Master's Thesis

Proton Decay in SUSY SU(5) GUTs Revisited after Discovery of the Higgs Boson

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概要

2012 年 7 月の大型ハドロン衝突型加速器 (LHC) 実験におけるヒッグス粒子の発見によって、素粒子標準模型 (標準模型) の全てのパラメーターが既知となった。この模型が現在知られている多くの素粒子現象を説明する一方で、ニュートリノ質量の起源に関する説明ができない、暗黒物質候補となる粒子の欠如、電荷の量子化の理由がない、などの理由で拡張も期待されている。

超対称性大統一理論はそのような拡張模型の一つの候補として盛んに研究されている。大統一理論では、三種類のゲージ相互作用が高いエネルギースケール ($\sim 10^{16}$GeV) で統一している。このゲージ群の統一による力の統一のみならず、クォークや電子、ニュートリノという物質粒子も統一的に記述される。超対称性は標準模型粒子と同じ表現で異なるスピン統計性を持つ粒子を導入する対称性であり、この導入によって力の統一が良くなる。

大統一を破れによって獲得する大統一で現れる粒子の質量は非常に重く、加速器実験ではこれらを直接探索することは不可能である。一方、超対称大統一理論では、クォークと電子、ニュートリノを混ぜる相互作用のため、核子崩壊という現象が予言される。そのため核子崩壊は、高エネルギーでの力の大統一を検証する一つの方法である。

標準模型を最小に拡張した最小超対称 $SU(5)$ 大統一模型では、陽子が荷電 K 中間子と反ニュートリノへ崩壊するモードに関して、その寿命が 10^{30} 年程度と予言される。この予言値は観測的制限と矛盾しているため、このモードを生成する質量次元 5 の有効演算子を、抑制または禁止するように模型を拡張しなければならなかった。

近年の LHC 実験による超対称粒子の直接探索の結果や、ヒッグス粒子の質量を再現する、最も簡潔な超対称模型の一つに、超対称性の破れのスケールが高い可能性が注目されている。本研究では、高いスケールで破れる超対称性を持つ最小超対称 $SU(5)$ 大統一模型において、大統一スケールで現れる粒子に関する質量の評価、及び質量次元 5 の演算子に起因した陽子の寿命を定量的に評価した。その結果、高スケール超対称性のもとで大統一が改善すること、寿命の短い崩壊モードが実験と矛盾せず、将来実験において観測される可能性があることを明らかにした。

Abstract

The standard model for particle physics (the standard model: the SM) describes the strong interaction, the weak interaction and the electromagnetic interaction. The strong interaction is the gauge interaction which describes dynamics of quarks and gluons, which construct nucleons; protons and neutrons. β decay ($n \to p + e^+ + \overline{\nu}$) is well explained by using the weak boson which obtains a mass by spontaneously symmetry breaking. The SM based on these gauge interactions explains many experimental results and phenomena.

In July 2012, the SM Higgs boson was discovered at the Large Hadron Collider (LHC): that is, all particles of the SM were discovered up until now. On the other hands, this phenomenological model is expected to be extended for some reasons: no existence of candidates of the dark matter, the origin of the neutrino mass, the reason for the charge quantization etc.

A candidate of the physics beyond the standard model (BSM) is the supersymmetric grand unified theory (SUSY GUT). In GUT, three gauge interactions are unified at very high energy ($\sim 10^{16}$GeV). Furthermore, the matter fields which are quarks and leptons are also unified. Supersymmetry (SUSY) introduces the copies which have the same charge as the SM particles and have the opposite spin-statistics against the SM particles. It is also well-known that SUSY improves the gauge coupling unification.

Since the energy scale of GUT is much high, the masses of the new particles which obtain mass due to SSB of GUT are much heavy. Thus, it is impossible to detect these particles directly in collider experiments. However, SUSY GUTs predict nucleon decay due to interactions mixing quarks and leptons. Nucleon decay is one of the ways to verify the unification of the gauge interactions. In the Super-Kamiokande experiment, however, there is no evidence of nucleon decay.

In the minimal SUSY $SU(5)$ GUT, which is the minimal GUT extension of the SM, the partial lifetime of proton is about 10^{30} years for the mode $p \to K^+ + \overline{\nu}$. On the other hand, an experimental lower bound for this mode is 5.9×10^{33} years. Since the theoretical prediction is contradicted this experimental results, the dimension five effective operators which give rise to this decay must be suppressed or forbidden by some mechanism.

The high-scale SUSY breaking mechanism gains attention as the most simple SUSY model which explains the recent SUSY search experiments at collider and the mass of the SM Higgs.

In this work, we have investigated the minimal SUSY $SU(5)$ GUT with the high-scale SUSY breaking mechanism; in particular, we have evaluated the mass spectrum of the GUT particles and the proton lifetime caused by the dimension-five operators quantitatively. As a results, we have revealed that the unification is improved in terms of the mass of GUT-particles; that is, the unification of the standard model gauge interactions is improved without large threshold corrections at GUT scale. Moreover, we also have revealed that the dimension-five operators is not need to suppress or forbid; this decay mode may be discovered in the future experiment at Hyper-Kamiokande. This work is based on these papers; [1, 2].

Contents

1 Introduction

In July 2012, the standard model Higgs boson was discovered by the collider experiment at the Large Hadron Collider (LHC) [3, 4]. While the standard model for the particle physics (SM) [5, 6, 7] explains many phenomena, it is expected that the SM will be extended in some point of view. We have been able to understand the SM more deeply than before due to the discovery of the SM Higgs boson. This discovery not only confirms that the SM is the best to describe the particle phenomena below several TeV scale but also helps us to explore the physics beyond the SM (BSM). Namely, it is also possible for us to investigate the BSM more realistically by treating the mass of the SM Higgs as the input parameter. Supersymmetric grand unified theories (SUSY GUTs) have been investigated as candidates of BSMs.

Supersymmetry (SUSY) has interesting properties. One of them is the improvement of the gauge coupling unification. There are three gauge interactions in the SM, which are strong interaction and electroweak interactions. Though it seems that these gauge couplings are unified at very high energy region ($10^{14} \sim 10^{17}$ GeV) without SUSY, they are unified more precisely in supersymmetric theories at $\sim 10^{16}$ GeV [8]. Furthermore, there exist candidates of the dark matter since the lightest SUSY particles are stable thanks to a discrete symmetry "R-parity".

Collider experiments tell us that the observed Higgs boson has the mass of about 125 GeV, which is heavier than the tree-level mass of the lighter neutral Higgs boson in the minimal supersymmetric standard model (MSSM), and also there is no signal of the superpartners; especially the masses of superpartners which have the color charge are severely constrained. High-scale SUSY and Split SUSY scenarios have gotten a lot of attention recently since the 125 GeV Higgs boson is realized by large quantum corrections [9] and the scalar partners are much heavier than the SM particles. Scalar partners of the SM fermions in these scenarios, which have the masses of about 10^2 TeV, are heavier than those in low-scale SUSY breaking scenarios. On the other hand, the mass scale of gauginos (and also higgsinos in some model) is several TeV. It is pointed out that the gauge coupling unification well work in spite of the decoupled scalar partners [10, 11]. SUSY flavor problems can be also cured due to the heavy scalar fermions.

SUSY GUTs well solve some problems and questions in the SM. For instance, the reason why the electric charge is quantized is explained by unifying the SM gauge group into the simple group. Also the anomaly cancellation in the SM may be understood in some GUT models. However, the energy scale of the SUSY GUTs is around 10^{16}GeV which is much higher than the electroweak scale ($\sim 10^2$GeV). We hence can not detect any GUT particles directly by collider physics.

Though, it is possible to detect some evidence indirectly. Nucleon decay is predicted as a typical phenomenon in GUTs. Since quarks and leptons are embedded in multiplets which are transformed properly under the unified gauge group in SUSY GUTs, the baryon number is no longer a conserved charge.

The minimal SUSY $SU(5)$ GUT is one of such SUSY GUTs, which is the most minimal extended theory of the SM. It is well known that the minimal SUSY $SU(5)$ GUT has been excluded due to predicting too short proton lifetime [12, 13]. This short lifetime is given rise to by the color-triplet Higgs multiplet which is the $SU(5)$ partner of the MSSM Higgs; main decay mode is proton to charged K meson and anti-neutrino. Thus, the interactions which give rise to the terrible nucleon decay have to be suppressed or forbidden by some mechanism. For instance, the missing-partner model is possible to suppress these operators by imposing the Peccei-Quinn symmetry [14]. As another example, the theory with global $U(1)$ R symmetry is able to be evaded from experimental constraints since the terrible interactions are forbidden by this symmetry [15]. The higher-dimensional operators which are suppressed by Planck mass are also unfavorable interactions in supersymmetric theories in point of view of nucleon decay. Accordingly, it is supposed to be suppressed by flavor symmetry, which is referred to as the Froggatt-Nielsen mechanism [16], as reference in [17]. However, as we will show later, the minimal SUSY $SU(5)$ GUT makes proton lifetime prediction longer in the high-scale SUSY breaking scenario.

This thesis is organized as follows. In Sec. 2, we review the SM and its success with particle physics briefly. Then, we introduce supersymmetry as an allowed space-time symmetry and its breaking mechanisms. In addition, we will minimally extend the SM to the supersymmetric one. In particular, the most simplest model which explains the recent collider experimental results, "the high-scale SUSY breaking" is reviewed in the last of this section. In next section, we will show the introduction of the minimal supersymmetric $SU(5)$ grand unified theory (SUSY $SU(5)$ GUT). We evaluate the GUT scale mass spectrum by using threshold corrections in Sec. 4 in the high-scale SUSY breaking scenario. In this section, we will review matching conditions at some thresholds. Then, our numerical analysis by using 2-loop level renormalization group equations shows that the GUT-scale particles have the same order ($\sim 10^{16}$GeV) masses in the high-scale SUSY breaking scenario. In Sec. 5, we will show that harmful proton decay mode can be evaded from the recent experimental limit in the high-scale SUSY breaking scenario. In the last section, we will summarize and discuss our results.

2 Standard Model and its Supersymmetric Extension

In this section, we review the standard model (SM) and its supersymmetric extensions. First of all, we introduce the standard model in particle physics. All of the parameters in the SM are already measured by various experiments. And then, we introduce supersymmetry and its fascinating properties. If we adopt the supersymmetric (SUSY) theory as an extension of the SM, SUSY must be broken above electroweak scale since there is no direct evidence by the collider experiment. We briefly introduce the mechanisms for SUSY breaking. Next we will introduce the minimal supersymmetric standard model (MSSM) as one of the candidate of the extension of the SM. The advantage and disadvantage of MSSM are introduced after that. Finally, we explain the High-scale SUSY breaking scenario briefly.

2.1 The Standard Model in Particle Physics

Gauge invariance is one of the requirements in particle physics. Gauge transformation is defined as the local phase transformation. If ϕ is a field belonging to a fundamental representation under a gauge group, the gauge transformation for ϕ is defined as follows*:

$$\phi \to U\phi, \quad (U = e^{i\alpha_a T^a}), \tag{2.1.1}$$

where T^a is the "generator" of gauge symmetry and the index a is summed up. Since this symmetry is local one, derivative terms of this field are not transformed covariantly under this transformation. In order to make these terms covariant under gauge transformation, we must introduce the covariant derivative. The covariant derivative is defined as,

$$D_\mu = \partial_\mu - ig\mathcal{A}_\mu, \quad (\mathcal{A}_\mu = A_\mu^a T^a), \tag{2.1.2}$$

where ∂_μ is an ordinary differential operator and $\mathcal{A}_\mu$ is called a gauge field, and g is a gauge coupling constant. The number of the roman indices of the gauge field A_μ^a is equal to the number of the generators, namely these gauge fields belong to the adjoint representation of this gauge group. Under the gauge transformation, these gauge fields are defined as the fields transforming as follows

$$\mathcal{A}_\mu \to U\mathcal{A}_\mu U^{-1} + \frac{i}{g} U\partial_\mu U^{-1}. \tag{2.1.3}$$

Therefore, the covariant derivative of the field ϕ is transformed as

$$\begin{aligned} D_\mu\phi =& \partial_\mu\phi - ig\mathcal{A}_\mu\phi \\ \to& \partial_\mu(U\phi) - ig\left[U\mathcal{A}_\mu U^{-1} + \frac{i}{g}U\partial_\mu U^{-1}\right]U\phi \\ =& U(D_\mu\phi). \end{aligned} \tag{2.1.4}$$

*There is no difference whether the spin-statistics of this field ϕ is the Bose-Einstein or the Fermi-Dirac. However, this description is correct in the only case that this field is transformed as the fundamental representation under this gauge group.

For the gauge invariance, the kinetic terms for the gauge fields must be written as the gauge invariant form. However, since the gauge fields itself are not gauge invariant, we need to find the gauge invariant quantities. A quantity including only the gauge field, called field strength tensor, is defined as follows:

$$\mathcal{F}_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu - ig[\mathcal{A}_\mu, \mathcal{A}_\nu]. \tag{2.1.5}$$

This quantity is transformed under the gauge transformation as

$$\mathcal{F}_{\mu\nu} \to U \mathcal{F}_{\mu\nu} U^{-1} \quad : \mathcal{F}_{\mu\nu} = F^a_{\mu\nu} T^a. \tag{2.1.6}$$

Hence, the gauge invariant quantity must be proportional to $\mathrm{tr}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}$. Generators are normalized as $\mathrm{tr}(T^a T^b) = \frac{1}{2}\delta^{ab}$ for the case of the fundamental representation. In order to make the kinetic terms of gauge fields canonical, the coefficient of the gauge invariant quantity must be determined as

$$-\frac{1}{2}\mathrm{tr}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}. \tag{2.1.7}$$

The gauge symmetry of the standard model for particle physics is $SU(3)_C \times SU(2)_L \times U(1)_Y$. Scalar fields and fermion fields in the SM are transformed as the fundamental (anti-fundamental) representations or trivial representations under this gauge symmetry. The gauge representations of the bosonic sector of the SM are showed in Table 1. The SM has explained many experimental observables well.

The $SU(3)_C$ symmetry describes "the strong interaction" of which gluons are the corresponding gauge boson. There are the fermions called "quarks" which transform as the fundamental representation under this gauge theory. Quantum Chromodynamics (QCD) describes the dynamics of gluons and quarks. It is believed that QCD describes the confinement of quarks at low-energy since the QCD couplings become more strong and non-perturbative effects can not be negligible near the cut-off scale $\Lambda_{QCD} \sim 200$MeV. Below this energy scale, since the description with quarks and gluons is not correct, we must use the description with mesons and baryons which are called hadrons. Protons and neutrons that consist of light quarks, up and down quarks, form nuclei. These nucleons with masses of $938 \sim 940$MeV are heavy though the masses of light quarks are about several MeV. This mass problem was solved for the first time by Y. Nambu and G. Jona-Lasinio [18, 19] by using analogy with mass-acquirement mechanism of photon in superconductor, which is called "chiral symmetry breaking".

