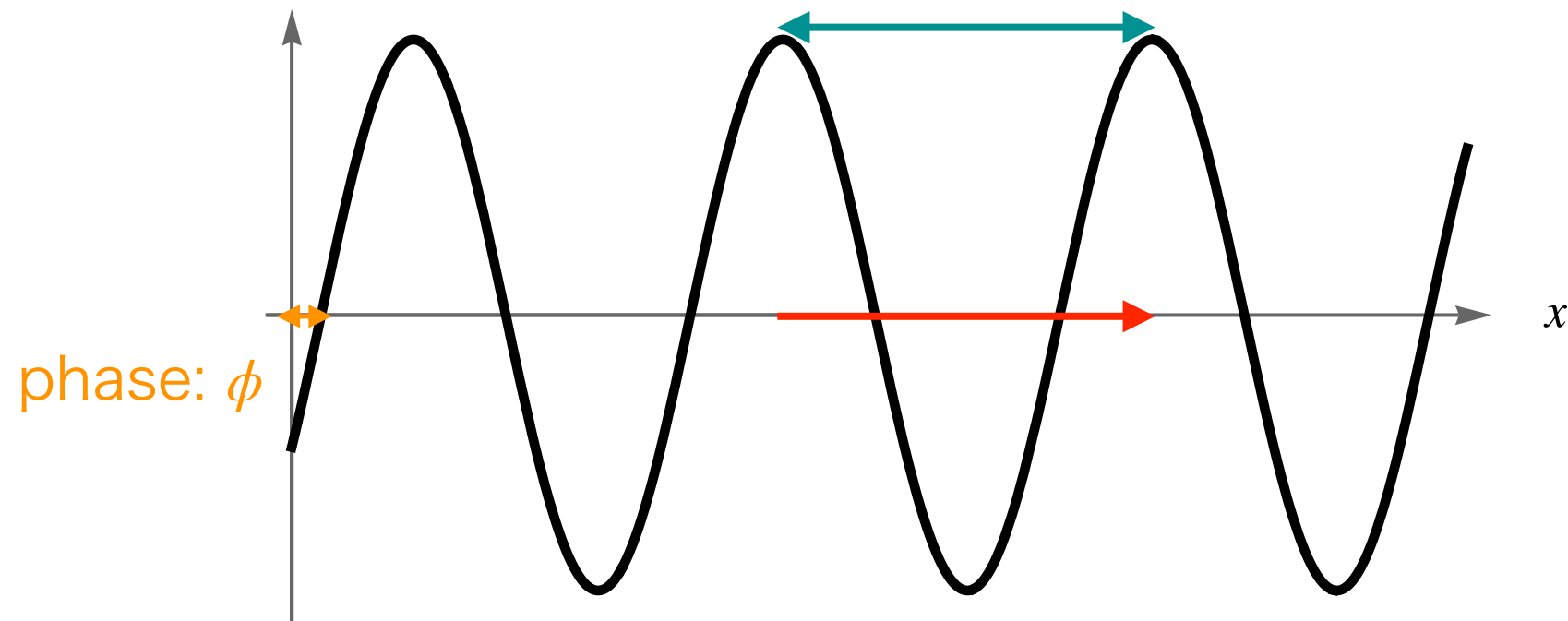
Properties of Wave

sine wave

$$A \sin(kx - \omega t + \phi)$$

wavelength: λ

(angular) wavenumber: $k = 2\pi\lambda^{-1}$

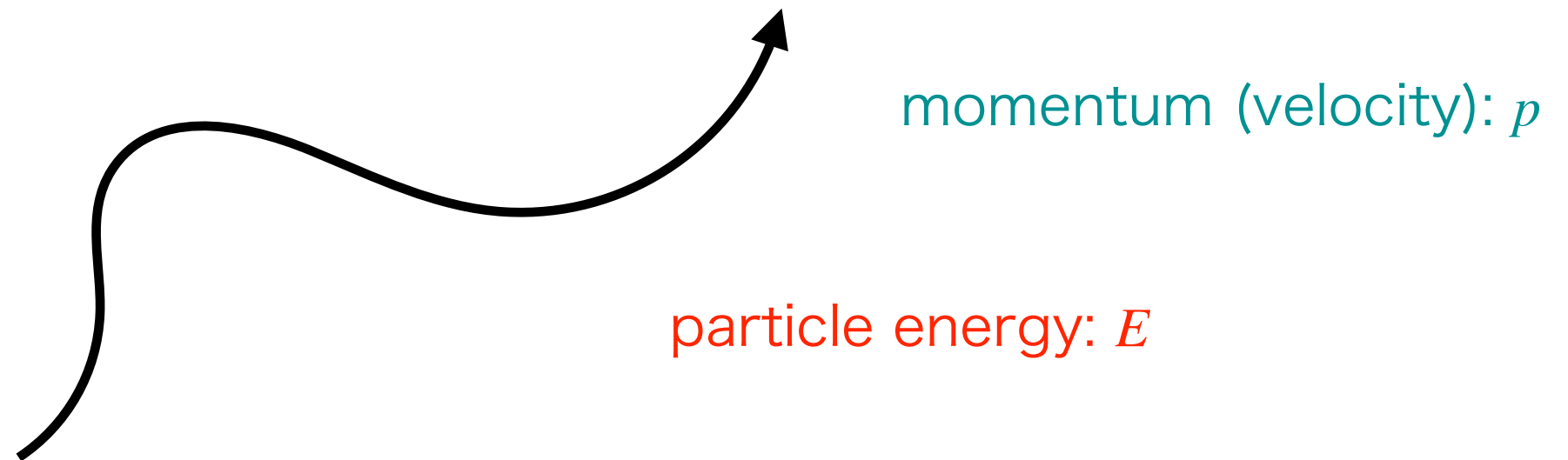


time-period: T

frequency: $\nu = T^{-1}$

angular frequency: $\omega = 2\pi\nu$

Properties of Particle



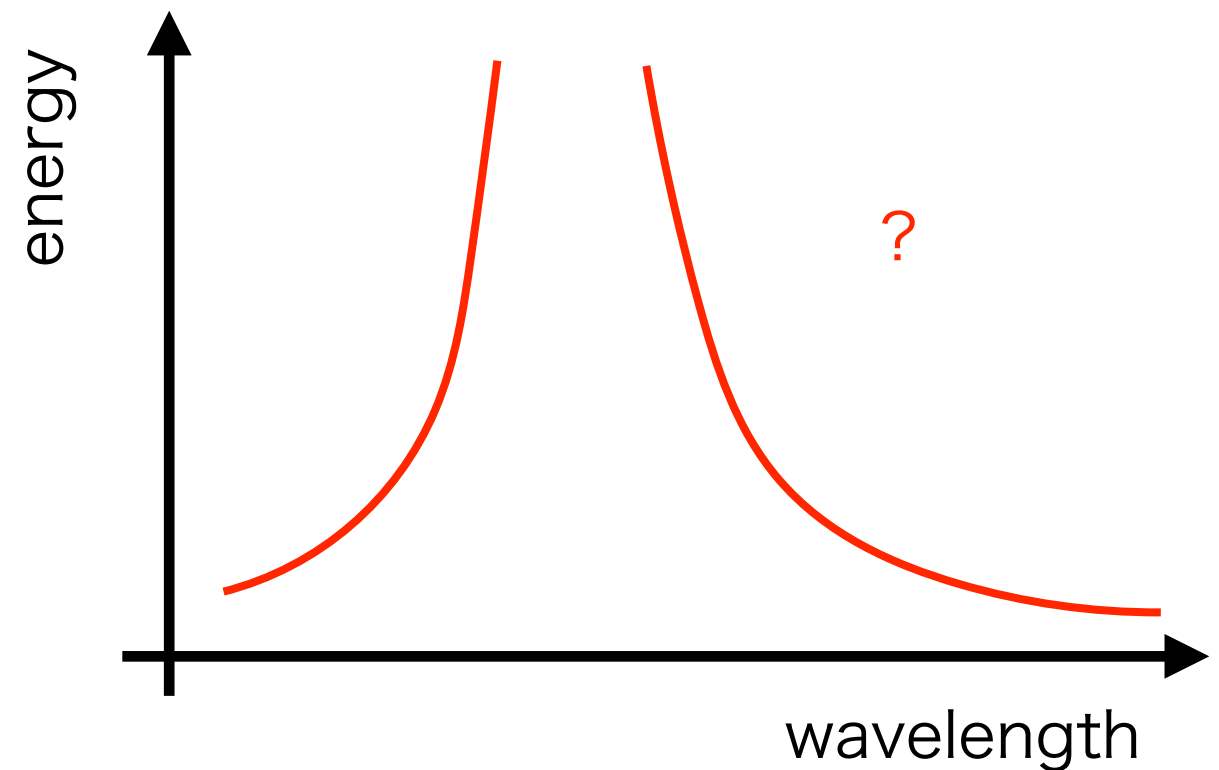
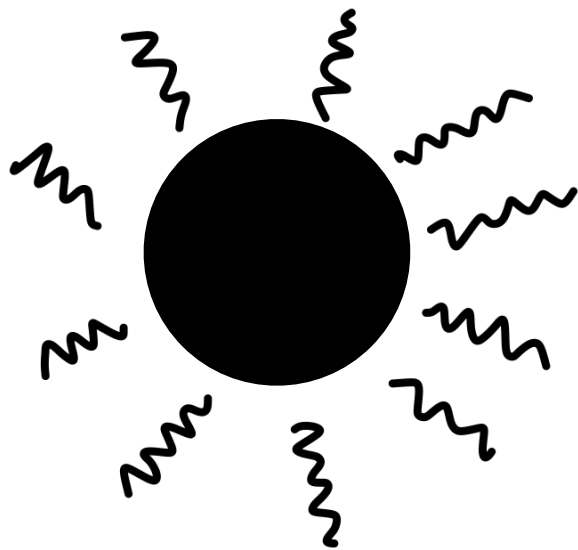
Given potential, the position of particle $x(t)$: deterministic

Number of particles: countable $n = 1, 2, \dots$

Black-Body Radiation

Black-body: ideal objects which absorb light with any wavelength
(no reflection) and also emit radiation

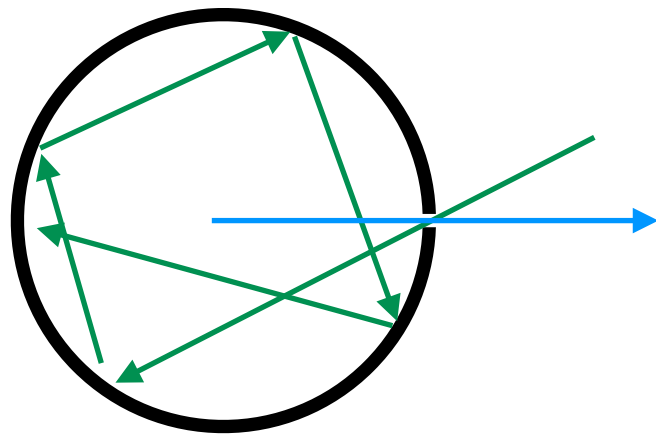
Q: What is energy distribution for black-body radiation?



Experiments for Black-body Radiation

Otto Lummer, Ferdinand Kurlbaum (1898)

Cavity with a hole



Spectrometer

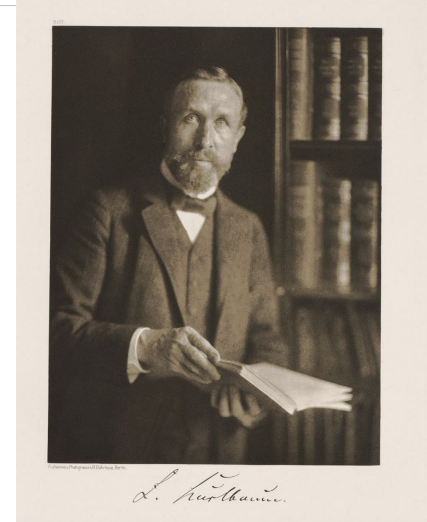
Temperature: T (a tunable parameter)

Approximate Black-body

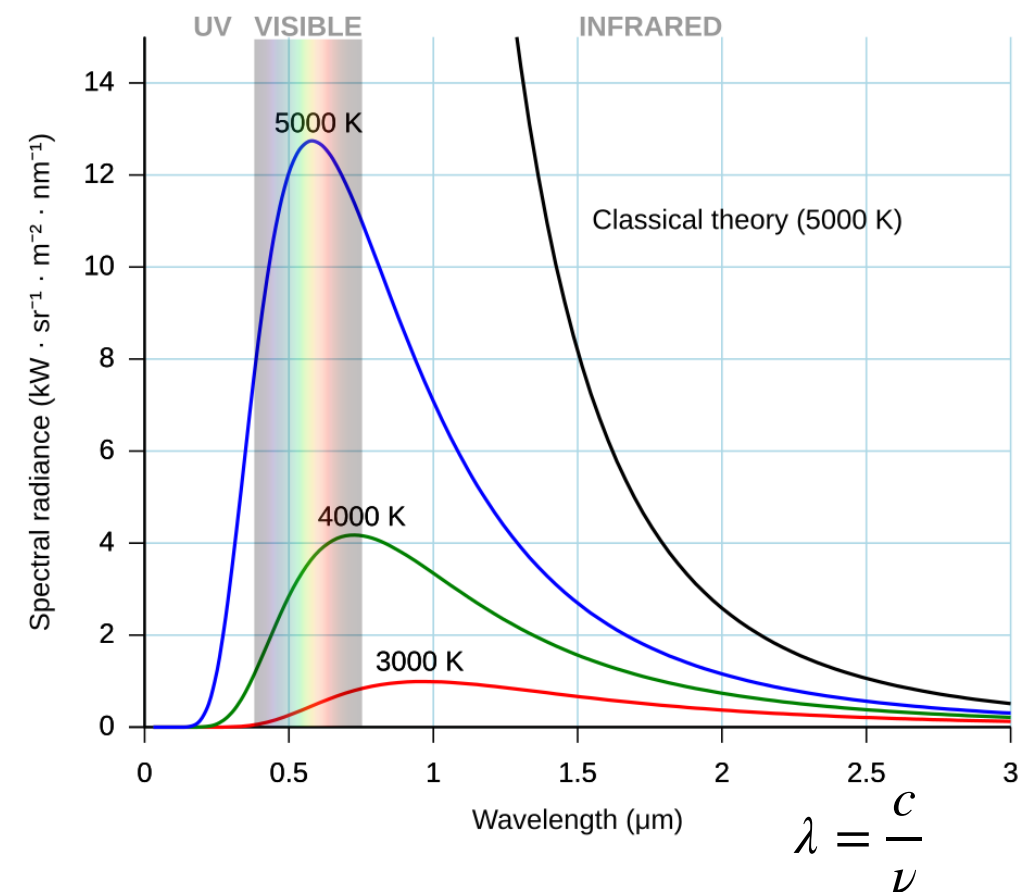
- absorbs radiation with any frequencies
- emitted radiation (thermalized by cavity)



Otto Lummer

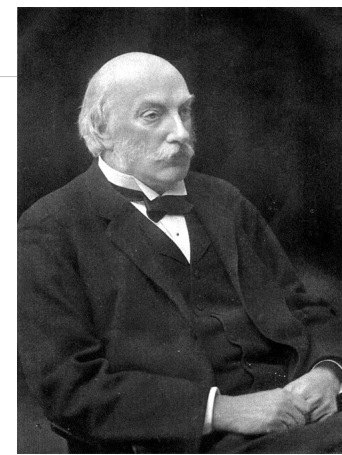


Ferdinand Kurlbaum

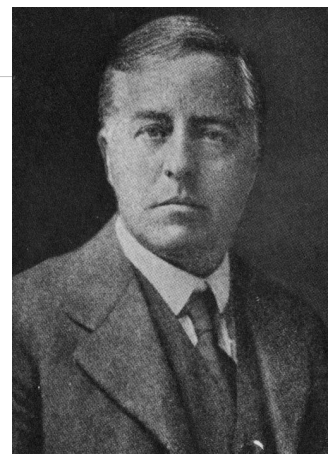


Rayleigh-Jeans Law

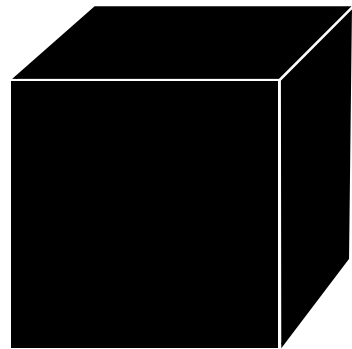
Assume the ideal cube system (with size of L^3)



Lord Rayleigh



John Jeans



the radiation field is the sum of $e^{iq \cdot x}$ with wavenumber $\mathbf{q} = \frac{2\pi}{L}\mathbf{n}$

3-vector (integer): $\mathbf{n} = (n_x, n_y, n_z)$

frequency: $\nu = \frac{c}{\lambda} = \frac{|\mathbf{n}|c}{L}$

#normal mode in $[\nu, \nu + d\nu]$

$$N(\nu)d\nu = 2 \times 4\pi |\mathbf{n}|^2 d|\mathbf{n}| = 8\pi \left(\frac{L}{c}\right)^3 \nu^2 d\nu$$

each energy of radiation (= harmonic oscillator: classical statistical mechanics)

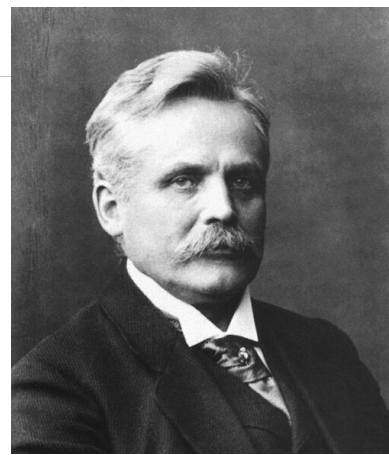
$$\epsilon(T) = k_B T$$

energy density in the radiation (Rayleigh-Jeans formula)

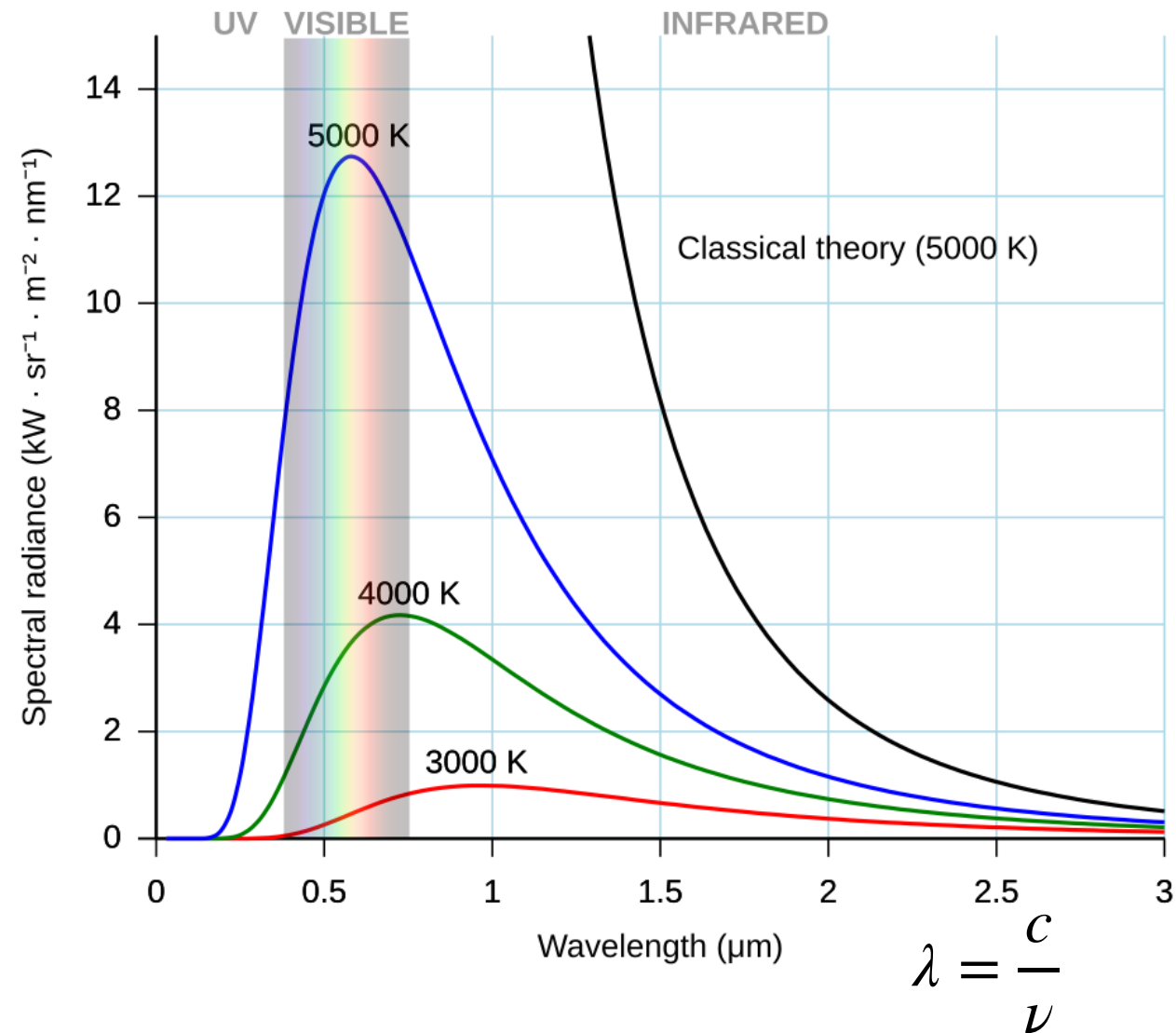
$$\rho(\nu, T)d\nu = \frac{\epsilon(T)N(\nu)d\nu}{L^3} = \frac{8\pi k_B T \nu^2 d\nu}{c^3}$$

total energy from black body is infinite (UV catastrophe)