The $SU(2)_L \times U(1)_Y$ is the unified gauge group of electroweak (EW) interactions. There are four gauge bosons corresponding to the generators of the gauge group $SU(2)_L \times U(1)_Y$. Three of these gauge bosons, called the weak bosons, couple to only left-handed fields. The other is $U(1)_Y$ gauge boson which couples to the fields having $U(1)_Y$ charge called hypercharge Y.

Table 1: Bosonic sector of the SM

	name	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
G_μ	gluon	**8**	**1**	0
W_μ	W-boson	**1**	**3**	0
B_μ	B-boson	**1**	**1**	0
H	Higgs	**1**	**2**	1/2

Weak interaction was found in the β decay of particles historically. For instance, it is well known that neutron decays into proton, electron and anti-neutrino, namely down-quark decays into up-quark, electron and anti-neutrino at elementally particle level. At the first time, this decay was described as four-fermi interactions proposed by E. Fermi which are non-renormalizable operators. Then, by introducing massive vector bosons, this operator is obtained as effective operator after integrating out massive vector bosons. These massive vector bosons are the weak bosons.

However gauge bosons must be massless particles due to gauge invariance. Then, the electroweak gauge group must be broken down to $U(1)_{EM}$ which is electromagnetic gauge group by spontaneously symmetry breaking (SSB), called the Higgs mechanism* [22]. The SSB of the EW gauge group is caused by the scalar boson which has vacuum expectation value (VEV), the so-called Higgs boson. The Higgs boson is $SU(2)_L$ doublet and has hypercharge 1/2. This mechanism is caused by the scalar potential of the Higgs boson as

$$V(H) = -\mu^2 H^\dagger H + \frac{\lambda}{2}(H^\dagger H)^2. \tag{2.1.8}$$

where μ^2 is positive. The minimum of this potential occurs at

$$\langle H\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}, \quad v = \sqrt{\frac{2\mu^2}{\lambda}}. \tag{2.1.9}$$

The VEV of the Higgs field is measured as $v = 246.22\text{GeV}$ [24]. This VEV gives rise to the masses of the gauge bosons through the covariant derivative as follows. The kinetic term of the Higgs boson is given by

$$\begin{aligned}(D_\mu H)^\dagger D^\mu H =& \partial_\mu H^\dagger \partial^\mu H \\ &+ igH^\dagger \tau^a W^a_\mu \partial^\mu H + ig' B_\mu H^\dagger \partial^\mu H + (\text{h.c.}) \\ &+ H^\dagger\left(g^2 \tau^a \tau^b W^a_\mu W^{b\mu} + g'^2 B_\mu B^\mu + 2gg'\tau^a W^a_\mu B^\mu\right) H,\end{aligned} \tag{2.1.10}$$

*Three research groups; by F.Brout and R.Englert [20]; by P.W.Higgs [21, 22]; and by G.S.Guralnik, C.R.Hagen, and T.W.B.Kibble [23], investigated this mechanism independently in 1964.

where W_μ^a and B_μ are the gauge bosons of $SU(2)_L$ and $U(1)_Y$; g and g' are the gauge couplings of $SU(2)_L$ and $U(1)_Y$, respectively. τ^a denotes the generators of $SU(2)_L$. After the Higgs field obtains VEV, the mass terms of the gauge bosons are written down as

$$\frac{1}{4}g^2v^2W_\mu^+W^{-\mu} + \frac{1}{8}v^2\sqrt{g^2+g'^2}Z_\mu Z^\mu \equiv m_W^2 W_\mu^+W^{-\mu} + \frac{1}{2}m_Z^2 Z_\mu Z^\mu, \tag{2.1.11}$$

where these are the mass eigenstates of the gauge fields defined as

$$\begin{aligned} W_\mu^\pm &= \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2), \\ Z_\mu &= \frac{1}{\sqrt{g^2+g'^2}}(gW_\mu^3 - g'B_\mu), \\ A_\mu &= \frac{1}{\sqrt{g^2+g'^2}}(g'W_\mu^3 + gB_\mu). \end{aligned} \tag{2.1.12}$$

$W_\mu^\pm$, Z_μ and A_μ are charged W-bosons, neutral Z-boson, and photon, respectively. By using this mass eigenstate of the gauge fields, the covariant derivative for a field belonging to a fundamental representation of $SU(2)_L$, with $U(1)_Y$ charge Y becomes;

$$\begin{aligned} D_\mu &= \partial_\mu - igA_\mu^a\tau^a - ig'YB_\mu \\ &= \partial_\mu - i\frac{g}{\sqrt{2}}\left(W_\mu^+\tau^+ + W_\mu^-\tau^-\right) - i\frac{1}{\sqrt{g^2+g'^2}}Z_\mu\left(g^2\tau^3 - g'^2Y\right) - i\frac{gg'}{\sqrt{g^2+g'^2}}A_\mu\left(\tau^3+Y\right), \end{aligned} \tag{2.1.13}$$

where $\tau^\pm \equiv (\tau^1 \pm i\tau^2)/2$. This means the electromagnetic charge is defined as the following form;

$$e \equiv \frac{gg'}{\sqrt{g^2+g'^2}}. \tag{2.1.14}$$

By some experiments, the masses of the weak bosons are confirmed as [24]

$$\begin{aligned} m_W &\equiv \frac{1}{2}gv = 80.385 \pm 0.015\text{GeV}, \\ m_Z &\equiv \frac{1}{2}\sqrt{g^2+g'^2}v = 91.1876 \pm 0.0021\text{GeV}. \end{aligned} \tag{2.1.15}$$

Also, the Weinberg angle defined as

$$\sin^2\theta_W \equiv \frac{g'^2}{g^2+g'^2}, \tag{2.1.16}$$

Table 2: gauge representation of fermions in the SM

	name	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
Q_L	left-handed quark doublet	**3**	**2**	$1/6$
L_L	left-handed lepton doublet	**1**	**2**	$-1/2$
u_R	right-handed up-type quark	**3**	**1**	$2/3$
d_R	right-handed down-type quark	**3**	**1**	$-1/3$
e_R	right-handed charged lepton	**1**	**1**	-1

is determined as $\sin^2\theta_W(m_Z) = 0.231$. The electromagnetic charge is related to the weak gauge coupling through the Weinberg angle: $e = g\sin\theta_W$.

Now we expand the Higgs potential around the VEV. We will work in the unitary gauge.

$$H = U(x)\begin{pmatrix} 0 \\ \dfrac{v+h(x)}{\sqrt{2}} \end{pmatrix} \tag{2.1.17}$$

with $U(x) = e^{i\xi(x)}$; $\xi(x) \equiv \xi^a(x)\tau^a$. We obtain the Higgs potential after we substitute this field for Eq. (2.1.8).

$$V(h) = \frac{1}{8}\lambda h^4(x) + \frac{1}{2}\lambda v h^3(x) + \frac{2\mu^2}{2}h^2(x) - \frac{1}{4}\lambda v^4. \tag{2.1.18}$$

Thus, we obtain the mass of the physical Higgs boson;

$$m_h^2 = 2\mu^2 = \lambda v^2 \tag{2.1.19}$$

Now, let us consider the fermion sector in the SM. There are various fields belonging to different representations of the standard model gauge group. These fields are summarized in Table 2. Q_L is a quark doublet field including left-handed up quark and down quark. L_L is a lepton doublet field including left-handed electron and neutrino. u_R, d_R, e_R are $SU(2)_L$ singlet right-handed up, down and electron field, respectively.

There is the structure called "generation" or "flavor" in the fermion sector. Gauge interactions of these fermions are flavor-blind interactions because these terms are given rise to by canonical kinetic terms as $i\overline{\psi}_j \not{D} \psi_j$ where j is flavor index. However, in the SM, since there is the scalar field, the Higgs field, this flavor symmetry is broken by the Yukawa couplings as follows.

$$\mathcal{L}_{\text{Yukawa}} = -Y^u_{ij}\epsilon_{ab}\overline{u}_{Ri}H^a Q^b_{Lj} + Y^d_{ij}\epsilon_{ab}\overline{d}_{Ri}(H^C)^a Q^b_{Lj} + Y^e_{ij}\epsilon_{ab}\overline{e}_{Ri}(H^C)^a L^b_{Lj}, \tag{2.1.20}$$

where H denotes the Higgs boson, and $H^C \equiv i\sigma_2 H^*$ is charge conjugation of H. $a, b, \ldots$ and ϵ_{ab} denote the indices of $SU(2)_L$ and totally anti-symmetric tensor, respectively.

When the Higgs boson obtains the vacuum expectation value, these Yukawa terms become the Dirac mass terms of quarks and leptons after redefining the flavor basis in order to diagonalize the Yukawa matrices.

$$\begin{aligned}\mathcal{L}_{\text{mass}} =& \frac{v}{\sqrt{2}}Y^u_{ij}\overline{u}_{Ri}u_{Lj} + \frac{v}{\sqrt{2}}Y^d_{ij}\overline{d}_{Ri}d_{Lj} + \frac{v}{\sqrt{2}}Y^e_{ij}\overline{e}_{Ri}e_{Lj} \\ =& \sum_i \left(m_{u_i}\overline{u}_{Ri}u_{Li} + m_{d_i}\overline{d}'_{Ri}d'_{Li} + m_{e_i}\overline{e}_{Ri}e_{Li} \right),\end{aligned} \tag{2.1.21}$$

How many generations are in the standard model? J. W. Cronin, V. L. Fitch and their collaborators discovered the evidence of CP violation through decay of K_L meson into two pions [25]. CP violation is caused by a complex phase in couplings. M. Kobayashi and T. Maskawa revealed that at least three generations were needed to remain complex phases by field re-definition [26]. The mass and weak eigenstates are related as

$$\begin{pmatrix} u_L \\ c_L \\ t_L \end{pmatrix}_{\text{weak}} = \begin{pmatrix} u_L \\ c_L \\ t_L \end{pmatrix}_{\text{mass}}, \quad \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}_{\text{weak}} = U \begin{pmatrix} d'_L \\ s'_L \\ b'_L \end{pmatrix}_{\text{mass}}, \quad \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}_{\text{weak}} = \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}_{\text{mass}}, \tag{2.1.22}$$

where U is called the Cabibbo-Kobayashi-Maskawa(CKM) matrix [26, 27].

$$U_{\text{CKM}} = \begin{pmatrix} U_{ud} & U_{us} & U_{ub} \\ U_{cd} & U_{cs} & U_{cb} \\ U_{td} & U_{ts} & U_{tb} \end{pmatrix}. \tag{2.1.23}$$

This matrix is parametrized by three real parameters "CKM angle" and one phase factor "KM phase".

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{2.1.24}$$

where $c_{ij} = \cos\theta_{ij}, s_{ij} = \sin\theta_{ij}$. In Table 3, we summarize the names of the fermions in the SM.

Table 3: Generation in the SM

	1^{st}	2^{nd}	3^{rd}
up-type	Up	Charm	Top
down-type	Down	Strange	Bottom
charged lepton	Electron	Muon	Tau
neutrino	Electron neutrino	Muon neutrino	Tau neutrino

Combining the above arguments, the Lagrangian for the SM is given as,

$$\begin{aligned}\mathcal{L}_{\text{SM}} =& -\frac{1}{2}\text{tr}G^{\mu\nu}G_{\mu\nu} - \frac{1}{2}\text{tr}W^{\mu\nu}W_{\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} \\ &+ \sum_i \left(\overline{Q}_{Li} i\not{D} Q_{Li} + \overline{L}_{Li} i\not{D} L_{Li} + \overline{u}_{Ri} i\not{D} u_{Ri} + \overline{d}_{Ri} i\not{D} d_{Ri} + \overline{e}_{Ri} i\not{D} e_{Ri}\right) + (D_\mu H)^\dagger (D^\mu H) \\ &- Y^u_{ij}\epsilon_{ab}\overline{u}_{Ri}H^a Q^b_{Lj} + Y^d_{ij}\epsilon_{ab}\overline{d}_{Ri}(H^C)^a Q^b_{Lj} + Y^e_{ij}\epsilon_{ab}\overline{e}_{Ri}(H^C)^a L^b_{Lj} \\ &+ \mu^2 H^\dagger H - \frac{\lambda}{2}(H^\dagger H)^2.\end{aligned} \tag{2.1.25}$$

where $G_{\mu\nu}$, $W_{\mu\nu}$, and $B_{\mu\nu}$ are the field strength tensors for the SM gauge fields.

Though the SM explains various particle phenomena well, the SM is expected to be extended for some reasons.

- No candidate of dark matter
- The origin of the neutrino mass
- The baryon asymmetry of the Universe
- Why hypercharge (in other words electric charge) is quantized
- Why does gauge anomaly cancel

The SM does not contain the candidate of dark matter, while it is confirmed that dark matter accounts for 26.5% of all components of the Universe [28].

Let us consider the neutrino sector in the SM. There is no evidence that there exists a right handed neutrino up until now. Thus, the Yukawa term of neutrino does not appear in the SM Lagrangian. From the neutrino oscillation experiments [29, 30, 31, 32, 33], however, it is confirmed that there are the masses of the neutrinos. Since neutrinos are electrically neutral, the Majorana mass term can be allowed. Then, the flavor eigenstate for the neutrinos is different from the mass eigenstate for neutrino. When we diagonalize the mass matrix for neutrinos, we obtain the mixing matrix called Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [34, 35, 36] as similar way as the CKM matrix. The parameters in the PMNS matrix (in particular, angles of this matrix) and the squared mass differences are revealed or constrained by some experiment; the measurement of $\sin^2 2\theta_{13}$ in Daya Bay [37], the absence of $\nu_\mu \to \nu_\tau$ is excluded at the 3.4 σ by OPERA experiment [38], the measurement of the solar ν_e disappearance (SNO [39, 40] and Super-Kamiokande [41]), neutrino oscillation (T2K [32, 33], K2K [31]), the measurement of $|\Delta m^2_{23}|$ and $\sin^2 2\theta_{23}$ via muon neutrino disappearance in MINOS [42], and others [Super-K (atmospheric $\nu_\mu, \bar{\nu}_\mu$) [43], KamLAND (reactor ν_e disappearance) [30], MINOS [44], and others].

The standard model does not also explain the baryon asymmetry in our Universe. Later, new observational result for baryon number density is obtained by Planck Collaboration [28]

$$\frac{n_B}{s} = \frac{n_b - n_{\bar{b}}}{s} = (8.6 \pm 0.1) \times 10^{-11} (68\%\text{C.L.}), \tag{2.1.26}$$

A.D.Sakharov pointed out that there were three conditions in order to make the baryon asymmetry which exists in our Universe [45], existence of baryon number violating interactions, existence of C and CP violation and the necessity of the non-equilibrium state in the history of the Universe.

Since the gauge symmetry of hypercharge is $U(1)_Y$, hypercharges of the SM particles need not to be quantized. As we had shown, all the SM particles have fractional $U(1)_Y$ charges.