Wien's Displacement Law



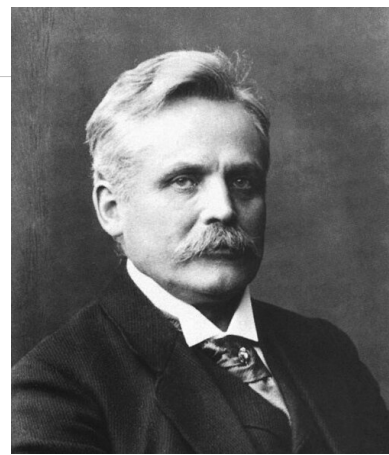
Wilhelm Wien



A guess from experimental results

peak frequency is proportional to temperature: $\nu_{\text{peak}} \propto T$

Wien's Displacement Law



Wilhelm Wien

A bit detail understanding:

pressure from electromagnetic radiation: $p = \frac{\rho}{3}$ (energy density ρ)

The first law of thermodynamics: $dE = -pdV$

total energy $E = \rho V$, $dE = \rho dV + Vd\rho$

$$\frac{d\rho}{\rho} = -\frac{4}{3} \frac{dV}{V}: \text{ and find } \rho V^{4/3} = \text{const.}$$

Stefan-Boltzmann Law: $\rho = \sigma T^4$ (σ : Stefan-Boltzmann constant)

Leads to $TV^{1/3} = \text{const.}$

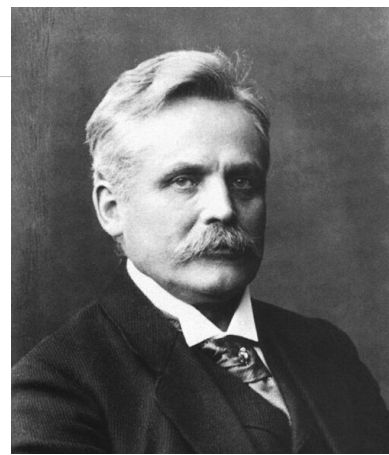
When black-body is adiabatically compressed: $V \rightarrow V' = \varepsilon V$,

temperature changes: $T \rightarrow T' = \varepsilon^{1/3} T$

wavelength also changes: $\lambda \rightarrow \lambda' = \varepsilon^{-1/3} \lambda$ (or frequency: $\nu \rightarrow \nu' = \varepsilon^{1/3} \nu$)

$\rightarrow \nu/T$ is invariant under adiabatic compression

Wien's Displacement Law



Wilhelm Wien

Energy density spectrum: $\rho(\nu, T)$

integrate over the spectrum: $\int d\nu \rho(\nu, T) = \rho(T) \propto T^4$ **Stefan-Boltzmann Law**

If the spectrum has the form of

$$\rho(\nu, T) = \nu^3 f\left(\frac{\nu}{T}\right) \Rightarrow \int d\nu \rho(\nu, T) = T^4 \int dx x^3 f(x)$$

Wien's assumption:

The existence probability (spectrum) under the thermal equilibrium for state with E

$$p(E) \propto e^{-E/k_B T} \quad \text{Boltzmann-type distribution}$$

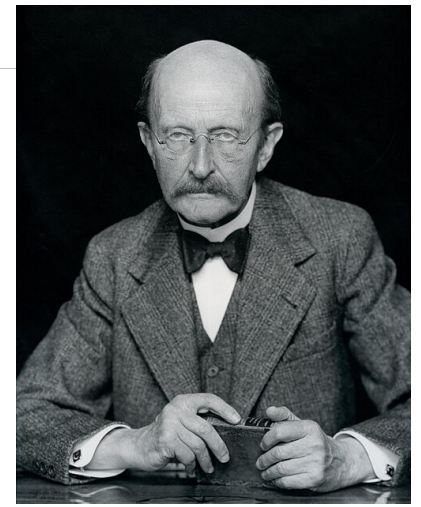
Oscillators energy: $E \propto \nu$ $\rho(\nu, T) = C \nu^3 f e^{-a\nu/k_B T}$

a is unknown parameter

C is normalization factor

Planck's Quantization

Radiation energy is not continuum: $E_n = nh\nu$



Max Planck

Planck's constant $h = 6.626 \times 10^{-34} \text{ J} \cdot \text{Hz}^{-1}$

Planck's treatment: replace the energy per harmonic oscillators

$$k_B T \rightarrow \frac{h\nu}{e^{h\nu/k_B T} - 1}$$

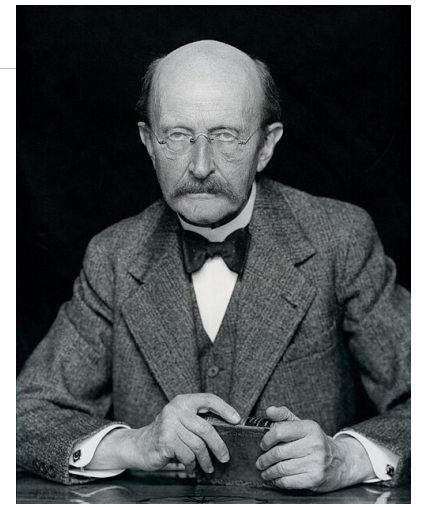
Classical Statistics

$$\epsilon(T) = \frac{\int dE E e^{-E/k_B T}}{\int dE e^{-E/k_B T}}$$

Planck's treatment

$$\epsilon(T) = \frac{\sum_n^\infty E_n e^{-E_n/k_B T}}{\sum_n^\infty e^{-E_n/k_B T}}$$

Planck's Quantization: Black-body Radiation



Max Planck

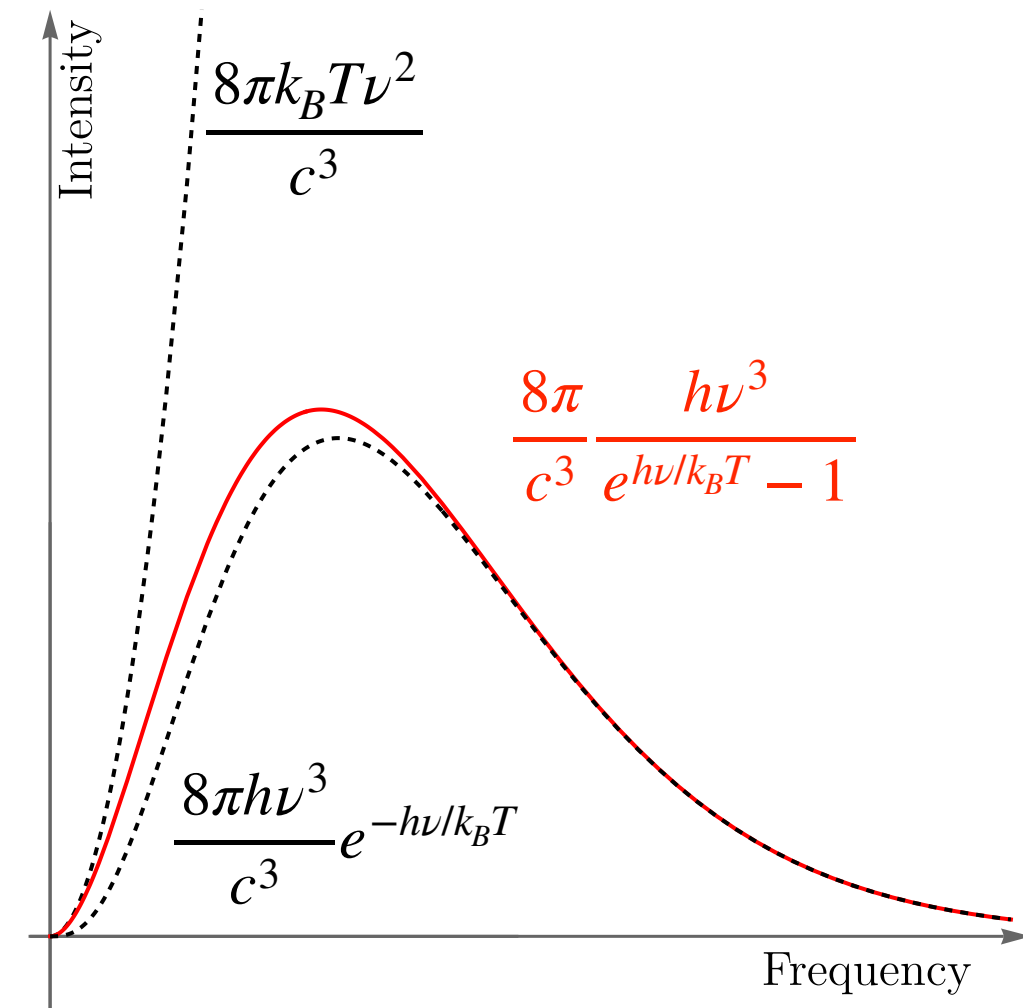
Planck's law $\rho(\nu, T)d\nu = \frac{8\pi}{c^3} \frac{h\nu^3 d\nu}{e^{h\nu/k_B T} - 1}$

Two frequency limits

$$\rho(\nu, T) \rightarrow \frac{8\pi\nu^2 k_B T}{c^3}, \quad (\nu/T \ll 1) \quad \text{Rayleigh-Jeans}$$

$$\rho(\nu, T) \rightarrow \frac{8\pi h\nu^3}{c^3} e^{-h\nu/k_B T}, \quad (\nu/T \gg 1) \quad \text{Wien}$$

Finiteness of total energy: $\int_0^\infty \rho(\nu, T) d\nu < \infty$



Application: Dulong-Petit Law

Solid state physics

atoms oscillating around lattice points (harmonic oscillator)

total energy per mole: $U = 3N_A \times k_B T$

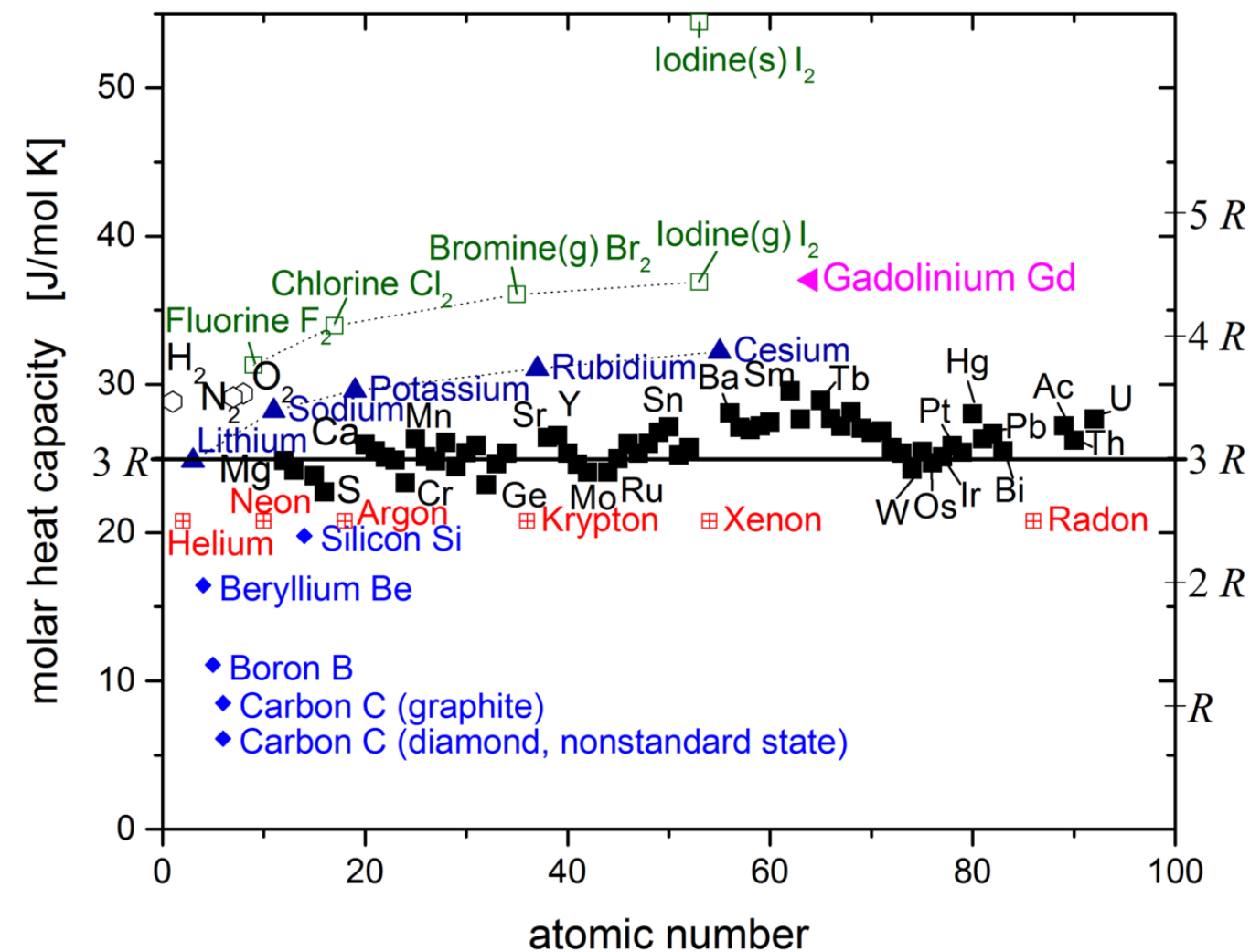
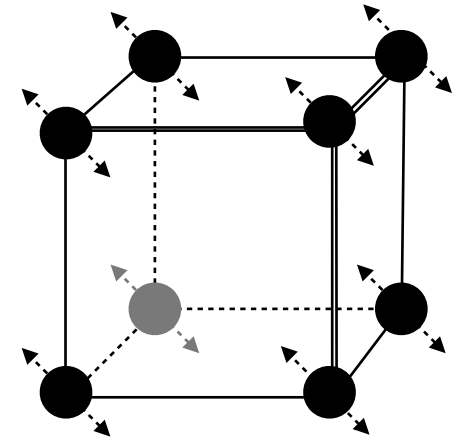
N_A : Avogadro's number

(Molar) Heat capacity

$$C \equiv \frac{dU}{dT} = 3k_B N_A = 3R$$

* gas constant R : $pV = nRT$

This law is consistent in various atoms



Breakdown of Dulong-Petit Law

deviate from Dulong-Petit prediction at **low T**

$$C \rightarrow 0 \text{ at } T \rightarrow 0$$

Planck's treatment

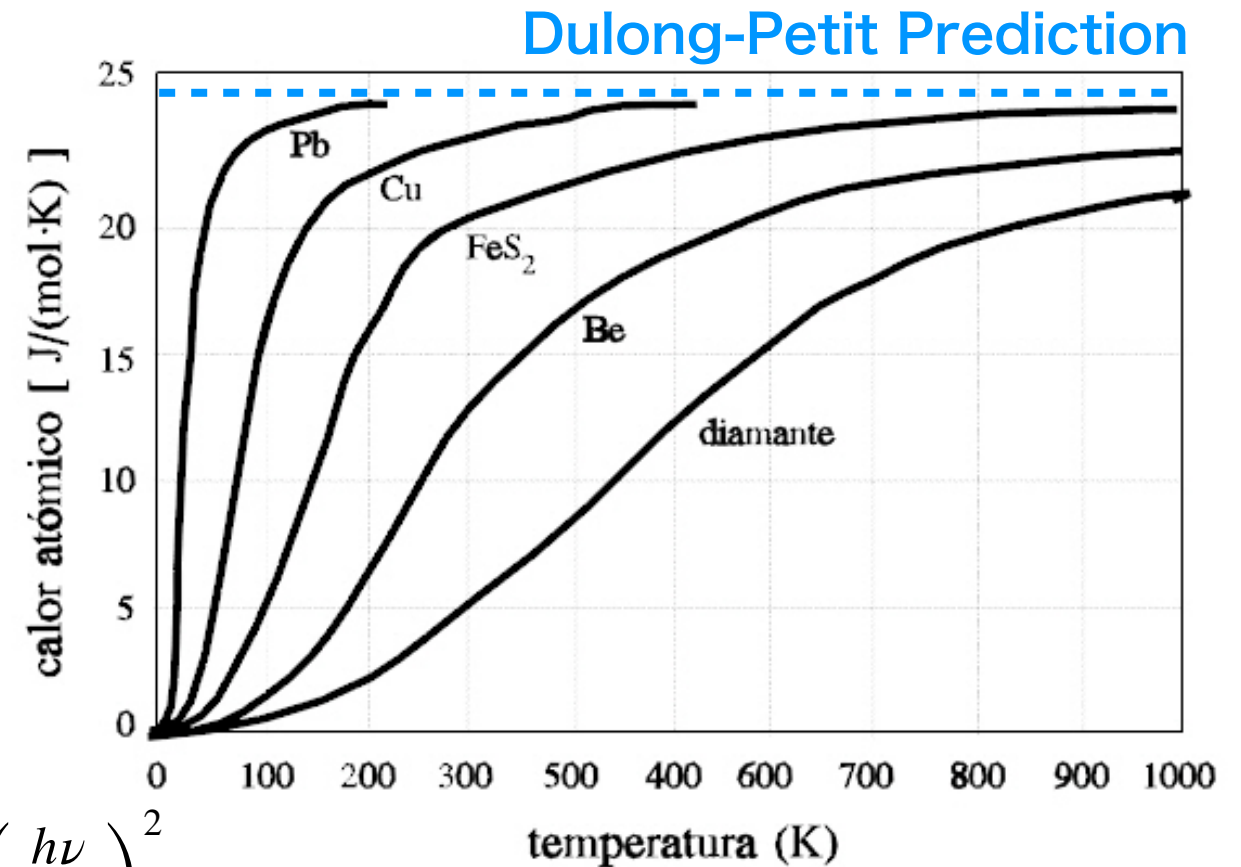
$$k_B T \rightarrow \frac{h\nu}{e^{h\nu/k_B T} - 1}$$

$$U = 3N_A \times \frac{h\nu}{e^{h\nu/k_B T} - 1} \quad C = 3N_A k_B \frac{e^{h\nu/k_B T}}{(e^{h\nu/k_B T} - 1)^2} \left(\frac{h\nu}{k_B T} \right)^2$$

Two temperature limits

$$C(\nu, T) \rightarrow 3N_A k_B, \quad (\nu/T \ll 1)$$

$$C(\nu, T) \rightarrow 3N_A k_B e^{-h\nu/k_B T} \left(\frac{h\nu}{k_B T} \right)^2, \quad (\nu/T \gg 1)$$

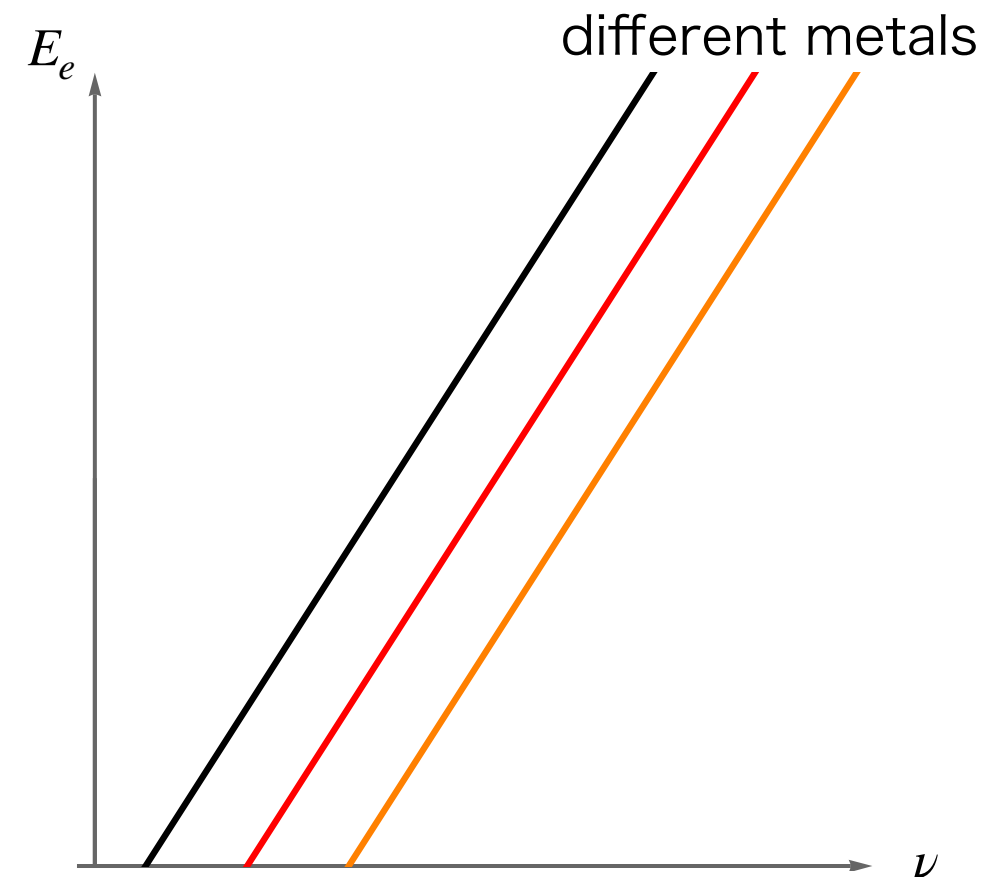


Photoelectric Effect

Radiation incidents and the exposed metal emits electrons

Emitted electron energy

- **not** depend on radiation intensity
- depends only on frequency
- **not** emitted below a threshold
- same slope for different metals