Also, gauge anomalies in the SM are completely cancelled even if we include the gravitational interaction. If the anomaly coefficient defined as

$$A^{abc} \equiv \text{tr}[t^a \{t^b, t^c\}], \tag{2.1.27}$$

equals zero, anomaly is cancelled. There are 'safe group' in 4 dimensions. For example, the gauge groups that have only real or pseudo real representation as $SU(2)$, $SO(2N+1)$ and $\mathrm{SO}(4N)$ with $N \geq 2$ or exceptional groups E_8 and others are the safe group. And a few other algebras that have neither real nor pseudo real representation have some representation for which anomaly coefficient vanishes, $SO(4N+2)$ with $N \geq 2$ or exceptional group E_6 [46]. Unfortunately the anomalies in $SU(N)$ with $N \geq 3$ are not cancelled automatically.

In order to explain these questions, the physics beyond the standard model (BSMs) are expected.

2.2 Supersymmetry

S.R.Coleman and J.Mandula explained that Poincaré generators (including energy-momentum generators P_μ and Lorentz rotation generators $M_{\mu\nu}$) and generators of compact Lie groups were only allowed as generators of the symmetry of S-matrix [47]. This no-go theorem is called "Coleman-Mandula theorem"

This theorem is assumed that relativistic local quantum field theory (QFT) is based on coordinates of spacetime x^μ. Therefore, if we introduce fermionic coordinates, this theorem is not applicable. The extended field theories which are based on both of the fermionic and bosonic coordinates are able to have additional symmetry called “Supersymmetry”*. In particular, in $\mathcal{N}=1$ SUSY extended theories, all of fields are extended to superfields which depend on ordinary bosonic coordinates x^μ and fermionic coordinates $\theta_\alpha, \bar{\theta}^{\dot{\alpha}}$. Thus superfields include both of bosonic component fields and fermionic ones.

In general, the unitary transformation of translation including the direction to the superspace is defined as:

$$G(x,\theta,\theta^\dagger) \equiv \exp\left[i(x^\mu \hat{P}_\mu + \theta\hat{Q} + \theta^\dagger \hat{Q}^\dagger)\right]. \tag{2.2.1}$$

where we define the translation operator in a superspace direction as follows:

$$\begin{aligned} \hat{Q}_\alpha &\equiv i\frac{\partial}{\partial\theta^\alpha} - (\sigma^\mu\theta^\dagger)_\alpha\partial_\mu, \quad \hat{Q}^\alpha \equiv i\frac{\partial}{\partial\theta_\alpha} - (\theta^\dagger\overline{\sigma}^\mu)^\alpha\partial_\mu, \\ \hat{Q}^{\dagger\dot{\alpha}} &\equiv i\frac{\partial}{\partial\theta^\dagger_{\dot{\alpha}}} - (\overline{\sigma}^\mu\theta)^{\dot{\alpha}}\partial_\mu, \quad \hat{Q}^\dagger_{\dot{\alpha}} \equiv i\frac{\partial}{\partial\theta^{\dagger\dot{\alpha}}} - (\theta\sigma^\mu)_{\dot{\alpha}}\partial_\mu. \end{aligned} \tag{2.2.2}$$

Now, we define the supersymmetric transformation as the translation in a superspace direction, and then, we obtain the supersymmetric transformation of the superfield $S(x^\mu,\theta,\theta^\dagger)$,

$$\begin{aligned} \sqrt{2}\delta_\epsilon S &\equiv -i(\epsilon\hat{Q} + \epsilon^\dagger\hat{Q}^\dagger)S(x^\mu,\theta,\theta^\dagger) \\ &= \left[\epsilon^\alpha\frac{\partial}{\partial\theta^\alpha} + \epsilon^\dagger_{\dot{\alpha}}\frac{\partial}{\partial\theta^\dagger_{\dot{\alpha}}} + i(\epsilon\sigma^\mu\theta^\dagger + \epsilon^\dagger\overline{\sigma}^\mu\theta)\partial_\mu\right] S(x^\mu,\theta,\theta^\dagger) \\ &= S(x^\mu + i(\epsilon\sigma^\mu\theta^\dagger + \epsilon^\dagger\overline{\sigma}^\mu\theta), \theta+\epsilon, \theta^\dagger+\epsilon^\dagger) - S(x^\mu,\theta,\theta^\dagger). \end{aligned} \tag{2.2.3}$$

In supersymmetric theories, matter fields and gauge fields of the standard model are embedded in the chiral(anti-chiral) superfields and vector superfields, respectively. Chiral (anti-chiral) superfields Φ ($\bar{\Phi}$) are constrained as

$$\bar{D}_{\dot{\alpha}}\Phi = 0, \quad (D_\alpha\Phi^\dagger = 0), \tag{2.2.4}$$

where $D(\bar{D})$ is chiral(anti-chiral) covariant derivative defined as:

$$\begin{aligned} D_\alpha &\equiv \frac{\partial}{\partial\theta^\alpha} - i(\sigma^\mu\theta^\dagger)_\alpha\partial_\mu, \quad D^\alpha \equiv \frac{\partial}{\partial\theta_\alpha} + i(\theta^\dagger\overline{\sigma}^\mu)^\alpha\partial_\mu, \\ D^{\dagger\dot{\alpha}} &\equiv \frac{\partial}{\partial\theta^\dagger_{\dot{\alpha}}} - i(\overline{\sigma}^\mu\theta)^{\dot{\alpha}}\partial_\mu, \quad D^\dagger_{\dot{\alpha}} \equiv \frac{\partial}{\partial\theta^{\dagger\dot{\alpha}}} + i(\theta\sigma^\mu)_{\dot{\alpha}}\partial_\mu. \end{aligned} \tag{2.2.5}$$

*Supersymmetry algebra is given in Appendix B.1.

Vector superfields V are real superfields $V = V^\dagger$. A chiral superfield is divided into component fields as

$$\begin{aligned}\Phi(x,\theta,\theta^\dagger) =&\phi(x) - i\theta^\dagger\bar{\sigma}^\mu\theta\partial_\mu\phi(x) - \frac{1}{4}\theta^2\theta^{\dagger 2}\partial^\mu\partial_\mu\phi(x) \\ &+ \theta\psi(x) + \frac{i}{\sqrt{2}}\theta^2\theta^\dagger\bar{\sigma}^\mu\partial_\mu\psi(x) + \theta^2 F(x),\end{aligned} \tag{2.2.6}$$

where $\phi(x)$ and $\psi(x)$ are a x-dependent complex scalar field and a Weyl fermion field, respectively. $F(x)$ is an auxiliary field which vanishes by using equations of motion and is needed to match the degrees of freedom both of bosonic and fermionic ones under off-shell condition.

Non-abelian supergauge transformation for chiral (anti-chiral) superfields is defined as*

$$\Phi \to \Phi' = e^{i\Lambda}\Phi, \quad \Phi^\dagger \to (\Phi^\dagger)' = \Phi^\dagger e^{-i\Lambda^\dagger}, \tag{2.2.7}$$

where $\Lambda = \Lambda^a T^a, \Lambda^\dagger = \Lambda^{\dagger a}T^a$. $\Lambda^a, \Lambda^{a\dagger}$ are chiral and anti-chiral (superspace dependent) parameter, respectively. T^a is generator of the non-abelian gauge group. Gauge vector superfields ($V = V^a T^a$) are transformed as follows;

$$e^V \to (e^V)' = e^{i\Lambda^\dagger}e^V e^{-i\Lambda}. \tag{2.2.8}$$

In terms of the supergauge transformation of vector superfield, the infinitesimal supergauge transformation is obtained as follows:

$$\begin{aligned}V &\to V + \mathcal{L}_{V/2}\cdot\left[-i(\Lambda+\Lambda^\dagger) + \coth\mathcal{L}_{V/2}\cdot i(\Lambda^\dagger - \Lambda)\right] \\ &= V - \frac{i}{2}\left[V, \Lambda+\Lambda^\dagger\right] - i(\Lambda - \Lambda^\dagger) + i\sum_{n=1}^{\infty}\frac{B_n}{(2n)!}\left[V,\left[V,\cdots\left[V,\Lambda^\dagger-\Lambda\right]\cdots\right]\right],\end{aligned} \tag{2.2.9}$$

where $\mathcal{L}_A \cdot B$ denotes the Lie bracket defined as

$$\mathcal{L}_A \cdot B = [A, B], \tag{2.2.10}$$

and B_n is the Bernoulli numbers defined by

$$\frac{x}{e^x - 1} = \sum_{n=0}^{\infty}\frac{B_n}{n!}x^n. \tag{2.2.11}$$

*Of course, Φ ($\Phi^\dagger$) belongs to fundamental (anti-fundamental) representation.

Thus, the gauge invariant action is obtained from the product of Φ and $\widetilde{\Phi} = \Phi^\dagger e^V$. Gauge fields in Yang-Mills theories are embedded in the vector superfield:

$$\begin{aligned} V(x,\theta,\theta^\dagger) =& a + \theta\xi + \theta^\dagger\xi^\dagger + \theta^2 b + \theta^{\dagger 2} b^* + \theta\sigma^\mu\theta^\dagger A_\mu \\ &+ \theta^{\dagger 2}\theta\left(\lambda + \frac{i}{2}\sigma^\mu\partial_\mu\xi^\dagger\right) + \theta^2\theta^\dagger\left(\lambda^\dagger + \frac{i}{2}\bar{\sigma}^\mu\partial_\mu\xi\right) + \theta^2\theta^{\dagger 2}\left(\frac{1}{2}D + \frac{1}{4}\partial_\mu\partial^\mu a\right). \end{aligned} \tag{2.2.12}$$

By using the supergauge transformation $\Lambda^\dagger - \Lambda$ defined as Eq. (2.2.9), we reduce the number of the component fields of vector superfield:

$$V_{WZ}(x,\theta,\theta^\dagger) = \theta^\dagger\overline{\sigma}^\mu\theta A_\mu(x) + \theta^{\dagger 2}\theta\lambda(x) + \theta^2\theta^\dagger\lambda^\dagger(x) + \frac{1}{2}\theta^2\theta^{\dagger 2}D(x), \tag{2.2.13}$$

$A_\mu(x)$ and $\lambda(x), \lambda^\dagger(x)$ are gauge fields and gaugino fields, respectively. $D(x)$ is an auxiliary field which has no kinetic term. In non-abelian supersymmetric Yang-Mills (SUSY YM) theories, V, Λ, and $\Lambda^\dagger$ denote the summation of the product of superfields and the generator of gauge group *i.e.* $V = 2g\sum_a V^a T^a, \Lambda = \sum_a \Lambda^a T^a$ and $\Lambda^\dagger = \sum_a \Lambda^{\dagger a} T^a$ where g is the gauge coupling of SUSY YM theory. Note that after the supergauge fixing which is called "Wess-Zumino gauge", we are able to use ordinary gauge transformation. In Eq. (2.2.9), we still have the freedom to do ordinary gauge transformation. When we set $\Lambda = \Lambda^\dagger$, this supergauge transformation becomes the ordinary gauge transformation as:

$$V \to V - i[V,\Lambda], \quad A^a_\mu \to A^a_\mu + g f^{abc} A^b_\mu \Lambda^c. \tag{2.2.14}$$

The kinetic term for gauge fields is given by constructing the field-strength chiral superfield as follows:

$$\mathcal{W}_\alpha = -\frac{1}{4}\overline{D}^2(e^{-V} D_\alpha e^V). \tag{2.2.15}$$

This superfield obviously satisfies chiral constraint. This is, of course, the summation of the product of superfields and generators of gauge group in non-abelian SUSY YM ($\mathcal{W}_\alpha = 2g\sum_a \mathcal{W}^a_\alpha T^a$). The field-strength chiral superfield transforms

$$\mathcal{W}_\alpha \to \mathcal{W}'_\alpha = e^{i\Lambda}\mathcal{W}_\alpha e^{-i\Lambda}, \tag{2.2.16}$$

under a supergauge transformation. Therefore, the gauge invariant kinetic term for gauge field is given by

$$\int d^2\theta \frac{1}{4kg^2}\mathrm{Tr}\mathcal{W}^\alpha\mathcal{W}_\alpha = D^a D^a + 2i\lambda^a\sigma^\mu\nabla_\mu\lambda^{a\dagger} - \frac{1}{2}F^{a\mu\nu}F^a_{\mu\nu} + \frac{i}{4}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}, \tag{2.2.17}$$

where generators are normalized as $\mathrm{Tr}T^aT^b = k\delta^{ab}$. The gauge coupling g and CP-violating angle Θ are combined into the holomorphic coupling defined as

$$\tau_a \equiv \frac{1}{g^2} - i\frac{\Theta}{8\pi^2}. \tag{2.2.18}$$

The kinetic term is obtained as

$$\frac{1}{16k}\int d^2\theta \tau_a \mathrm{Tr}\mathcal{W}^\alpha \mathcal{W}_\alpha + \text{h.c.}. \tag{2.2.19}$$

In general the gauge covariant kinetic terms of matter fields are given rise to by the Kähler potential $\mathcal{K}(\widetilde{\Phi}, \Phi)$ which is composed by the product of chiral superfields (Φ) and anti-chiral superfields ($\widetilde{\Phi} \equiv \Phi^\dagger e^V$). The interaction terms including such as Yukawa terms are generated from superpotential, $W(\Phi)$. The general Lagrangian for the non-renormalizable theory is given as

$$\mathcal{L} = \int d^4\theta\, \mathcal{K}(\widetilde{\Phi}, \Phi) + \left[\int d^2\theta\, W(\Phi) + \text{h.c.}\right] + \left[\int d^2\theta\, \frac{1}{16}\tau_a f_{ab}(\Phi)\mathcal{W}^{a\alpha}\mathcal{W}^b_\alpha + \text{h.c.}\right]. \tag{2.2.20}$$

where $f(\Phi)$ is holomorphic function of chiral superfields called "gauge kinetic function". In the renormalizable theory, the forms of these functions are determined as follows:

$$\begin{aligned} \mathcal{K}(\widetilde{\Phi}, \Phi) &= \widetilde{\Phi}^i \Phi_i, \\ W(\Phi) &= \frac{1}{2}M^{ij}\Phi_i\Phi_j + \frac{1}{3!}y^{ijk}\Phi_i\Phi_j\Phi_k, \\ f_{ab}(\Phi) &= \delta_{ab}, \end{aligned} \tag{2.2.21}$$

where $i, j, \ldots$ are labels of chiral(anti-chiral) superfields.

In the supersymmetric theories, there is a special property. This is "non-renormalization theorem". This theorem ensures that there is no vertex correction for the interaction which is generated from superpotential[48].

2.3 SUSY Breaking

Supersymmetric extensions of the standard model are fascinating in points of some phenomenological aspects. However, superpartners of the SM particles have not been directly discovered yet at any collider experiments. In particular, masses of SUSY colored particles have been constrained more strictly than before. At LHC experiments, lower bounds for masses of colored particles are around 1TeV [49].

Considering this fact, supersymmetry should be broken above electroweak scale. In general, effects of SUSY breaking can appear in a mass difference between superpartners, and also in a difference between the interactions which are associated with each other by supersymmetry.