Photoelectric Effect

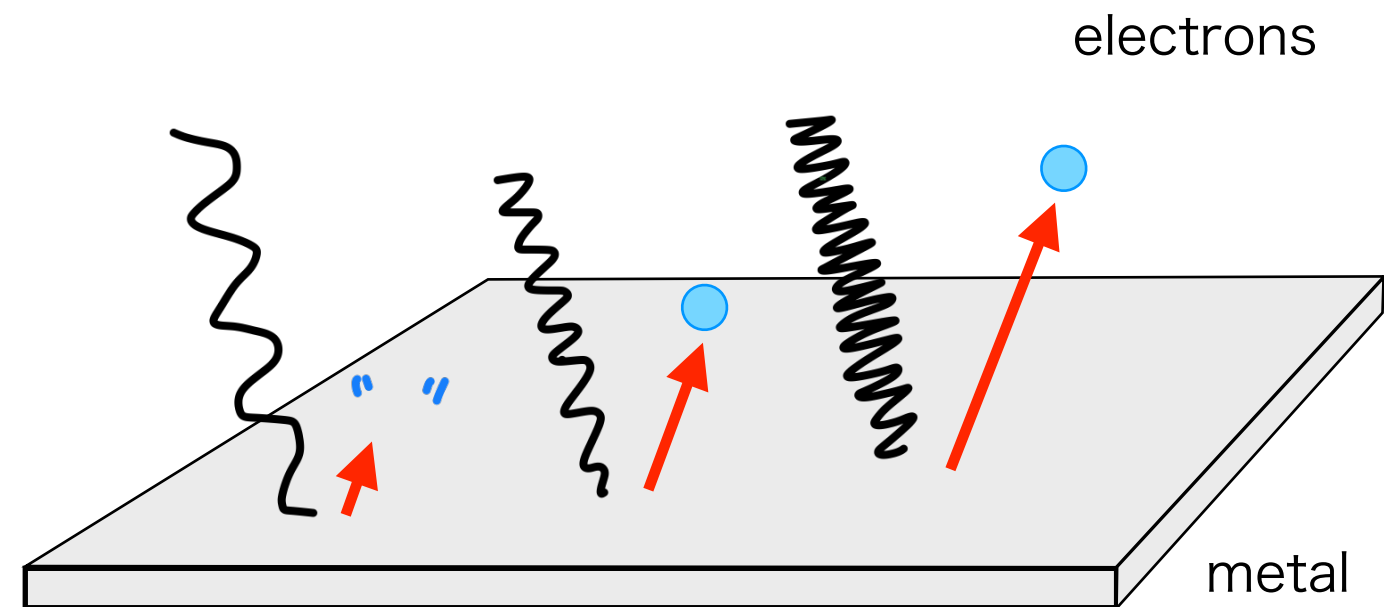
Einstein's idea

Radiation with frequency ν

= collection of particle (of light) with energy $E = h\nu$

Energy of emitted electrons

$$E_e = h(\nu - \nu_{\min})$$



Compton Scattering

Radiation: (elastic) scattering does not change its frequency

e.g.) mirror does not change color (= freq.) of objects

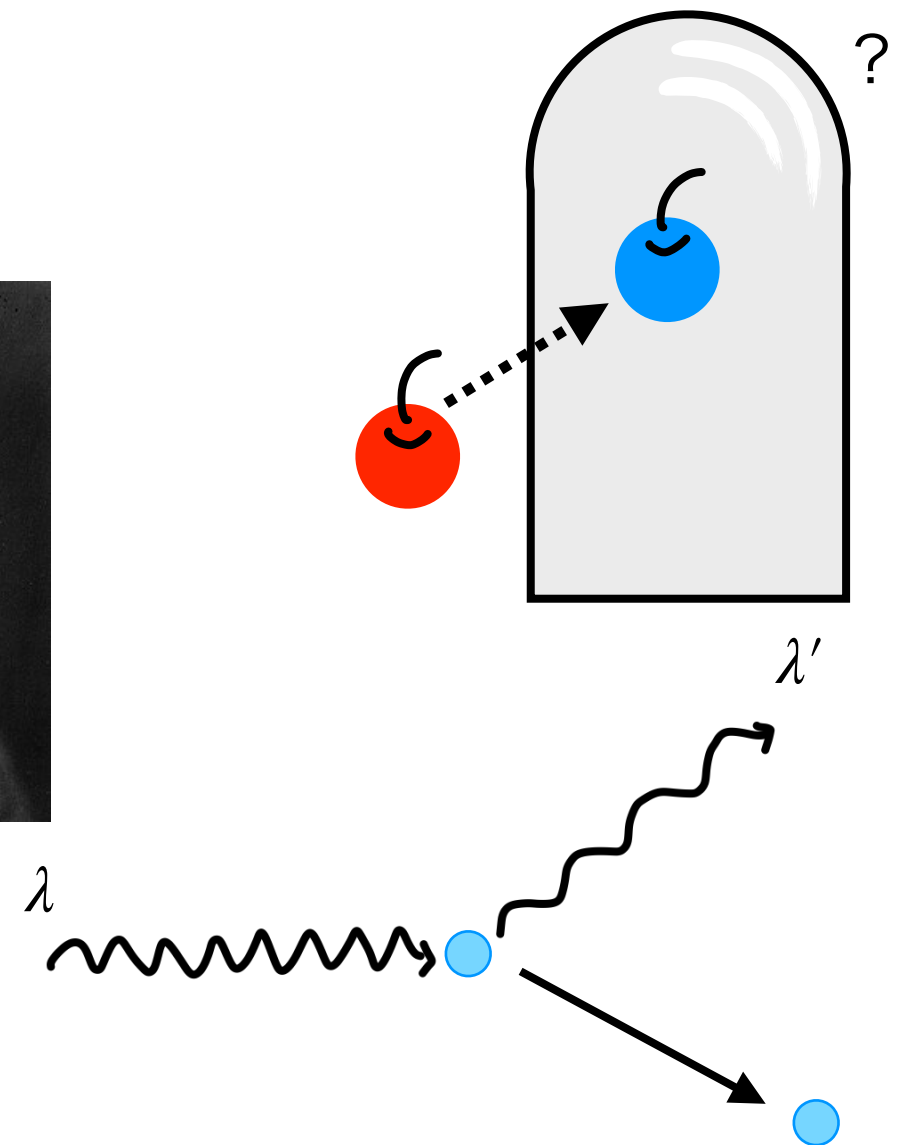
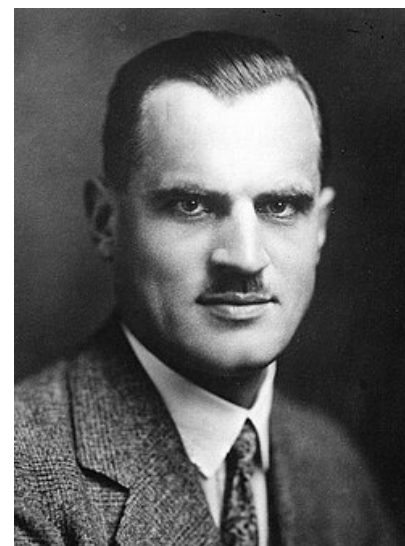
A. Compton (1923)

X-ray scattering with an electron
= **high energy radiation**

$$\lambda' - \lambda = \frac{h}{m_e c} (1 - \cos \theta)$$

interpreted as an electron and "photon" scattering

exercise) show this relation by assuming light as a particle

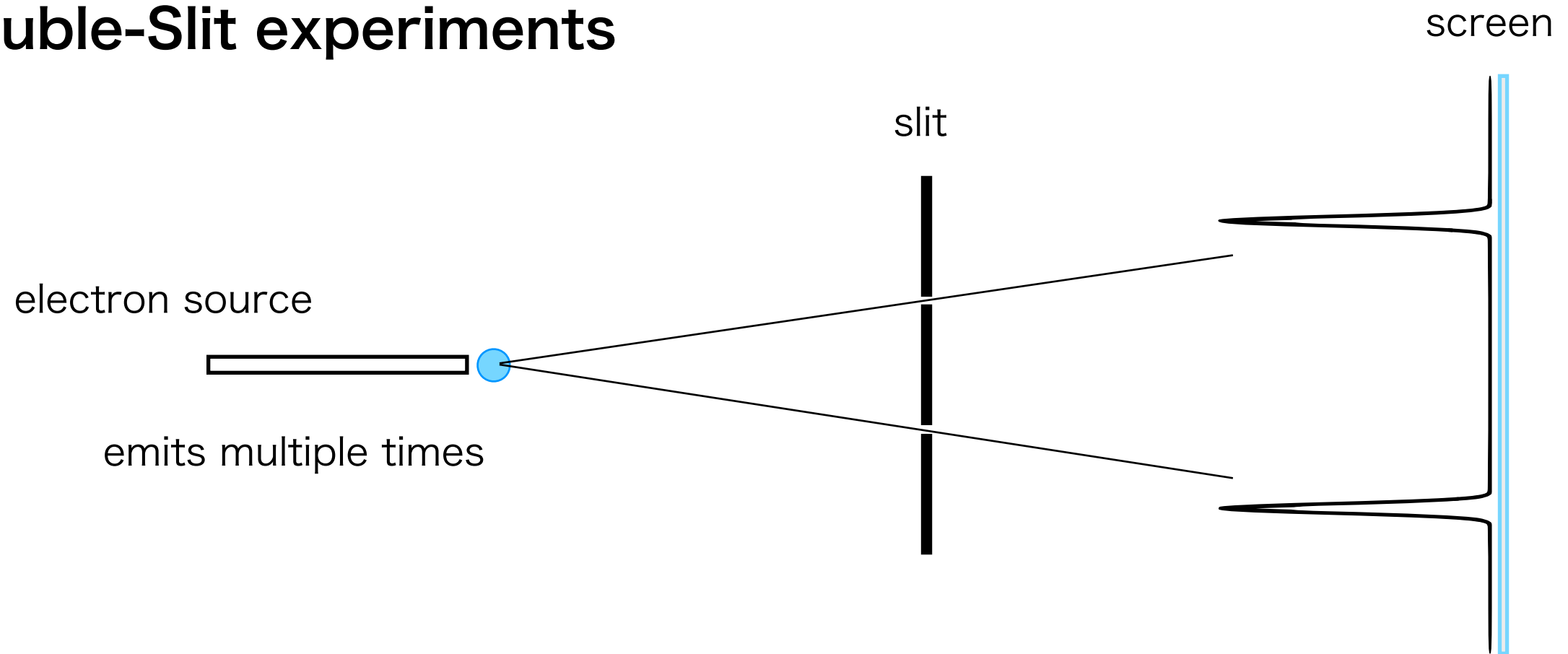


Old Quantum Mechanics



Particle to Wave

Double-Slit experiments



if single electron behaves as a classical particle:

electrons hit **two** specific places on screen

Double-Slit experiments

Not really

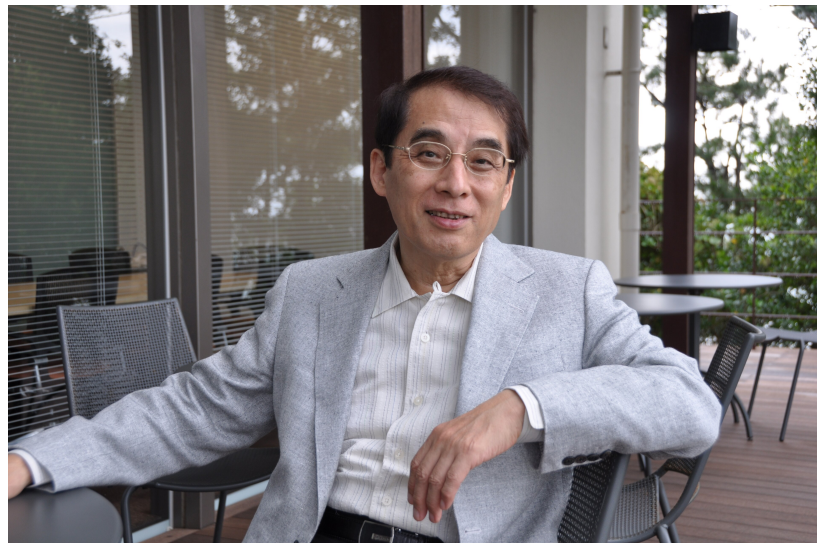
interference patterns are found

1961: C. Jönsson, (electron beam)

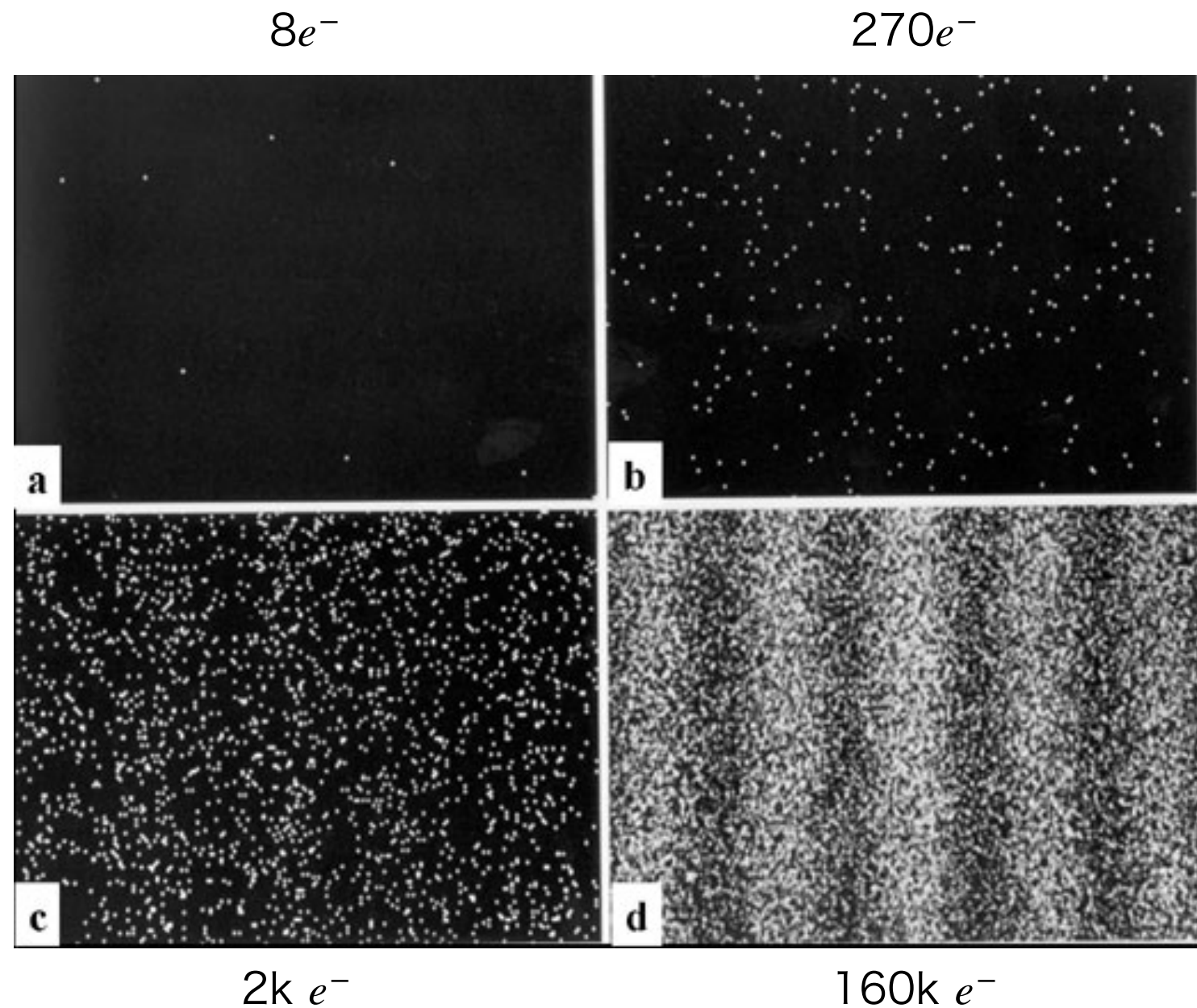
1974: G. F. Missiroli, G. Pozzi, P. G. Merli,
-> single electron "beam"

1989: A. Tonomura, et al

-> visualizing the accumulation of patterns

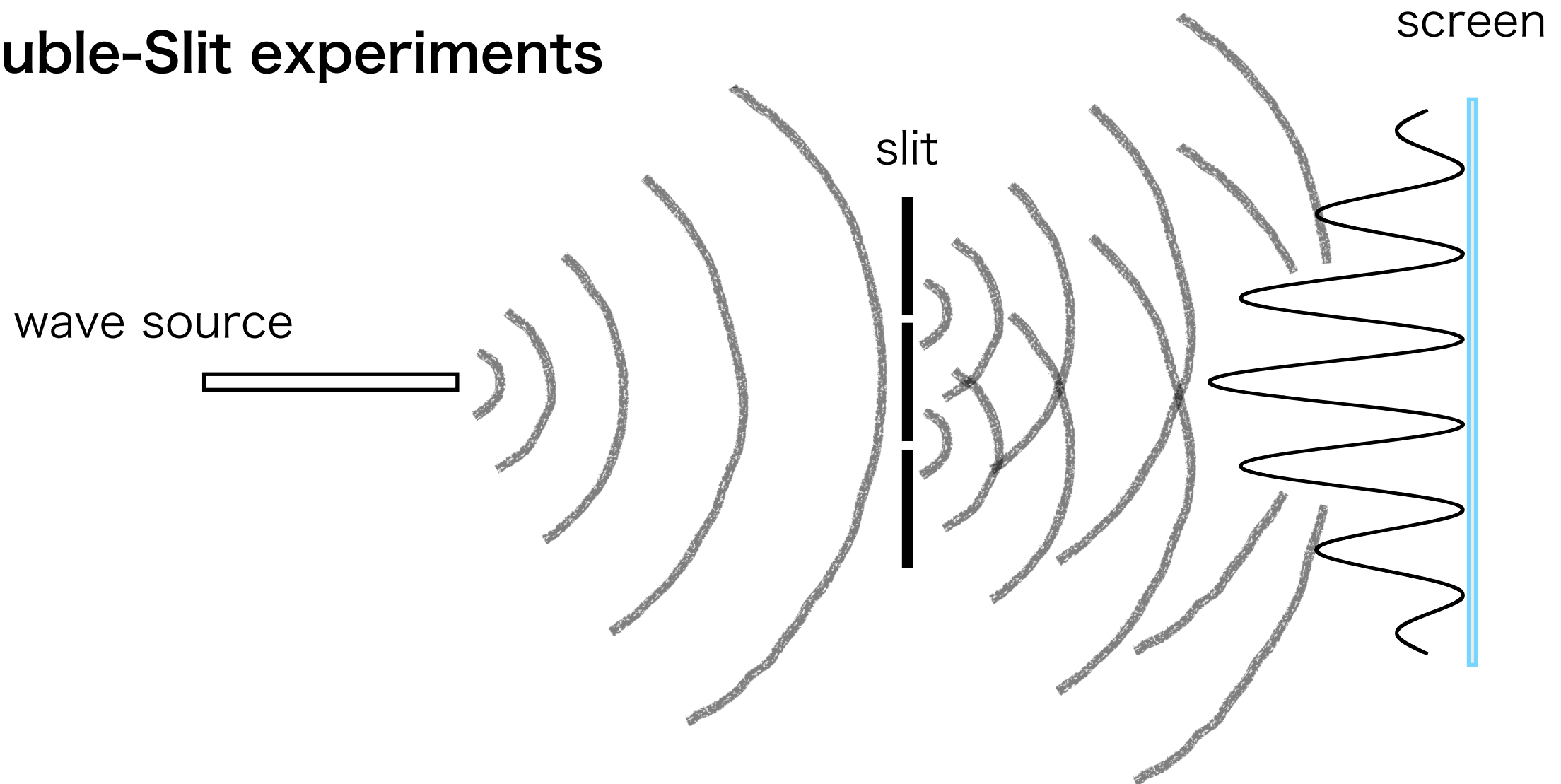


Akira Tonomura



[Hitachi Webpage](#)

Double-Slit experiments



Same as the interference of waves

single electron: appears as matter (hits at a specific position)

"existence" probability: interferes like wave

de Broglie Waves

A hypothesis for "matter" wave

Wave-like behavior of the moving particle (with momentum p)