In the supersymmetric algebra, generators of SUSY and translation are related as expressed in Appendix B.1.

$$\{Q_\alpha, Q^\dagger_{\dot\beta}\} = 2i\sigma^\mu_{\alpha\dot\beta}P_\mu, \tag{2.3.1}$$

where $Q_\alpha, Q^\dagger_{\dot\beta}$ and P_μ are generators of SUSY and translation, respectively. Since Hamiltonian is the zeroth component of the translation generator, we have

$$H = \frac{1}{4}\left(Q_1Q_1^\dagger + Q_2Q_2^\dagger + Q_1^\dagger Q_1 + Q_2^\dagger Q_2\right). \tag{2.3.2}$$

That is, supersymmetric vacuum $|0\rangle$ satisfies the following equality.

$$\langle 0|H|0\rangle = \frac{1}{4}\sum_{\text{all } Q} |Q\,|0\rangle|^2 = 0 \tag{2.3.3}$$

Thus, in supersymmetric theories, the minimum of potential should be zero. If the minimum of potential is not zero, SUSY is obviously broken. In supersymmetric theories, scalar potential is given as

$$\begin{aligned} V(\phi) &= \sum_i F_i F^{i*} + \frac{1}{2}\sum_a D^a D^a \\ &= \sum_i \left|\frac{\partial W}{\partial \Phi_i}\right|_{\Phi=\phi} + \frac{1}{2}\sum_a (\phi^{i*}T^a\phi_i)^2, \end{aligned} \tag{2.3.4}$$

where the first and second terms are obtained from F- and D-terms, respectively. This means that the breaking of supersymmetry is related to the non-zero vacuum expectation values (VEVs) of auxiliary fields.

While SUSY must be broken at electroweak scale at least, quadratic divergences such as quantum corrections for mass terms of scalar fields are unfavorable for introducing supersymmetry. Thus, SUSY must be broken softly, which means that there is no dimensionless coupling in SUSY breaking sector.

One of the ideas for soft SUSY breaking is that our world is divided into visible sector and hidden sectors*. In visible sector, we live and we observe all of particle phenomena. On the other hand, in hidden sector, supersymmetry is dynamically broken and its effect is communicated by the fields which live in both of visible and hidden sector. These fields are called "messengers".

*There is a convenient method in order to obtain the soft parameters [50, 51, 52]. At first we extend all of the coupling constants to the spurion superfields. If the higher-components of these spurion superfields have non-zero VEVs, then SUSY is softly breaking and the soft parameters are proportional to VEVs of these higher-components.

The gravity interaction is one of candidates of messengers. This mediation mechanism is called "Planck-Mediated SUSY Breaking"(PMSB) [53, 54, 55, 56, 57, 58]. In this mechanism, SUSY breaking is mediated through Planck-suppressed higher-dimensional operators such as

$$\int d^4\theta \frac{XX^\dagger}{M_{Pl}^2}\Phi^\dagger\Phi, \tag{2.3.5}$$

where M_{Pl} is the Planck-mass ($\sim 10^{18}$GeV). If higher-components of X have VEVs, scalar component of Φ obtains soft mass which is different from mass of fermion component of Φ. Similarly, gaugino mass term is obtained from higher-dimensional operator as the following form.

$$\frac{1}{16}\int d^2\theta \frac{X}{M_{Pl}}\mathcal{W}^a\mathcal{W}^b, \tag{2.3.6}$$

We can find that a typical scale of soft SUSY breaking is obtained as $M_{\rm soft} = \langle F_X\rangle / M_{Pl}$ in this mechanism where $\langle F_X\rangle$ is the VEV of the F-term of X.

One of other possibilities of soft SUSY breaking mechanism is "Anomaly-Mediated SUSY Breaking" (AMSB)[59, 60]. In this model, SUSY breaking is given rise to through superconformal anomaly. We will explain details of this mechanism in Appendix C.2. Let us consider properties of this model. In this model, gaugino mass terms are induced with 1-loop suppression since these soft parameters are obtained through the breaking of scale symmetry, which means that these parameters must depend on beta functions and anomalous dimensions. For example, we can obtain the masses of gauginos which are superpartners of the gauge bosons as follows:

$$M_a = -\frac{\beta(g_a)}{g_a}m_{3/2}, \tag{2.3.7}$$

where a is the label of the gauge group and $\beta(g_a)$ is β function of the gauge coupling g_a. We have the masses of the MSSM gaugino including the contribution from the Higgs-higgsino loop.

$$\begin{aligned}
M_1 &= \frac{b_1 g_1^2}{16\pi^2}\left[m_{3/2} + \frac{\mu_H}{11}\sin 2\beta \frac{m_A^2}{\mu_H^2 - m_A^2}\ln\frac{\mu_H^2}{m_A^2}\right], \\
M_2 &= \frac{b_2 g_2^2}{16\pi^2}\left[m_{3/2} + \mu_H \sin 2\beta \frac{m_A^2}{\mu_H^2 - m_A^2}\ln\frac{\mu_H^2}{m_A^2}\right], \\
M_3 &= \frac{b_3 g_3^2}{16\pi^2}m_{3/2},
\end{aligned} \tag{2.3.8}$$

where μ_H and m_A are the masses of the higgsino and the pseudo-scalar Higgs, respectively. $m_{3/2}$ is the mass of gravitino.

2.4 The Minimal Supersymmetric Standard Model

The supersymmetric extension of the standard model [61, 62, 63, 64] is the one of the BSMs. Supersymmetry for the particle physics requires that the partner particles (superpartner) of the SM whose difference is only spin-statistics of these particles.

Now, we explain the minimal supersymmetric extension of the standard model (the minimal supersymmetric standard model: MSSM). To make Yukawa terms supersymmetric, we must assign chiral superfields as Table 4. Superpotential cannot include both of chiral superfield and its complex conjugate (anti-chiral superfield). Thus, two Higgs doublets are needed to be introduced in supersymmetric theories. One of two is coupled to up-type quark superfield, and another is coupled to down-type and electron-type superfields.

Table 4: Chiral superfields in MSSM

	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	Z_{2R}
Q	$\mathbf{3}$	$\mathbf{2}$	$1/6$	-1
L	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	-1
U^C	$\overline{\mathbf{3}}$	$\mathbf{1}$	$-2/3$	-1
D^C	$\overline{\mathbf{3}}$	$\mathbf{1}$	$1/3$	-1
E^C	$\mathbf{1}$	$\mathbf{1}$	1	-1
H_u	$\mathbf{1}$	$\mathbf{2}$	$1/2$	1
H_d	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	1

$$W_{\text{Yukawa}} = Y^u_{ij} H_u U^C_i Q_{Lj} - Y^d_{ij} H_d D^C_i Q_{Lj} - Y^e_{ij} H_d E^C_i L_{Lj}, \tag{2.4.1}$$

where $i, j, \ldots$ are flavor indices. In this notation, the indices of $SU(3)_C$ and $SU(2)_L$ are not written down explicitly. In particular the indices of $SU(2)_L$ are contracted by anti-symmetric tensor such as the case of the SM Lagrangian.

If we assume a Z_2-parity called "R-parity" invariance which assigns SM particles and these superpartners to parity-even and parity-odd, respectively, as Table 4, the lightest superpartner can be a dark mater candidate. Furthermore, the baryon number violating and the lepton number violating operators as

$$\begin{aligned} W_{\Delta L=1} &= \frac{1}{2}\lambda^{ijk} L_i L_j E^C_k + \lambda'^{ijk} L_i Q_j D^C_k + \mu^i L_i H_u, \\ W_{\Delta B=1} &= \lambda^{ijk}_B U^C_i D^C_j D^C_k, \end{aligned} \tag{2.4.2}$$

are forbidden by this parity. Although these superpotentials generate the most terrible nucleon decay interactions induced by exchanging sfermions at tree level, these are prohibited by this parity.

Since mass terms of the standard model fermions appear after electroweak symmetry breaking, the supersymmetric mass terms are absent in superpotential of MSSM. Only the supersymmetric mass term for Higgs superfields is allowed as

$$W_{\text{mass}} = \mu H_u H_d. \tag{2.4.3}$$

The full superpotential for MSSM is obtained from the summation of above two superpotentials.

$$W_{\text{MSSM}} = W_{\text{mass}} + W_{\text{Yukawa}} \tag{2.4.4}$$

This superpotential generates a lot of new interactions in addition to the Yukawa interactions in the standard model Lagrangian.

In this minimal extension of the standard model, the scalar potential of Higgs boson is given rise to by supersymmetric interactions and soft breaking terms as the following form:

$$\begin{aligned} V =&(|\mu|^2 + m_{H_u}^2)|H_u^0|^2 + (|\mu|^2 + m_{H_d}^2)|H_d^0|^2 \\ &- [b\mu H_u^0 H_d^0 + \text{h.c.}] + \frac{1}{8}(g^2 + g'^2)(|H_u^0|^2 - |H_d^0|^2)^2, \end{aligned} \tag{2.4.5}$$

where there are two neutral Higgs components. Namely, by taking the linear combination of these scalar fields as

$$\begin{aligned} H_u^0 &= v_u + \frac{1}{\sqrt{2}} h^0 \sin\beta + \frac{1}{\sqrt{2}} H^0 \cos\beta + \ldots, \\ H_d^0 &= v_d + \frac{1}{\sqrt{2}} h^0 \cos\beta + \frac{1}{\sqrt{2}} H^0 \sin\beta + \ldots. \end{aligned} \tag{2.4.6}$$

where h^0 and H^0 are neutral Higgs bosons. In particular, the lighter Higgs boson h^0 is identified with the SM Higgs boson. the quartic coupling of the SM Higgs in scalar potential is obtained as

$$\lambda = \frac{1}{4}(g^2 + g'^2)\cos^2 2\beta, \tag{2.4.7}$$

at tree-level and the mass of the SM Higgs boson is given by

$$m_{h^0}^2 = \frac{1}{2}\left(m_{A^0}^2 + m_Z^2 - \sqrt{(m_{A^0}^2 - m_Z^2)^2 + 4m_Z^2 m_{A^0}^2 \sin^2 2\beta}\right), \tag{2.4.8}$$

where m_{A^0} is the mass of the CP-odd neutral Higgs defined as $m_{A^0}^2 = 2b\mu/\sin 2\beta$. This tells us that the Higgs mass is bounded from above as

$$m_{h_0} < m_Z|\cos 2\beta|. \tag{2.4.9}$$

The mass of the observed Higgs boson is measured as 125.9 ± 0.4GeV, which is larger than the mass of Z-boson. This means that large quantum corrections to the mass of the SM Higgs boson are needed in order to realize the observed mass.

There is another problem called the μ problem which is a kind of fine-tuning problems. In the MSSM, dimensionful parameter μ in vector-like mass term is supersymmetric one. Thus, μ has the same order of the cutoff scale of MSSM or above. On the other hands, the soft masses $m_{H_u}^2, m_{H_d}^2$ have the same order of the SUSY breaking scale. These parameters must be cancelled and generate proper negative squared mass in order that the electroweak symmetry breaking must be given rise to. Though, there is no reason why the supersymmetric dimensionful parameter has the same order of the SUSY breaking scale. This means that there may be some mechanism to generate the μ term near the SUSY breaking scale.

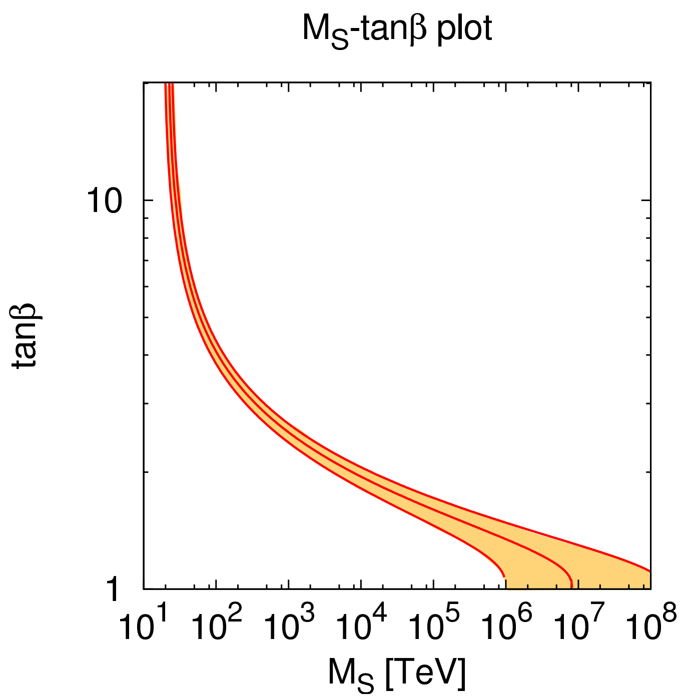


Figure 1: Higgs mass in high-scale SUSY scenario. $m_h = 125.9 \pm 0.4$GeV, $\alpha_S(m_Z) = 0.118 \pm 0.007$ and pole mass $m_t = 173.07 \pm 0.6 \pm 0.8$ GeV. The center line show the region the observational center value is realized. Error bar indicate the input error of Higgs mass, strong coupling and top quark pole mass. We use RGEs for quartic coupling and Yukawa coupling are at 2-loop level; and threshold correction for quartic coupling is at 1-loop level. In this numerical calculation, we set A-terms to be 0.

2.5 High-Scale SUSY breaking

Discovery of the SM Higgs boson tells us that the observed mass of the SM Higgs boson is larger than the prediction in MSSM at tree level. We have two possibilities to make mass of the SM Higgs heavy; one of them is to raise the quartic coupling constant of Higgs potential at tree-level, another is large quantum corrections to the mass of Higgs boson. The quartic coupling at tree-level can be enlarged in the next-to minimal supersymmetric standard model (NMSSM)*. Large quantum corrections can be given rise to as follows:

- Extra matters which are gauge singlet fields or vector-like matters [65],
- SUSY breaking scale is higher than several TeV [9, 66],
- Large A-terms [65].

*In NMSSM, μ term is given rise to by VEV of the new gauge-singlet superfield S. The scalar component of S mixes with CP-even Higgs h^0 and H^0.

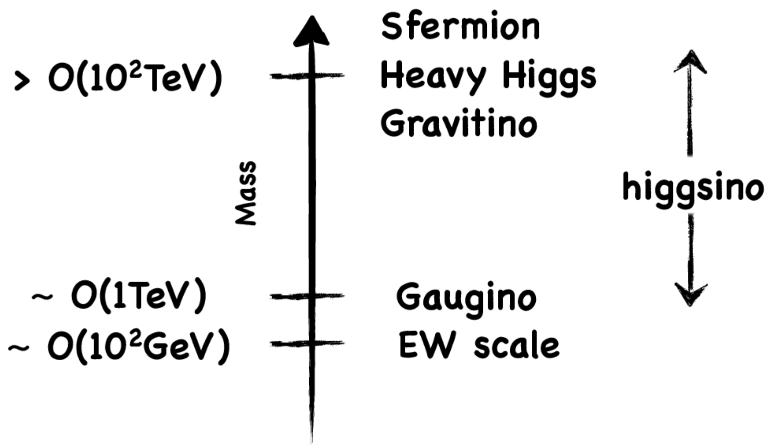


Figure 2: Mass spectrum in High-scale SUSY breaking scenario

High-scale supersymmetry is one of these models where the observed mass of Higgs boson can be realized [9, 66]†. The region where the observed Higgs boson is realized is shown in Fig. 1. This is the most simplest and fascinating phenomenological model. Since the breaking scale can be larger than several TeV, higher dimensional operators which cause SUSY flavor problems can be sufficiently suppressed by these mass scales.

In this model, we only assume that there is no gauge singlet field in SUSY breaking sector. The mass terms of sfermions are given rise to by higher-dimensional operators as the following form.

$$\int d^4\theta \frac{XX^\dagger}{\Lambda^2}\Phi^\dagger\Phi, \tag{2.5.1}$$

where X lives in hidden sector and has non-zero VEVs of higher-component. However, the gauge invariant action does not contain higher-dimensional operators which generate gaugino mass terms and A-terms as,

$$\int d^2\theta \frac{X}{\Lambda}\mathcal{W}^a\mathcal{W}^b, \quad \int d^2\theta \frac{X}{\Lambda}\lambda^{ijk}\Phi_i\Phi_j\Phi_k. \tag{2.5.2}$$

The leading terms for gaugino masses and A-terms are obtained through AMSB in this model. Thus, the masses of gauginos will be proportional to the mass scale of the gravitino with 1-loop suppression. The mass spectrum will be split among heavy scalar particles, gravitino and gauginos.

The mass scale of the higgsino depends on the details of models. In some models, the higgsino mass can be the same order as gravitino or SUSY breaking scale discussed by [66, 68]*.

†Recently J. Feng and his collaborators estimated the mass of the Higgs boson at three-loop level [67].

*In these references, arguments about higgsino mass (or μ-problem) are based on a mechanism called "Giudice-Masiero mechanism"[69].

On the other hand, in the other models, low-scale μ-term is given rise to by various ways. One is that higher dimensional operator obtained from R-symmetry breaking mechanism. When we assign R-charge 2 to c, R-invariant term generating higgsino mass is given as the following form:

$$\int d^4\theta \frac{XX^\dagger}{\Lambda^5} c D_\alpha H_u D^\alpha H_d \tag{2.5.3}$$

generates μ-term [70]. Another is caused by Giudice-Masiero mechanism which is imposed R-symmetry and its breaking at the energy lower than Planck scale. Anyway, we treat the mass scale of higgsino as a parameter when we carry out some calculation.

Combining all information as mentioned above, the mass spectrum of the simple model based on the high-scale SUSY breaking is obtained as in Fig. 2.

- All the masses of scalar components of chiral supermultiplet are set to be above 10^2TeV.
- And also we set a gravitino to be 10^2TeV, and gauginos to be several TeV through AMSB.
- The mass of higgsinos is treated as a model parameter.

3 Minimal SUSY SU(5) Grand Unified Theory

Grand unified theories (GUTs) explain some questions in the SM. In GUTs, the SM gauge group is embedded in a large and a simple group, for examples $SU(5)$, $SO(10)$, and E_6. This means that running gauge couplings can be unified at GUT scale, $M_{GUT} \sim 10^{16\sim 17}$GeV. Since the SM gauge group is unified as a simple group, after the symmetry breaking of unified gauge group, the $U(1)_Y$ charge of particles is quantized. The cancellation of the gauge anomaly can be expressed because of unified gauge group.

GUT was first investigated by H. Georgi and S. L. Glashow in 1974 [71]. They identified the minimal unified gauge group with $SU(5)$ by imposing that the rank of unified gauge group is four and this gauge group has complex representation. In $SU(5)$ GUT, the anomaly coefficient defined in Eq. (2.1.27) of a **5** representation is equivalent to that of a **10** representation. In the $SU(5)$ GUT, since matter fields of the SM are embedded in $\bar{\mathbf{5}} \oplus \mathbf{10}$ representation, gauge anomaly is completely cancelled. Then, the supersymmetric extension of the minimal $SU(5)$ GUT was independently investigated by N. Sakai [72] and S. Dimopoulos and H. Georgi [73]. As mentioned above, in the supersymmetric extension of grand unified theories, it is well-known that gauge coupling unification can be improved as in Fig. 3 [8].

In this section, we briefly review the minimal supersymmetric $SU(5)$ grand unified theory (the minimal SUSY $SU(5)$ GUT).

3.1 Field Contents

The MSSM superfields are embedded in the SU(5) symmetric superfields. First, the left-handed quark doublet Q^{ar}, the charge conjugated right-handed up-type quark U_a^C, and the charge conjugated right-handed charged lepton E^C are embedded in the anti-symmetric 10 dimensional representation chiral superfield Ψ:

$$\Psi^{\alpha\beta}(\mathbf{10}) = \frac{1}{\sqrt{2}} \begin{pmatrix} \epsilon^{abc} U_c^C & Q^{ar} \\ -Q^{sb} & \epsilon^{sr} E^C \end{pmatrix}, \tag{3.1.1}$$

$\alpha, \beta, \cdots = 1, 2, \cdots 5$ represent the $SU(5)$ indices. The roman indices ($a, b, c, \cdots = 1, 2, 3$) denote the $SU(3)_C$ indices and the roman indices ($r, s, \cdots = 1, 2$) denote the $SU(2)_L$ indices. The other MSSM chiral superfields are embedded into the anti-fundamental ($\bar{\mathbf{5}}$) representation chiral supermultiplet Φ.

$$\Phi_\alpha(\bar{\mathbf{5}}) = \begin{pmatrix} D_a^C \\ \epsilon_{rs} L^s \end{pmatrix}, \tag{3.1.2}$$

D_a^C is the charge-conjugated right-handed down-type quark. L^s is the left-handed lepton doublet. Two Higgs doublets in MSSM, H_u and H_d, are embedded in chiral superfields belonging to a pair of **5** and $\bar{\mathbf{5}}$ representation of $SU(5)$ with the color-triplet Higgs multiplets

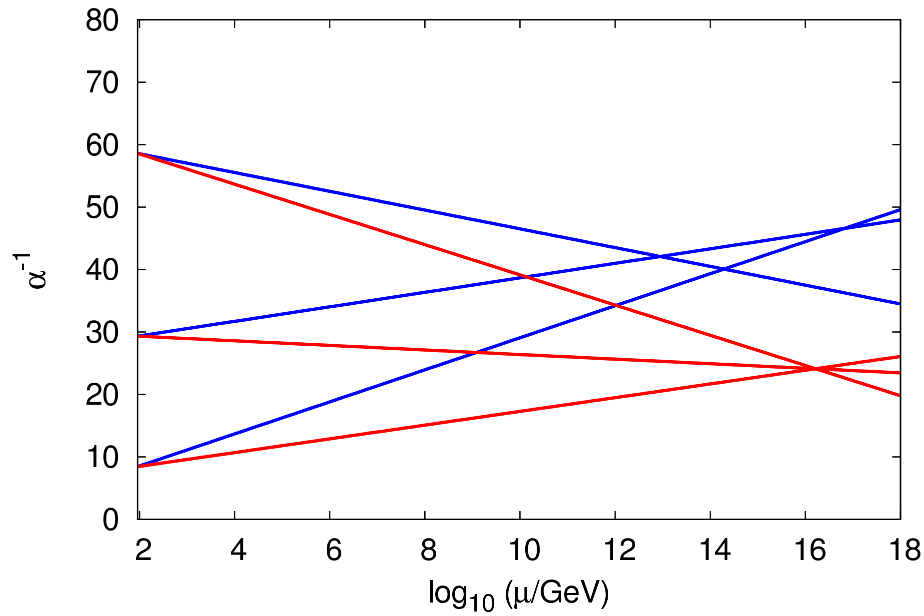


Figure 3: The unification of the gauge couplings. Blue line represent the non-SUSY scenario. Red line corresponds to the case that all of the sparticles are included at the electroweak scale. We use the 1-loop RGEs for drawing this figure. This figure shows that the unification is improved in the SUSY GUTs.

H_C and $\overline{H}_C$.

$$H(\mathbf{5}) = \begin{pmatrix} H_C^a \\ H_u^r \end{pmatrix}, \quad \overline{H}(\bar{\mathbf{5}}) = \begin{pmatrix} \overline{H}_{Ca} \\ \epsilon_{rs} H_d^s \end{pmatrix}, \tag{3.1.3}$$

There exists the additional Higgs field which breaks unified gauge group up to the standard model gauge group; the so-called adjoint Higgs superfield Σ. Σ transforms as the adjoint representation under the $SU(5)$ gauge symmetry.

$$\Sigma^\alpha_\beta(\mathbf{24}) = \begin{pmatrix} \Sigma_8 & \Sigma_{(3,2)} \\ \Sigma_{(3^*,2)} & \Sigma_3 \end{pmatrix} + \frac{1}{\sqrt{2}}\frac{1}{\sqrt{30}} \begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix} \Sigma_{24} \tag{3.1.4}$$

Now, we will check the $SU(5)$ gauge transformation of each field. $U = \exp(i\theta^a T^a)$ is a unitary transformation of $SU(5)$. T^a are the generators of $SU(5)$, and θ^a are transformation parameters. Each field transforms as follows

$$\begin{aligned} \Phi_\alpha &\to \Phi_\beta U^{\dagger\,\beta}_\alpha, \\ \Psi^{\alpha\beta} &\to U^\alpha_{\ \gamma} U^\beta_{\ \delta} \Psi^{\gamma\delta}, \\ \Sigma^\alpha_{\ \beta} &\to (U\Sigma U^\dagger)^\alpha_{\ \beta}. \end{aligned} \tag{3.1.5}$$

The infinitesimal gauge transformation for these field is obtained as:

$$\begin{aligned}
&\Phi_\alpha \to \Phi_\alpha - i\theta^a \Phi_\beta (T^a)^\beta_{\ \alpha}, \\
&\Psi^{\alpha\beta} \to \Psi^{\alpha\beta} + i\theta^a \left\{ (T^a)^\alpha_{\ \gamma} \Psi^{\gamma\beta} + (T^a)^\beta_{\ \gamma} \Psi^{\alpha\gamma} \right\}, \\
&\Sigma^\alpha_{\ \beta} \to \Sigma^\alpha_{\ \beta} + i\theta^a \left[T^a, \Sigma\right]^\alpha_{\ \beta}.
\end{aligned} \tag{3.1.6}$$

Let us consider the case that the adjoint Higgs superfield breaks the unified gauge group as $SU(5) \to SU(3)_C \times SU(2)_L \times U(1)_Y$ by the vacuum expectation value (VEV). Thus the adjoint Higgs superfield must have the VEV which is invariant under the SM gauge group. The adjoint Higgs superfield Σ must satisfy the traceless condition, that is, $\mathrm{Tr}\Sigma = 0$. Thus, the VEV of the adjoint Higgs must have the form as:

$$\langle \Sigma \rangle = V \begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix}. \tag{3.1.7}$$

Let us consider differences between the unified $U(1)$ charge and the $U(1)_Y$ hypercharge. The generator of the unified $U(1)$ gauge group is normalized as $\mathrm{tr}(T_{U(1)} T_{U(1)}) = 1/2$ and has the form of

$$T_{U(1)} = -\frac{1}{2\sqrt{15}} \begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix}. \tag{3.1.8}$$

The hypercharge generator T_Y is proportional to $T_{\mathrm{U(1)}}$. We determine this constant of proportionality in order to be consistent with the hypercharges of MSSM fields.

$$T_{U(1)} = \sqrt{\frac{3}{5}} T_Y \tag{3.1.9}$$

Finally, the canonically normalized kinetic terms* are obtained from the gauge invariant Kähler potential which is defined as

$$\mathcal{K} = \Phi_\alpha \left(e^{-2g\mathcal{V}_5}\right)^\alpha_{\ \beta} \Phi^{*\beta} + \Psi^*_{\alpha\beta} \left(e^{2g\mathcal{V}_5}\right)^\alpha_{\ \delta} \left(e^{2g\mathcal{V}_5}\right)^\beta_{\ \gamma} \Psi^{\gamma\delta} + 2\Sigma^\alpha_{\ \beta} \left(e^{2g\mathcal{V}_5}\right)^\beta_{\ \gamma} \Sigma^\gamma_{\ \delta} \left(e^{-2g\mathcal{V}_5}\right)^\delta_{\ \alpha}. \tag{3.1.10}$$

*These fields are normalized as $SU(5)$ fields but not as MSSM superfields.

Table 5: $SU(5)$ superfields decomposition

	$SU(5)$	$(SU(3)_C, SU(2)_L)_Y$
Φ	$\bar{\mathbf{5}}$	$(\bar{\mathbf{3}},\mathbf{1})_{1/3}\oplus(\mathbf{1},\mathbf{2})_{-1/2}$
Ψ	$\mathbf{10}$	$(\bar{\mathbf{3}},\mathbf{2})_{1/6}\oplus(\bar{\mathbf{3}},\mathbf{1})_{-2/3}\oplus(\mathbf{1},\mathbf{1})_{1}$
Σ	$\mathbf{24}$	$(\mathbf{8},\mathbf{1})_0\oplus(\mathbf{1},\mathbf{3})_0\oplus(\bar{\mathbf{3}},\mathbf{2})_{-5/6}\oplus(\mathbf{3},\mathbf{2})_{5/6}\oplus(\mathbf{1},\mathbf{1})_0$
H	$\mathbf{5}$	$(\mathbf{3},\mathbf{1})_{-1/3}\oplus(\mathbf{1},\mathbf{2})_{1/2}$
$\overline{H}$	$\bar{\mathbf{5}}$	$(\bar{\mathbf{3}},\mathbf{1})_{1/3}\oplus(\mathbf{1},\mathbf{2})_{-1/2}$
$\mathcal{V}_5$	$\mathbf{24}$	$(\mathbf{8},\mathbf{1})_0\oplus(\mathbf{1},\mathbf{3})_0\oplus(\bar{\mathbf{3}},\mathbf{2})_{-5/6}\oplus(\mathbf{3},\mathbf{2})_{5/6}\oplus(\mathbf{1},\mathbf{1})_0$

where $\mathcal{V}_5$ and g_5 are the vector superfield and the gauge coupling constant of $SU(5)$, respectively. The gauge (vector) superfields are defined as

$$\mathcal{V}_5 = \frac{1}{\sqrt{2}}\begin{pmatrix} G^a{}_b - \dfrac{2}{\sqrt{30}}B\delta^a{}_b & X^{\dagger a}{}_r \\ X^s{}_b & W^s{}_r + \dfrac{3}{\sqrt{30}}B\delta^s{}_r \end{pmatrix}, \tag{3.1.11}$$

where $G^a{}_b, W^s{}_r$, and B are the vector superfields for $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge fields. These gauge fields are defined as;

$$\frac{1}{\sqrt{2}}G^a{}_b = G^A(T_3^A)^a{}_b, \quad \frac{1}{\sqrt{2}}W^s{}_r = W^A(T_2^A)^s{}_r, \tag{3.1.12}$$

where T_2^A and T_3^A denote the generators of $SU(2)_L$ and $SU(3)_C$, respectively. The so-called X-type gauge superfields (X and $X^\dagger$) are the additional fields in the $SU(5)$ GUT.