de Broglie wavelength $\lambda = \frac{h}{p}$

in terms of wave number: $k = \frac{2\pi}{\lambda}$ $p = \hbar k$



Louis de Broglie

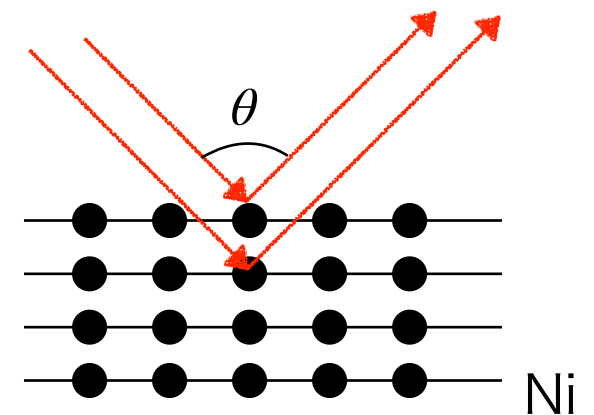
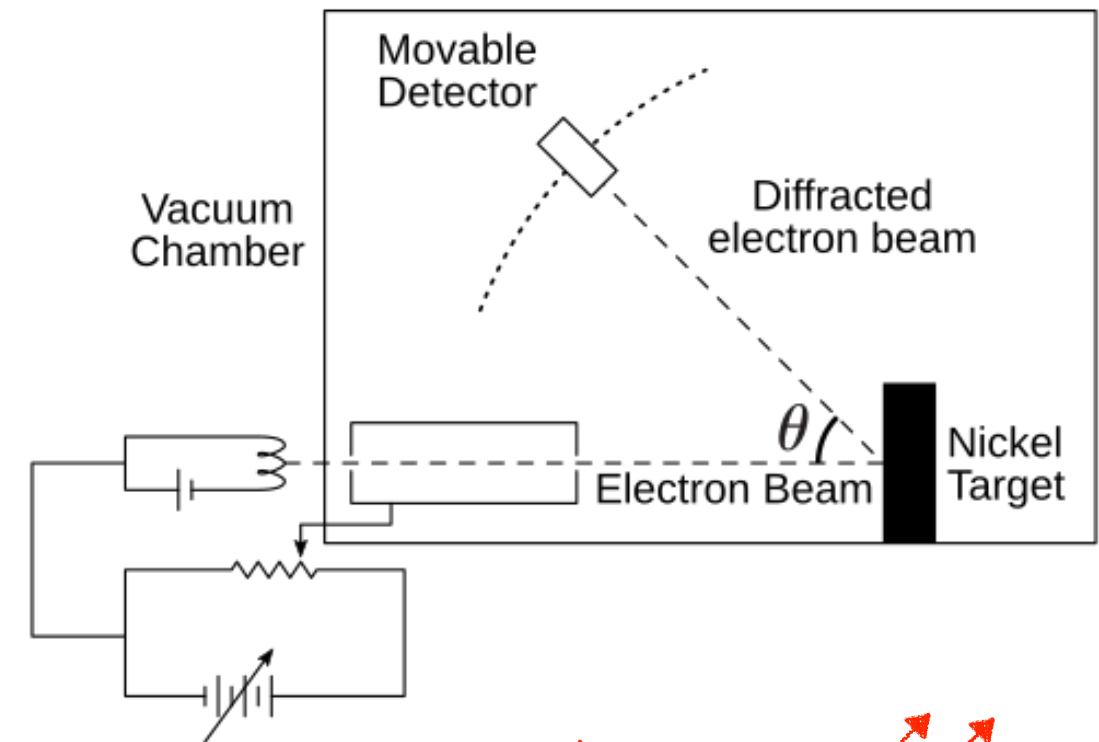
Davisson-Germer Experiment

(1923 - 1927)

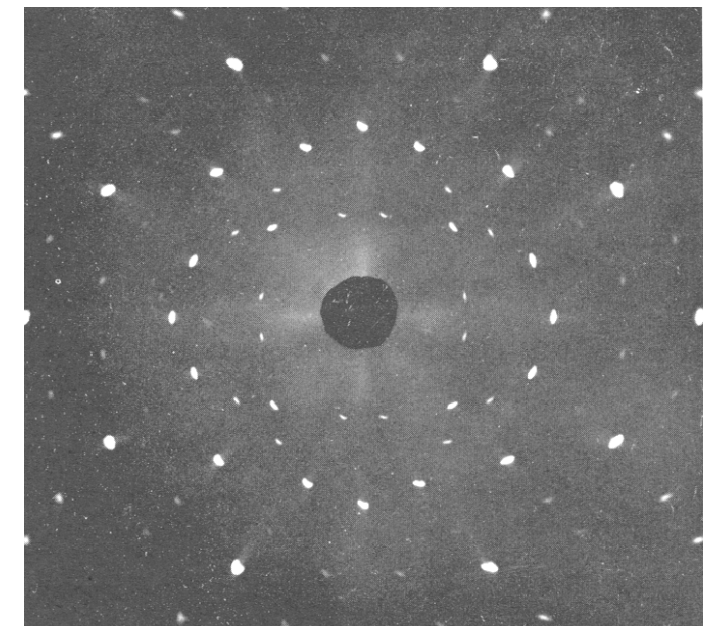
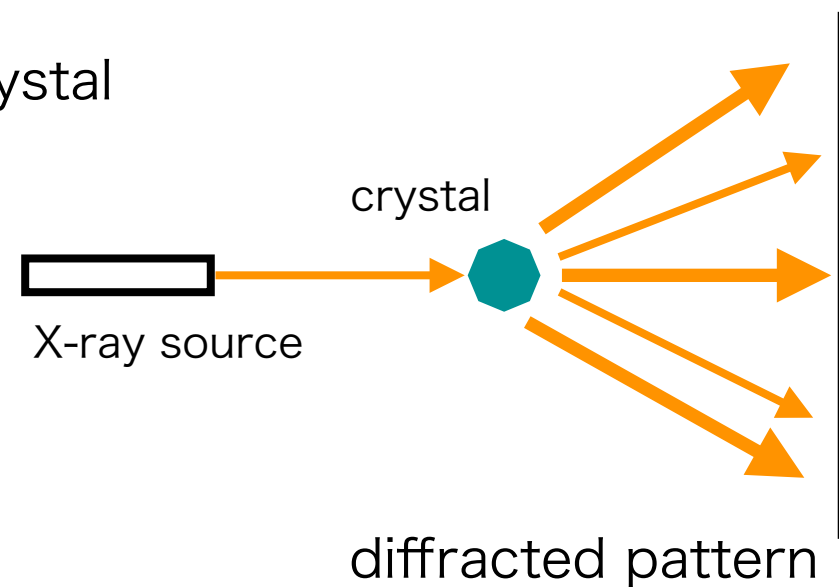
Relatively old experiment
showing an electron wave property

electron beam targeted on Nickel:
diffraction pattern observed

a property as wave!



Similar as X-ray diffraction by crystal



Particle-Wave Duality

- Until 20th Century, we knew enough elements:
 Electron (particle) and **Light** (wave)
- We have only looked at **one sides** of them
- **Particle** properties of **light** ("photon"):
 - Black-body radiation, photoelectric effect, Compton scattering, ...
 - Planck's quantization of radiation: $E = nh\nu$
- **Wave** properties of **electron** ("de Broglie wave"):
 - Double-Slit, Davisson-Germer, ...
 - de Broglie wave: $p = \hbar k$
- Wave and Particle are not separable objects beyond classical mechanics

Old Quantum Mechanics



Atomic Spectra

Atomic Spectra Problems

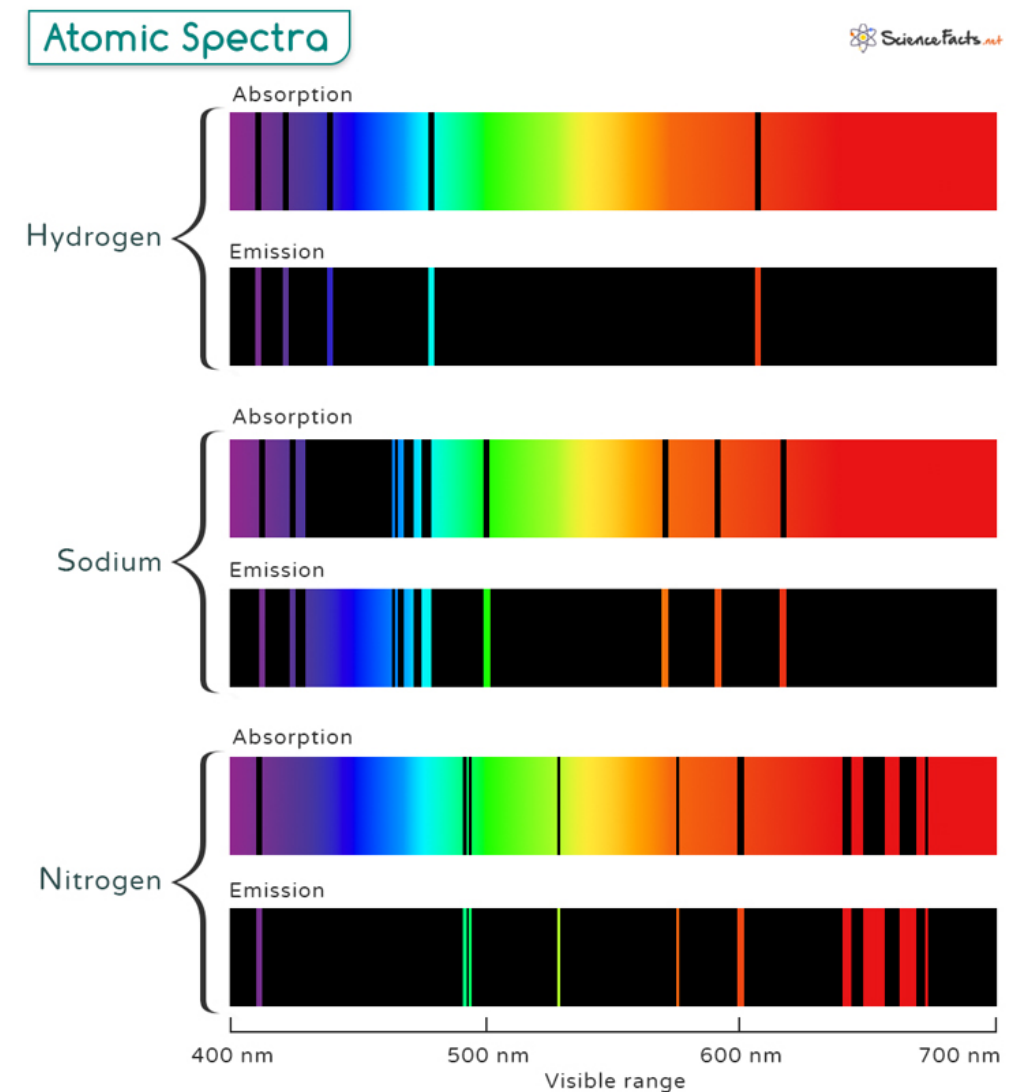
Until 20th Century, another problem confronted physicists.

Hot atomic gas: emit and absorb only at **certain** frequencies

the patterns of frequencies
depend on the elements

=> useful to find the elements constitution of stars
such as helium in the spectrum of the Sun

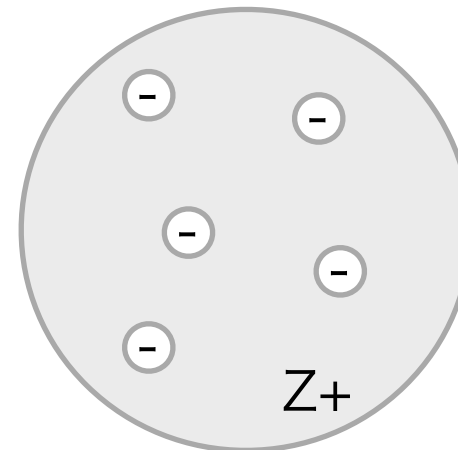
But, why??