We summarize the field contents of the $SU(5)$ GUT and the decomposition of these fields to the SM gauge representations in Table 5.

3.2 Superpotential

The general renormalizable superpotential of $SU(5)$ GUT is written as

$$\begin{aligned} W = & \frac{f}{3}\mathrm{Tr}\Sigma^3 + \frac{m}{2}\mathrm{Tr}\Sigma^2 + \lambda \overline{H}_\alpha(\Sigma^\alpha_\beta + a\delta^\alpha_\beta)H^\beta \\ & + \frac{h^{ij}}{4}\epsilon_{\alpha\beta\gamma\delta\epsilon}\Psi^{\alpha\beta}_i\Psi^{\gamma\delta}_j H^\epsilon + \sqrt{2}f^{ij}\Psi^{\alpha\beta}_i\Phi_{j\alpha}\overline{H}_\beta \\ & + \sum_i \Phi_{i\alpha}H^\alpha + c^{ijk}\Psi^{\alpha\beta}_i\Phi_{j\alpha}\Phi_{k\beta} + c^{ij}\Psi^{\alpha\beta}_i\overline{H}_\alpha\overline{H}_\beta. \end{aligned} \tag{3.2.1}$$

The first line describes the self-interaction terms of the adjoint Higgs superfield and the interaction terms of the adjoint Higgs superfield and $\mathbf{5}, \overline{\mathbf{5}}$ Higgs superfield. In the second line, coefficients of these terms are determined in order to realize the MSSM Yukawa terms in terms of the canonically normalized MSSM superfields. The third line describes the other gauge invariant terms.

We impose the R-parity invariance to our Lagrangian. The transformation properties of each supermultiplet under the R-parity are defined as below,

$$\Phi(x,\theta) \rightarrow -\Phi(x,-\theta) \qquad \text{for the matter chiral superfield,} \tag{3.2.2}$$

$$\Phi(x,\theta) \rightarrow \Phi(x,-\theta) \qquad \text{for the Higgs chiral superfield.} \tag{3.2.3}$$

This transformation property means even parity for SM particles and odd parity for their superpartners. By imposing the R-parity invariance, all of the terms in the third line in Eq. (3.2.1) are vanished.

Now, we consider the relations between each coefficients in the superpotential. If the adjoint higgs has the VEV Eq. (3.1.7), the adjoint higgs sector becomes as follows.

$$W_\Sigma = \frac{f}{3}\mathrm{Tr}\Sigma^3 + \frac{m}{2}\mathrm{Tr}\Sigma^2 = 30\left(-\frac{f}{3}V^3 + \frac{mV^2}{2}\right), \tag{3.2.4}$$

In order not to break supersymmetry in the GUT breaking vacuum, the minimum of this superpotential must be zero. For this condition, these coefficients, f and m, must satisfy an equality obtained as:

$$\frac{\partial W_\Sigma}{\partial V} = 30(-fV^2 + mV) = 0 \quad \Leftrightarrow \quad m = fV. \tag{3.2.5}$$

After the adjoint Higgs boson obtains the VEV, the $SU(5)$ Higgs boson and the adjoint higgs boson sector become

$$\begin{aligned} W_{h-\Sigma} =& \lambda \overline{H}_\alpha(\Sigma^\alpha_\beta + a\delta^\alpha_\beta)H^\beta \\ \longrightarrow & \lambda \overline{H}_\alpha \begin{pmatrix} 2V+a & & & & \\ & 2V+a & & & \\ & & 2V+a & & \\ & & & -3V+a & \\ & & & & -3V+a \end{pmatrix} H^\beta. \end{aligned} \tag{3.2.6}$$

Since V has the order of GUT scale ($\sim 10^{16}$GeV), we assume $a = 3V$ in order that the mass term of the SM higgs is equal to 0. This assumption is the so-called doublet-triplet splitting problem: there is no reason why the color-triplet Higgs multiplets are much heavier than the MSSM Higgs doublets. Finally the superpotential above the GUT scale is written down as:

$$\begin{aligned} W = & \frac{f}{3}\mathrm{Tr}\Sigma^3 + \frac{fV}{2}\mathrm{Tr}\Sigma^2 + \lambda \overline{H}_\alpha(\Sigma^\alpha_\beta + 3V\delta^\alpha_\beta)H^\beta \\ & + \frac{h^{ij}}{4}\epsilon_{\alpha\beta\gamma\delta\epsilon}\Psi_i^{\alpha\beta}\Psi_j^{\gamma\delta}H^\epsilon + \sqrt{2}f^{ij}\Psi_i^{\alpha\beta}\Phi_{j\alpha}\overline{H}_\beta. \end{aligned} \tag{3.2.7}$$

Now, we consider the degrees of freedom of Yukawa couplings in $SU(5)$ GUTs. h^{ij} is in $\mathbb{C}^6$ parameter space since this is symmetric matrix. And f^{ij} is in $\mathbb{C}^9$ parameter space. On the other hand, if there is no Yukawa term, we have degrees of freedom of the choice of the flavor basis of Φ, Ψ. This is global $\mathrm{U}(3) \times \mathrm{U}(3)$ flavor symmetry, namely there are $9 \times 2 = 18$ degrees of freedom of field re-definition.

Therefore, the physical degrees of freedom of the Yukawa couplings is $(6+9) \times 2 - 9 \times 2 = 12$. We can reparametrize these Yukawa matrices $Y_{ij} = h_{ij} + f_{ij}$ by using $U(3) \times U(3)$ flavor symmetry. $\mathbf{Y}$ is transformed by using flavor rotation as follows:

$$e^{i\alpha_1}e^{i\beta_{1\mu}T^\mu}\mathbf{Y}e^{i\beta_{2\mu}T^\mu}e^{i\alpha_2}, \tag{3.2.8}$$

where $\alpha_i, \beta_{i\mu}(i = 1,2; \mu = 1,2,\cdots,8)$ are real parameters, and T^μ are the Gell-Mann matrices.

After using flavor rotations, we make h^{ij} diagonal and only f^{ij} having off-diagonal elements.

$$h^{ij} = (h^i e^{i\varphi_i})\delta^{ij}, \tag{3.2.9}$$
$$f^{ij} = V^*_{ij}f^j. \tag{3.2.10}$$

Since h^{ij} and f^{ij} are Yukawa matrices, V_{ij} should become CKM matrix after h^{ij} is diagonalized. V_{ij} has four parameters (three CKM angles and KM complex phase). h^i, f^i are real parameters (tag is, there are six parameters) corresponding to masses of up-type quarks, charged leptons, and down-type quarks, respectively. Besides the SM parameters, there are extra degrees of freedom which are interpreted as additional phases. These additional phases can be combined with the diagonal elements in h^{ij} matrix or f^{ij} matrix. Since there are two additional phase parameters, we impose that $\varphi_1 + \varphi_2 + \varphi_3 = 0$. Thus, in this parametrization, there are $4 + 6 + (3-1) = 12$ parameters. Under this re-parametrization, the mass eigenstates of matter multiplet are obtained as:

$$\begin{aligned}
&\Psi_i \ni {}^t Q_i \equiv (U_i, D'_i) = (U_i, V_{ij} D_j), \\
&\Psi_i \ni e^{-i\varphi_i} U_i^C, \\
&\Psi_i \ni V_{ij} E_j^C, \\
&\Phi_i \ni D_i^C, \\
&\Phi_i \ni {}^t L_i \equiv (N_i, E_i).
\end{aligned} \tag{3.2.11}$$

Finally, let us decompose the Yukawa terms in the minimal SUSY $SU(5)$ GUT into the Yukawa terms in MSSM.

$$\begin{aligned}
W_{\text{Yukawa}} &= \frac{h^{ij}}{4} \epsilon_{\alpha\beta\gamma\delta\epsilon} \Psi_i^{\alpha\beta} \Psi_j^{\gamma\delta} H^\epsilon + \sqrt{2} f^{ij} \Psi_i^{\alpha\beta} \Phi_{j\alpha} \overline{H}_\beta \\
&= h^i V_{ij} U_{ia}^C E_j^C H_C^a - \frac{1}{2} h^i e^{i\varphi_i} \epsilon_{rs} \epsilon_{abc} (Q_i^{ra} Q_i^{sb}) H_C^c + h^i \epsilon_{rs} U_{ia}^C (Q_i^{ra} H_u^s) \\
&\quad + V_{ij}^* f^j e^{-i\varphi_i} \epsilon^{abc} U_{ia}^C D_{jb}^C \overline{H}_{Cc} - V_{ij}^* f^j \epsilon_{rs} Q_i^{ra} L_j^s \overline{H}_{Ca} \\
&\quad + V_{ij}^* f^j \epsilon_{rs} Q_i^{ra} H_d^s D_{ja}^C + f^i \epsilon_{rs} E_i^C L_i^r H_d^s
\end{aligned} \tag{3.2.12}$$

where $a, b, c, \ldots$ are color indices, $r, s, \ldots$ are $\mathrm{SU}(2)_L$ indices, and $i, j, \ldots$ are indices of generations.

3.3 Mass Spectrum of GUT Particles

After the adjoint Higgs boson has the VEV which breaks the grand unified gauge group to the SM gauge group, some of particles is obtained heavy masses whose scale is around GUT scale ($\sim 10^{16}$GeV). The massive particles are the color-triplet Higgs multiplets, the X-type gauge superfields, and the adjoint Higgs boson itself.

The mass term of the X-type gauge superfields is obtained from the covariant derivative of the adjoint Higgs boson.

$$D_\mu \Sigma = \partial_\mu \Sigma + ig \left[A_\mu, \Sigma \right] = D_\mu \Sigma' + ig \left[A_\mu, \langle \Sigma \rangle \right], \tag{3.3.1}$$

where the adjoint Higgs field is expanded around VEV $\langle \Sigma \rangle$, $\Sigma = \Sigma' + \langle \Sigma \rangle$. The form of VEV is written down in Eq. (3.1.7) explicitly.

$$\left[A_\mu, \langle \Sigma \rangle \right] = \frac{5V}{\sqrt{2}} \begin{pmatrix} 0 & -X_\mu^\dagger \\ X_\mu & 0 \end{pmatrix} \tag{3.3.2}$$

Thus, the mass terms of the X-type gauge superfields are obtained as follows;

$$\operatorname{Tr} \left(D_\mu \Sigma D^\mu \Sigma \right) = \operatorname{Tr} \left(D_\mu \Sigma' D^\mu \Sigma' \right) + 50 g^2 V^2 X^\dagger X + (\text{interaction terms}). \tag{3.3.3}$$

Therefore, the mass of the X-type gauge superfields is $M_X = 5\sqrt{2}gV$. The part of the adjoint Higgs boson in superpotential is expanded around VEV as,

$$\frac{f}{3}\mathrm{Tr}\Sigma^3 + \frac{fV}{2}\mathrm{Tr}\Sigma^2 = \frac{f}{3}\mathrm{Tr}(\Sigma' + \langle\Sigma\rangle)^3 + \frac{fV}{2}\mathrm{Tr}(\Sigma' + \langle\Sigma\rangle)^2. \tag{3.3.4}$$

If we want to know the mass of adjoint higgs, we have to see the scalar mass term or the fermion mass term. When we concentrate on scalar mass term, we should calculate the potential $|\partial W/\partial\Sigma^a|^2$. However the adjoint higgsino mass is easy to calculate.

$$\mathcal{L} \supset -\frac{1}{2}W^{ij}\psi_i\psi_j, \qquad \left(W^{ij} = \left.\frac{\partial^2 W}{\partial\Phi_i\partial\Phi_j}\right|_{\Phi\to\phi} \right). \tag{3.3.5}$$

The quadratic term in superpotential is

$$\begin{aligned} f\mathrm{Tr}\Sigma'^2 \langle\Sigma\rangle + \frac{fV}{2}\mathrm{Tr}\Sigma'^2 &= fV\mathrm{Tr}\Sigma'^2 \left(\begin{array}{c|c} 5/2 & 0 \\ \hline 0 & -5/2 \end{array} \right) \\ &= \frac{5fV}{2}\left(\mathrm{Tr}_3\Sigma_8\Sigma_8 - \mathrm{Tr}_2\Sigma_3\Sigma_3\right) - \frac{1}{4}fV\Sigma_{24}\Sigma_{24}. \end{aligned} \tag{3.3.6}$$

By using normalization of generators, $\mathrm{Tr}T^aT^b = \frac{1}{2}\delta^{ab}$, the mass terms are obtained as,

$$\frac{5}{4}fV\Sigma_8^a\Sigma_8^a - \frac{5}{4}fV\Sigma_3^a\Sigma_3^a - \frac{1}{4}fV\Sigma_{24}\Sigma_{24}. \tag{3.3.7}$$

Therefore, the masses of the adjoint higgsino (= the masses of the adjoint Higgs boson) are

$$M_{\Sigma_8} = M_{\Sigma_3} = \frac{5}{2}fV, \qquad M_{\Sigma_{24}} = \frac{1}{2}fV. \tag{3.3.8}$$

The vector-like mass term of the color-triplet Higgs is obtained from the interaction term between the color-triplet Higgs and the adjoint Higgs after the grand unified gauge group is broken down to the standard model gauge group spontaneously.

$$\begin{aligned} W_{h-\Sigma} &= \lambda\overline{H}_\alpha(\Sigma^\alpha_\beta + 3V\delta^\alpha_\beta)H^\beta \\ &\to 5\lambda V\overline{H}_{Ca}H^a_C \end{aligned} \tag{3.3.9}$$

Therefore, the mass of the color-triplet Higgs is

$$M_{H_C} = 5\lambda V. \tag{3.3.10}$$

4 GUT Mass Spectrum from Threshold Corrections

In this section, we assume that the standard model gauge group is embedded in the large group, $SU(5)$. Below the energy scale at which the unified gauge group is broken, we use the effective theory in which the heavy massive particles are integrated out in terms of the path integral. The coupling constants and the field normalizations in the effective theories include the effects of massive particles. These effects are called "threshold corrections". Details of construction methods about effective theories and threshold corrections are described in Appendix D.

First, we show the general treatments for the threshold corrections for gauge couplings and give the threshold corrections at each scale; the GUT scale, the SUSY breaking scale and the gaugino threshold. Then, we shows the constraint on the GUT-particle mass spectrum and our results for this constraint in high-scale SUSY breaking model.

4.1 Threshold Corrections from Integrating out the Heavy Massive Particles

Now, we assume that the standard model gauge group is unified at GUT scale ($\sim 10^{16}$GeV). The relation between the gauge couplings of the standard gauge group and the unified gauge group at a renormalization scale Λ is obtained as follows:

$$\alpha_i^{-1}(\Lambda) = \alpha_G^{-1}(\Lambda) - 4\pi\lambda_i(\Lambda). \tag{4.1.1}$$

where α_i and α_G are defined as $g_i^2/4\pi$ and $g_G^2/4\pi$; g_i and g_G are the gauge couplings of the standard model and the unified gauge coupling, respectively. λ_i are called threshold corrections for the gauge coupling g_i at GUT scale.

Let us consider the case that the matter contents of a theory change into the other matter contents at a renormalization scale μ and the gauge group of this theory is not change at μ. At this scale, the relation of the gauge couplings between these two theories is

$$\alpha_i'^{-1}(\mu) = \alpha_i^{-1}(\mu) - 4\pi\lambda_i'(\mu). \tag{4.1.2}$$

The effects of heavy massive particles are included in these threshold corrections. At 1-loop level, this threshold correction is obtained as

$$\lambda_i(\mu) = \frac{1}{48\pi^2}\left[-21\mathrm{Tr}\left(t_{iV}^2 \ln\frac{M_V}{\mu}\right) + 8\mathrm{Tr}\left(t_{iF}^2 \ln\frac{M_F}{\mu}\right) + \mathrm{Tr}\left(t_{iS}^2 \ln\frac{M_S}{\mu}\right)\right], \tag{4.1.3}$$

where M_V, M_F, and M_S are the masses of a massive gauge boson, a massive Dirac fermion, and a massive real scalar. We use $\overline{\mathrm{DR}}$ scheme* when we derive this threshold correction. The

*In this scheme, we require that supersymmetry is preserved by using the dimensional reduction when we regularize the loop integrals [74]. However, in this scheme, the supersymmetry can be broken at the higher-loop corrections[75, 76, 77].

1st term is obtained from the massive vector boson (including the vector boson, the ghost, and the Nambu-Goldstone (NG) boson). The contributions of the massive Dirac fermion and the massive real scalar boson are described in the 2nd and the 3rd terms, respectively. In this expression, t_{iV}, t_{iF} and t_{iS} are the generators of the massive vector boson, the massive Dirac fermions, and the real scalar bosons, respectively.

In SUSY GUTs, since massive fields which are associated with by supersymmetric transformation are integrated out at the GUT scale, it is useful to use the threshold correction in terms of superfields. Therefore we write down the threshold corrections in terms of a vector boson (including only vector boson and ghost), an absorbed complex NG boson, a Weyl fermion and a complex scalar.

$$\begin{aligned}&4\pi\lambda_i(\mu) = \\ &\frac{1}{2\pi}\left[-\frac{11}{3}\mathrm{Tr}\left(t_{iV}^2 \ln\frac{M_V}{\mu}\right) + \frac{1}{3}\mathrm{Tr}\left(t_{iV}^2 \ln\frac{M_V}{\mu}\right) + \frac{2}{3}\mathrm{Tr}\left(t_{iF}^2 \ln\frac{M_F}{\mu}\right) + \frac{1}{3}\mathrm{Tr}\left(t_{iS}^2 \ln\frac{M_S}{\mu}\right)\right]\end{aligned} \tag{4.1.4}$$

The 1st term is the contribution obtained from the massive vector boson and ghost. The contribution from the absorbed NG boson is described in the 2nd terms. The massive Weyl fermion and the massive complex scalar boson contribute to the threshold correction such as the 3rd term and the 4th term, respectively.

For vector superfields, there are a gauge field and its superpartners "gaugino" which are two Weyl fermions. There are also a NG boson and its superpartner in an absorbed Nambu-Goldstone chiral superfield. The threshold correction for the vector superfields is given by

$$\begin{aligned}&4\pi\lambda_i(\mu) \\ &= \frac{1}{2\pi}\left[-\frac{11}{3}\mathrm{Tr}\left(t_{iV}^2 \ln\frac{M_V}{\mu}\right) + \frac{2}{3}\mathrm{Tr}\left(t_{iV}^2 \ln\frac{M_V}{\mu}\right) + \frac{1}{3}\mathrm{Tr}\left(t_{iS}^2 \ln\frac{M_S}{\mu}\right) + \frac{2}{3}\mathrm{Tr}\left(t_{iS}^2 \ln\frac{M_S}{\mu}\right)\right] \\ &= \frac{1}{2\pi}(-3C + T)\ln\frac{M_V}{\mu} \\ &= \frac{1}{2\pi}(-2C)\ln\frac{M_V}{\mu}.\end{aligned} \tag{4.1.5}$$

where $C = \mathrm{Tr}\ t_{iV}^2$ and $T = \mathrm{Tr}\ t_{iS}^2$. In the second equality, we assume that each superfield which belongs to the same representation and is integrated out has the same mass. Actually, in the minimal SUSY $SU(5)$ GUT, these mass matrices are proportional to the unity. Since the trace of the squared matrices (that is, the Casimir operator) for the massive gauge boson and for the absorbed NG boson are the same, we set $C = T$ at the last equality.

For chiral superfields, there are a complex scalar field and its superpartners which are two Weyl fermions. Therefore the threshold correction for a chiral superfield is given by

$$\begin{aligned} 4\pi\lambda_i(\mu) &= \frac{1}{2\pi}\left[\frac{1}{3}\mathrm{Tr}\left(t_{iS}^2 \ln\frac{M_S}{\mu}\right) + \frac{2}{3}\mathrm{Tr}\left(t_{iS}^2 \ln\frac{M_S}{\mu}\right)\right] \\ &= \frac{1}{2\pi} T \ln\frac{M_V}{\mu}. \end{aligned} \tag{4.1.6}$$

4.1.1 Threshold Corrections at the GUT Scale

The heavy gauge boson (X-type gauge superfields) belongs to the fundamental representation in $SU(3)_C$ and also $SU(2)_L$. Thus, the trace of each Casimir operator is $1/2$. The hypercharge of this boson is $5/6$. The contribution of the X-type gauge superfields is obtained as

$$\begin{aligned} 4\pi\lambda_3^{\mathrm{V}}(\mu) &= \frac{1}{2\pi}\left(-2\times\frac{1}{2}\right)\ln\frac{M_X}{\mu}\times 2\times 2 = \frac{1}{2\pi}\cdot(-4)\ln\frac{M_X}{\mu}, \\ 4\pi\lambda_2^{\mathrm{V}}(\mu) &= \frac{1}{2\pi}\left(-2\times\frac{1}{2}\right)\ln\frac{M_X}{\mu}\times 2\times 3 = \frac{1}{2\pi}\cdot(-6)\ln\frac{M_X}{\mu}, \\ 4\pi\lambda_1^{\mathrm{V}}(\mu) &= \frac{1}{2\pi}\frac{3}{5}\left(-2\times\left(\frac{5}{6}\right)^2\right)\ln\frac{M_X}{\mu}\times 2\times 3\times 2 = \frac{1}{2\pi}\cdot(-10)\ln\frac{M_X}{\mu}, \end{aligned} \tag{4.1.7}$$

where the subscripts $1, 2, 3$ are correspond to gauge groups, unified $U(1)$, $SU(2)_L$ and $SU(3)_C$, respectively. M_X is the mass of the X-type gauge superfields. The factor 2 is appeared since the X-type gauge superfields is complex field. Since the unified $U(1)$ charge is different from $U(1)_Y$ regarding normalization, the threshold correction for the unified $U(1)$ is multiplied by $3/5$.

The adjoint Higgs boson $\Sigma(\mathbf{24})$ is divided into $\Sigma_8, \Sigma_3, \Sigma_{(3,2)}, \Sigma_{(\bar{3},2)}$, and Σ_{24} under the standard model gauge group. However $\Sigma_{(3,2)}$ and $\Sigma_{(\bar{3},2)}$ are not considered since they are eaten by the longitudinal component of the heavy gauge boson. Σ_8 is the adjoint representation field in $SU(3)_C$, is the trivial representation in $SU(2)_L$, and is neutral in $U(1)$. Similarly, Σ_3 is the adjoint representation field in $SU(2)_L$, is the trivial representation in $SU(3)_C$, and is neutral in $U(1)$. Σ_{24} is gauge singlet field. Therefore the contribution from integrating out the heavy adjoint Higgs field is given by

$$\begin{aligned} 4\pi\lambda_3^{\Sigma}(\mu) &= \frac{1}{2\pi}\cdot 3\cdot\ln\frac{M_\Sigma}{\mu}, \\ 4\pi\lambda_2^{\Sigma}(\mu) &= \frac{1}{2\pi}\cdot 2\cdot\ln\frac{M_\Sigma}{\mu}, \end{aligned} \tag{4.1.8}$$

where M_Σ is the masses of Σ_8, Σ_3. Let us consider the contribution from integrating out the color-triplet Higgs multiplet which belongs to the fundamental representation in $SU(3)_C$

and whose hypercharge is $\pm 1/3$. There are two color-triplet Higgs multiplets $H_C, \overline{H}_C$ in minimal SUSY $SU(5)$ GUT.

$$\begin{aligned} 4\pi\lambda_3^C(\mu) &= \frac{1}{2\pi}\frac{1}{2}\ln\frac{M_{H_C}}{\mu}\times 2, \\ 4\pi\lambda_1^C(\mu) &= \frac{1}{2\pi}\frac{3}{5}\left(\frac{1}{3}\right)^2\ln\frac{M_{H_C}}{\mu}\times 2\times 3 = \frac{1}{2\pi}\frac{2}{5}\ln\frac{M_{H_C}}{\mu}, \end{aligned} \tag{4.1.9}$$

where M_{H_C} is the mass of the color-triplet Higgs multiplets. Finally, the relation between the standard model gauge couplings and the unified gauge coupling at renormalization scale μ is expressed as;

$$\begin{aligned} \frac{1}{\alpha_3^s(\mu)} &= \frac{1}{\alpha_G(\mu)} - \frac{1}{2\pi}\left[-4\ln\frac{M_X}{\mu} + 3\ln\frac{M_\Sigma}{\mu} + \ln\frac{M_{H_C}}{\mu}\right], \\ \frac{1}{\alpha_2^s(\mu)} &= \frac{1}{\alpha_G(\mu)} - \frac{1}{2\pi}\left[-6\ln\frac{M_X}{\mu} + 2\ln\frac{M_\Sigma}{\mu}\right], \\ \frac{1}{\alpha_1^s(\mu)} &= \frac{1}{\alpha_G(\mu)} - \frac{1}{2\pi}\left[-10\ln\frac{M_X}{\mu} + \frac{2}{5}\ln\frac{M_{H_C}}{\mu}\right], \end{aligned} \tag{4.1.10}$$

where superscript s denotes the gauge couplings in the supersymmetric standard model.

4.1.2 Threshold Corrections from the SUSY Particles

For a complex scalar, threshold correction is given by

$$4\pi\lambda_i'(\mu) = \frac{1}{2\pi}\cdot\frac{1}{3}\cdot T\ln\frac{M_S}{\mu}. \tag{4.1.11}$$

Many complex scalar fields are included in supersymmetric theories, which are left-handed squarks, left-handed sleptons, right-handed up-type squark, right-handed down-type squark, and right-handed electron-type sleptons. Threshold corrections below their mass scales are obtained as;

$$\begin{aligned} 4\pi\lambda_3'(\mu) &= \frac{1}{2\pi}\left(\frac{1}{3}\ln\frac{M_{\tilde{Q}}}{\mu} + \frac{1}{6}\ln\frac{M_{\widetilde{U}^C}}{\mu} + \frac{1}{6}\ln\frac{M_{\widetilde{D}^C}}{\mu}\right), \\ 4\pi\lambda_2'(\mu) &= \frac{1}{2\pi}\left(\frac{1}{2}\ln\frac{M_{\tilde{Q}}}{\mu} + \frac{1}{6}\ln\frac{M_{\widetilde{L}}}{\mu}\right) \\ 4\pi\lambda_1'(\mu), &= \frac{1}{2\pi}\left(\frac{1}{30}\ln\frac{M_{\tilde{Q}}}{\mu} + \frac{1}{10}\ln\frac{M_{\widetilde{L}}}{\mu} + \frac{4}{15}\ln\frac{M_{\widetilde{U}^C}}{\mu} + \frac{1}{15}\ln\frac{M_{\widetilde{D}^C}}{\mu} + \frac{1}{5}\ln\frac{M_{\widetilde{E}^C}}{\mu}\right). \end{aligned} \tag{4.1.12}$$

$M_{\tilde{Q}}$ and $M_{\widetilde{L}}$ denote the masses of the left-handed squark and slepton, and $M_{\widetilde{U}^C}, M_{\widetilde{D}^C}$, and $M_{\widetilde{E}^C}$ denote the masses of the right-handed up squark, down squark, and selectron. In supersymmetric extension of the standard model, the heavy Higgs boson is also introduced in addition to the standard model Higgs boson. Since this heavy boson is the $SU(3)_C$ singlet, the $SU(2)_L$ doublet, and its hypercharge is $1/2$, the contributions from heavy Higgs is obtained as follows:

$$\begin{aligned} 4\pi\lambda_2'(\mu) &= \frac{1}{2\pi}\cdot\frac{1}{6}\ln\frac{M_H}{\mu}, \\ 4\pi\lambda_1'(\mu) &= \frac{1}{2\pi}\cdot\frac{1}{10}\ln\frac{M_H}{\mu}, \end{aligned} \tag{4.1.13}$$

where M_H is the mass of the heavy Higgs boson. In the supersymmetric standard model, many Weyl spinors are also included. The threshold correction for a Weyl spinor is given by

$$4\pi\lambda_i'(\mu) = \frac{1}{2\pi}\cdot\frac{2}{3}\cdot\mathrm{Tr}(t_{iF}^2)\ln\frac{M_F}{\mu}. \tag{4.1.14}$$

at renormalization scale μ. Since $\mathrm{Tr}(t_{iF}^2)$ is equal to N for the gaugino of $SU(N)$ and is equal to 0 for the abelian gaugino, threshold corrections for integrating out gauginos are given by

$$\begin{aligned} 4\pi\lambda_3'(\mu) &= \frac{1}{2\pi}\cdot 2\ln\frac{M_3}{\mu}, \\ 4\pi\lambda_2'(\mu) &= \frac{1}{2\pi}\cdot\frac{4}{3}\ln\frac{M_2}{\mu}, \\ 4\pi\lambda_1'(\mu) &= 0, \end{aligned} \tag{4.1.15}$$

where M_1, M_2 and M_3 denote the masses of binos, winos, and gluinos, respectively. The higgsinos are also Weyl spinors including in the supersymmetric standard model. The contributions from higgsinos which are superpartners of two doublet Higgs bosons are expressed as;

$$\begin{aligned} 4\pi\lambda_2'(\mu) &= \frac{1}{2\pi}\cdot\frac{2}{3}\ln\frac{M_{\widetilde{H}}}{\mu}, \\ 4\pi\lambda_1'(\mu) &= \frac{1}{2\pi}\cdot\frac{2}{5}\ln\frac{M_{\widetilde{H}}}{\mu}. \end{aligned} \tag{4.1.16}$$