Atomic Structure

important to understand the atomic spectra

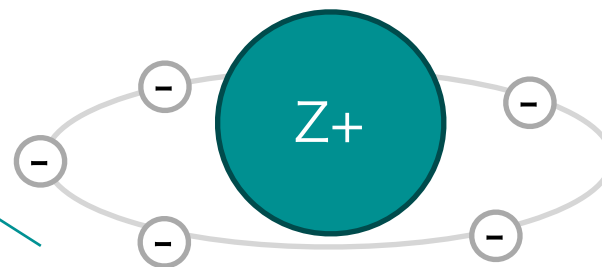
atom



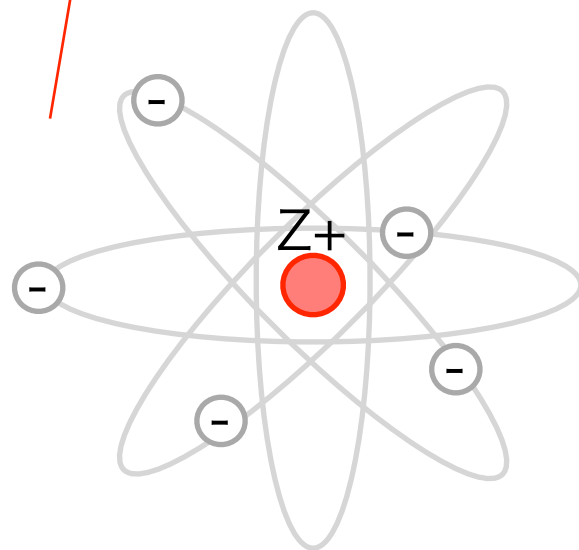
J.J. Thomson
a discoverer of electrons
Pudding Model



Christmas pudding



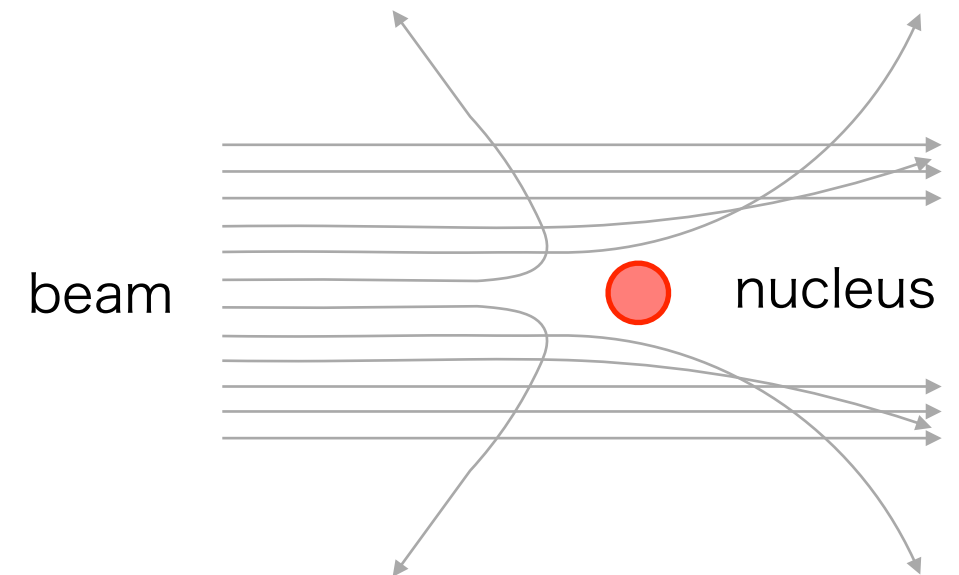
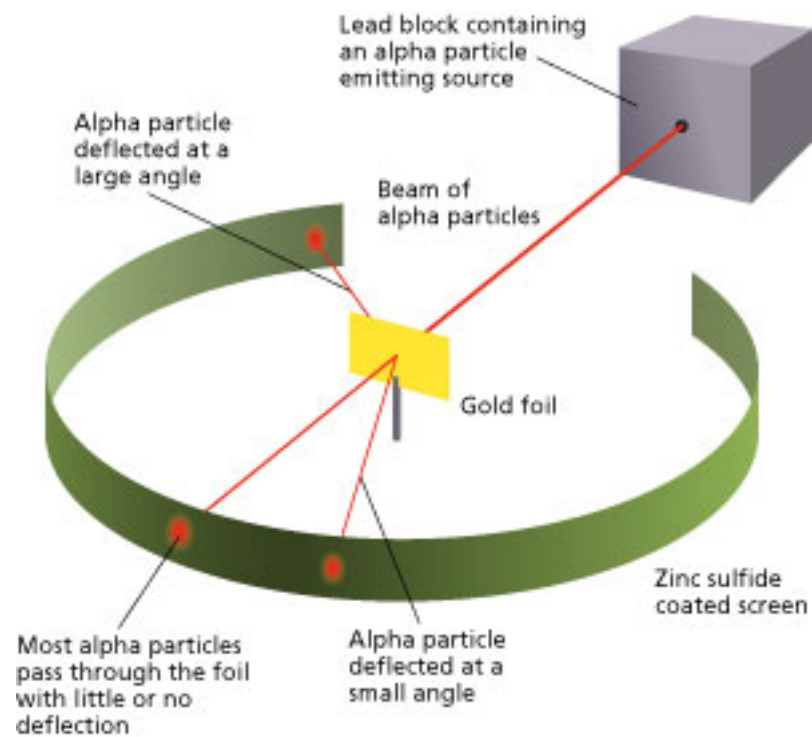
H. Nagaoka
Saturn Model



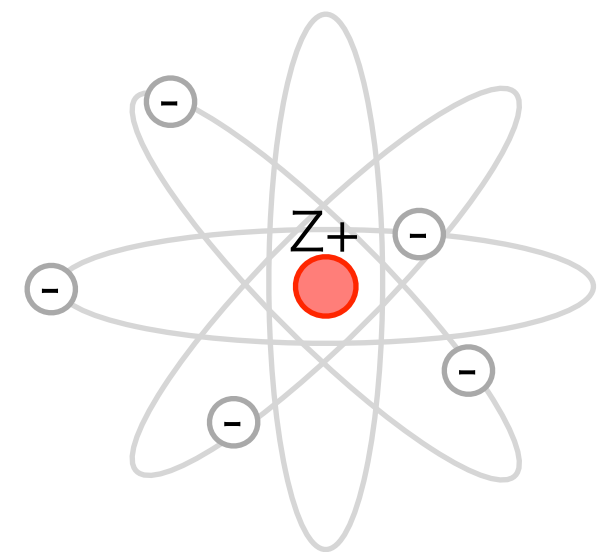
E. Rutherford
Planetary Model

Atomic Structure

Rutherford scattering experiment

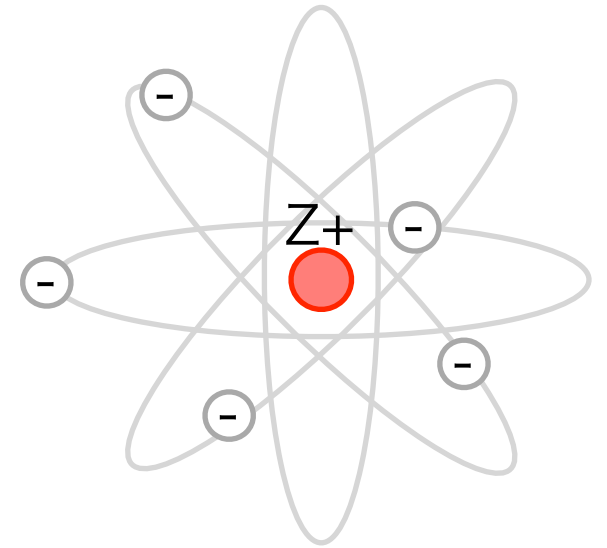


positive charge (nucleus) is localized in very compact space at center.
Are electrons orbitally running?



Atomic stability

If electrons run in an orbit:
electrons are accelerated by the electric field of nucleus



From electromagnetic theory:
electrons lose their energy by radiation "Bremsstrahlung (制動輻射)" and fall into the center

Rutherford Atomic structure is **unstable** in electromagnetism + mechanics.
How could atoms remain stable against the radiation?

Bohr's idea

Planck's constant

$$h = 6.626 \times 10^{-34} \text{ J} \cdot \text{Hz}^{-1}$$

same dimensions as angular momentum $J = \text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2}$
 $\text{Hz} = \text{s}^{-1}$

Bohr's guess: angular momentum of a bounded electron is discretized

Bohr's quantization condition $m_e v r = n \hbar, \quad (n = 1, 2, \dots)$

A generalization to elliptical orbits: **Bohr-Sommerfeld quantization condition**

$$\oint p_a dq_a = n_a h, \quad (n_a = 1, 2, \dots)$$

Discrete Energy Levels

Bohr's quantization condition $m_e v r = n \hbar$, $(n = 1, 2, \dots)$

The equation for centripetal force (向心力): $\frac{m_e v^2}{r} = \frac{Z e^2}{4 \pi r^2}$ (Heaviside-Lorentz Unit)

The orbital electron energy: $E = \frac{m_e v^2}{2} - \frac{Z e^2}{4 \pi r}$

these equation give $v = \frac{Z e^2}{4 \pi n \hbar}$, $r = \frac{4 \pi n^2 \hbar^2}{Z m_e e^2}$, $E = -\frac{Z^2 m_e}{2 n^2 \hbar^2} \left(\frac{e^2}{4 \pi} \right)^2$

the emitted radiation carries the energy of ΔE from transition $n \rightarrow n'$

$$\Delta E = -\frac{Z^2 m_e}{2 \hbar^2} \left(\frac{e^2}{4 \pi} \right)^2 \left(\frac{1}{n^2} - \frac{1}{n'^2} \right)$$

Discrete Energy Levels

Fine structure constant: $\alpha = \frac{e^2}{4\pi\hbar c} \simeq \frac{1}{137}$ (Heaviside-Lorentz Unit)

Electron mass: $m_e c^2 = 511 \text{ keV}$ $1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$

Energy level of electron

$$E = -\frac{Z^2 m_e}{2n^2 \hbar^2} \left(\frac{e^2}{4\pi} \right)^2 \simeq -\frac{13.6 Z^2 \text{ eV}}{n^2}$$

-> consistent with Hydrogen energy spectrum

Einstein's relations for radiation energy $\Delta E = h\nu = \hbar\omega$: emitted angular frequency

$$\omega = \frac{\Delta E}{\hbar} = -\frac{Z^2 m_e}{2\hbar^3} \left(\frac{e^2}{4\pi} \right)^2 \left(\frac{1}{n^2} - \frac{1}{n'^2} \right)$$

-> consistent with Balmer's formula (empirical)

$$\nu = Rc \left(\frac{1}{n^2} - \frac{1}{n'^2} \right)$$

Atomic Spectra

- There are several hints beyond classical mechanics:
 - **Absorption and emission lines of atoms**
 - **Rutherford scattering** (compressed core of atoms = nucleus)
- **Bohr(-Sommerfeld) Quantization condition:** $m_e v r = n \hbar$, $(n = 1, 2, \dots)$
 - > well-match for hydrogen spectrum

Summary

- 19th - early 20th centuries
 - several evidences for beyond classical mechanics
 - trials and errors for explaining these phenomena
- Key observations:
 - wave-particle dualities
 - Bohr-Sommerfeld quantization
 - **Quantum/Quanta**: minimum quantities which have above properties
- Not enough to be systematic (kind of symptomatic therapy: 对症治疗)
- Beginning of Quantum Mechanics

Next Lecture

- Several Formulations of Quantum Mechanics
 - Wave Mechanics (Erwin Schrödinger)
 - Matrix Mechanics (Werner Heisenberg)
- Equivalence of these formulations (by Paul Dirac)
- Several properties of quantum mechanics
- Probabilistic interpretation of quantum mechanics