$M_{\widetilde{H}}$ denotes the masses of the higgsino. Let us summarize threshold correction from SUSY particles. In the high-scale SUSY breaking model, the squarks, the sleptons, the higgsinos and the heavy higgses are integrated out at the SUSY breaking scale. The effects of these heavy particles are included through the threshold corrections which are given by

$$\frac{1}{\alpha_3'(\mu)} = \frac{1}{\alpha_3^s(\mu)} - \frac{1}{2\pi}\left[\frac{1}{3}N_g\ln\frac{M_{\widetilde{Q}}}{\mu} + \frac{1}{6}N_g\ln\frac{M_{\widetilde{U}^C}}{\mu} + \frac{1}{6}N_g\ln\frac{M_{\widetilde{D}^C}}{\mu}\right], \tag{4.1.17}$$

$$\frac{1}{\alpha'_2(\mu)} = \frac{1}{\alpha^s_2(\mu)} - \frac{1}{2\pi}\left[\frac{1}{2}N_g \ln\frac{M_{\widetilde{Q}}}{\mu} + \frac{1}{6}N_g \ln\frac{M_{\widetilde{L}}}{\mu} + \frac{2}{3}\ln\frac{M_{\widetilde{H}}}{\mu} + \frac{1}{6}\ln\frac{M_H}{\mu}\right], \tag{4.1.18}$$

$$\begin{aligned}\frac{1}{\alpha'_1(\mu)} = \frac{1}{\alpha^s_1(\mu)} - \frac{1}{2\pi}\Bigg[&\frac{1}{30}N_g \ln\frac{M_{\widetilde{Q}}}{\mu} + \frac{4}{15}N_g \ln\frac{M_{\widetilde{U}^C}}{\mu} + \frac{1}{15}N_g \ln\frac{M_{\widetilde{D}^C}}{\mu} \\ &+\frac{1}{10}N_g \ln\frac{M_{\widetilde{L}}}{\mu} + \frac{1}{5}N_g \ln\frac{M_{\widetilde{E}^C}}{\mu} + \frac{2}{5}\ln\frac{M_{\widetilde{H}}}{\mu} + \frac{1}{10}\ln\frac{M_H}{\mu}\Bigg].\end{aligned} \tag{4.1.19}$$

where N_g is the number of generations. The couplings with prime denote the gauge couplings in the standard model with the gauginos. Now, we assume that the heavy sparticles are degenerate in mass (M_S). Under this assumption, the relations between couplings are simplified as:

$$\begin{aligned}\frac{1}{\alpha'_3(\mu)} &= \frac{1}{\alpha^s_3(\mu)} - \frac{1}{2\pi}\frac{2}{3}N_g \ln\frac{M_S}{\mu}, \\ \frac{1}{\alpha'_2(\mu)} &= \frac{1}{\alpha^s_2(\mu)} - \frac{1}{2\pi}\left(\frac{2}{3}N_g + \frac{5}{6}\right)\ln\frac{M_S}{\mu} \\ \frac{1}{\alpha'_1(\mu)} &= \frac{1}{\alpha^s_1(\mu)} - \frac{1}{2\pi}\left(\frac{2}{3}N_g + \frac{1}{2}\right)\ln\frac{M_S}{\mu}.\end{aligned} \tag{4.1.20}$$

At the threshold where the energy scale gauginos are integrated out, the matching conditions for the gauge couplings are given by

$$\begin{aligned}\frac{1}{\alpha_3(\mu)} &= \frac{1}{\alpha'_3(\mu)} - \frac{1}{2\pi}2\ln\frac{M_3}{\mu}, \\ \frac{1}{\alpha_2(\mu)} &= \frac{1}{\alpha'_2(\mu)} - \frac{1}{2\pi}\frac{4}{3}\ln\frac{M_2}{\mu}, \\ \frac{1}{\alpha_1(\mu)} &= \frac{1}{\alpha'_1(\mu)}.\end{aligned} \tag{4.1.21}$$

where α_i denotes the standard model gauge couplings.

4.2 Constraints on Mass Spectrum of GUT-Scale Particles

In this subsection, we show that the mass spectrum of GUT-scale particles can be changed in the case of high-scale SUSY scenario. As we have explained in previous subsection, threshold corrections for the gauge couplings are the function of the renormalization scale and the mass scale of the particles which are integrated out. If we choose linear combinations of threshold corrections in Eq. (4.1.10) appropriately, the mass spectrum of the GUT scale particles which is independent from the unified gauge coupling at GUT scale is obtained as:

$$\begin{aligned}\frac{3}{\alpha_2^s(\mu)} - \frac{2}{\alpha_3^s(\mu)} - \frac{1}{\alpha_1^s(\mu)} &= \frac{1}{2\pi} \cdot \frac{12}{5} \ln \frac{M_{H_C}}{\mu}, \\ \frac{5}{\alpha_1^s(\mu)} - \frac{3}{\alpha_2^s(\mu)} - \frac{2}{\alpha_3^s(\mu)} &= \frac{1}{2\pi} \cdot 12 \ln \frac{M_X^2 M_\Sigma}{\mu^3}.\end{aligned} \tag{4.2.1}$$

Left-hand sides of these relations are given by gauge couplings of the MSSM at renormalization scale μ.

For understanding the sparticle mass dependence of the GUT-scale mass spectrum, we use 1-loop beta functions for the gauge couplings. RGEs at 1-loop level are given by

$$\frac{\mathrm{d}g_i}{\mathrm{d}\ln\mu} = \frac{b_i g_i^3}{16\pi^2}, \tag{4.2.2}$$

where b_i are determined as follows:

$$\begin{aligned}(b_1, b_2, b_3)_{\mathrm{SM}} &= \left(\frac{41}{10}, -\frac{19}{6}, -7\right), \\ (b_1, b_2, b_3)_{\mathrm{gaugino}} &= \left(\frac{41}{10}, -\frac{11}{6}, -5\right), \\ (b_1, b_2, b_3)_{\mathrm{MSSM}} &= \left(\frac{33}{5}, 1, -3\right).\end{aligned} \tag{4.2.3}$$

The subscript "SM" and "MSSM" denote that these coefficients are in the SM and in the MSSM, respectively. Similarly, "gaugino" means that these coefficients are in the theory which includes only the SM particles and gauginos. Then, we obtain the 1-loop result as:

$$\begin{aligned}\frac{3}{\alpha_2(m_Z)} - \frac{2}{\alpha_3(m_Z)} - \frac{1}{\alpha_1(m_Z)} &= \frac{1}{2\pi}\left[\frac{12}{5}\ln\frac{M_{H_C}}{m_Z} - 2\ln\frac{M_S}{m_Z} + 4\ln\frac{M_3}{M_2}\right], \\ \frac{5}{\alpha_1(m_Z)} - \frac{3}{\alpha_2(m_Z)} - \frac{2}{\alpha_3(m_Z)} &= \frac{1}{2\pi}\left[12\ln\frac{M_X^2 M_\Sigma}{m_Z^3} + 4\ln\frac{M_3 M_2}{m_Z^2}\right].\end{aligned} \tag{4.2.4}$$

From these relations, we can find out the following properties. First, the first line of Eq. (4.2.4) shows the sparticle mass dependence of the color-triplet Higgs multiplets. The mass of the color-triplet Higgs multiplets depends on the mass of the heavy scalars and the ratio of gaugino masses. That is, the mass of the color-triplet Higgs multiplets is proportional to $M_S^{5/6}$ and $(M_3/M_2)^{-5/3}$. This relation also implies that the color-triplet Higgs multiplets which is $SU(5)$ partner of the doublet Higgs bosons could become more massive when the heavy Higgs doublet obtains more heavier mass.

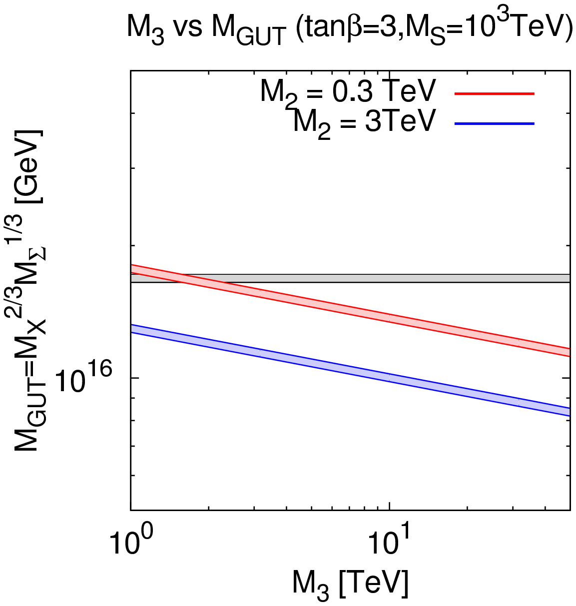


Figure 4: Gluino mass dependence of M_{GUT}. $\tan\beta$ is set to be 3 and the degenerate scalar mass is set to be 10^3 TeV. A black region describes the case of low-scale SUSY scenario that SUSY breaking scale is set to be 1TeV and the masses of gluino and wino are set to be 1TeV and 0.3TeV, respectively. Theoretical error coming from the strong coupring constant $\alpha_S(m_Z) = 0.1184(7)$ is also shown.

Another relation states that a mass combination such as $M_{GUT} \equiv M_X^{2/3} M_\Sigma^{1/3}$ is proportional to the product of the gaugino masses as $(M_3 M_2)^{-1/9}$. This relation also states that the sparticle mass does not contribute to this value.

In a realistic calculation, quark masses are estimated as the $\overline{\mathrm{MS}}$ masses except top quark mass. Also, we know gauge couplings as $\overline{\mathrm{MS}}$ values. Hence, if we use RGEs in the $\overline{\mathrm{DR}}$ scheme, we must transform these $\overline{\mathrm{MS}}$ values into the $\overline{\mathrm{DR}}$ ones. First, we will connect gauge couplings in the $\overline{\mathrm{MS}}$ scheme and those in the $\overline{\mathrm{DR}}$ scheme. These values are related by the following forms*:

$$\begin{aligned}
\frac{1}{\alpha_3^{\overline{\mathrm{DR}}}(\mu)} &= \frac{1}{\alpha_3^{\overline{\mathrm{MS}}}(\mu)} - \frac{1}{4\pi}, \\
\frac{1}{\alpha_2^{\overline{\mathrm{DR}}}(\mu)} &= \frac{1}{\alpha_2^{\overline{\mathrm{MS}}}(\mu)} - \frac{1}{6\pi}, \\
\frac{1}{\alpha_1^{\overline{\mathrm{DR}}}(\mu)} &= \frac{1}{\alpha_1^{\overline{\mathrm{MS}}}(\mu)}.
\end{aligned} \tag{4.2.5}$$

*These relations between the gauge couplings in the $\overline{\mathrm{MS}}$ and the $\overline{\mathrm{DR}}$ schemes are given in Appendix D.

Furthermore, masses of quarks are transformed as [78]:

$$
\begin{aligned}
m_q^{\overline{\mathrm{DR}}}(\mu) &= m_q^{\overline{\mathrm{MS}}}(\mu)\left[1-\frac{\alpha_S(\mu)}{3\pi}\right], \\
\frac{m_q^{\mathrm{pole}}}{m_q^{\overline{\mathrm{DR}}}(\mu)} &= 1+\frac{\alpha_S^{\overline{\mathrm{DR}}}(\mu)}{\pi}\left[\frac{5}{3}-\ln\left(\frac{m_q^{\overline{\mathrm{DR}}}(\mu)}{\mu}\right)^2\right].
\end{aligned}
\tag{4.2.6}
$$

By using all of these relations, we can use all of parameters as $\overline{\mathrm{DR}}$ couplings.

If we consider the threshold correction at the 1-loop level, we must treat beta functions of the gauge couplings at the 2-loop level. The 2-loop renormalization group equations for gauge couplings are described in Appendix E. In order to obtain the RG evolution of the gauge couplings, the 1-loop RGEs for Yukawa couplings are sufficient since these do not emerge in RGEs for gauge couplings until we consider them at the 2-loop level.

Since there are several threshold scales between the electroweak scale and the unification scale, running couplings should be matched properly at each threshold scales. Above the electroweak scale, we find the gaugino threshold is near several TeV. At this threshold, gauge couplings are matched as Eq. (4.1.21).

Next, at SUSY breaking threshold (at M_S), threshold corrections for the gauge couplings are given by Eq. (4.1.20). Yukawa couplings are matched as

$$
\begin{aligned}
\widetilde{U}(M_S) &= \frac{1}{\sin\beta}U(M_S), \\
\widetilde{Y}_j(M_S) &= \frac{1}{\cos\beta}Y_j(M_S) \qquad (j = D, E),
\end{aligned}
\tag{4.2.7}
$$

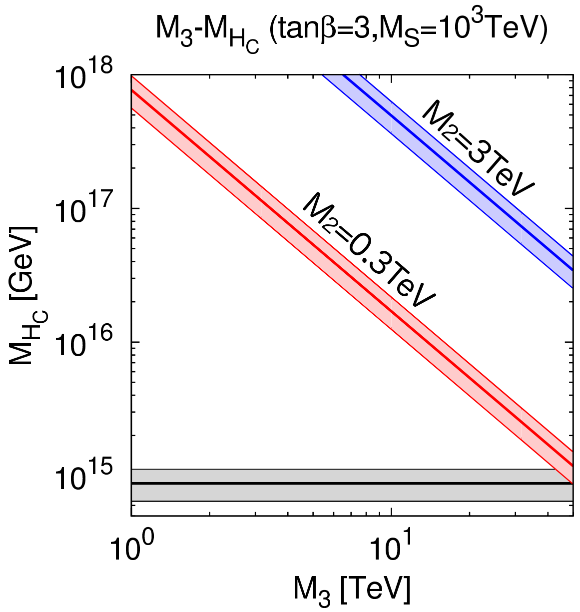
where Yukawa couplings in right-hand side are supersymmetric ones and those in left-hand side are the couplings in the standard mode with gauginos.

Numerical calculations for these GUT scale masses at the 2-loop level are exhibited in Fig. 4, Fig. 5, and Fig. 6. In these calculations, ambiguity of the strong gauge coupling is included as widths of theoretical lines.

First, in Fig. 4, we show the gluino mass dependence of M_{GUT}. A red line and a blue line correspond to the case that the mass of wino is set to be 0.3TeV and 3TeV, respectively. A black region describes the case of low-scale SUSY scenario that SUSY breaking scale is set to be 1TeV and the masses of gluino and wino are set to be 1TeV and 0.3TeV, respectively. In this parameter set, we obtain

$$
M_{\mathrm{GUT}} = (1.68 \pm 0.03) \times 10^{16}\ \mathrm{GeV}. \tag{4.2.8}
$$

Even if the breaking scale of supersymmetry is higher than the TeV scale, the GUT scale M_{GUT} will be barely changed from the low-scale SUSY scenario.



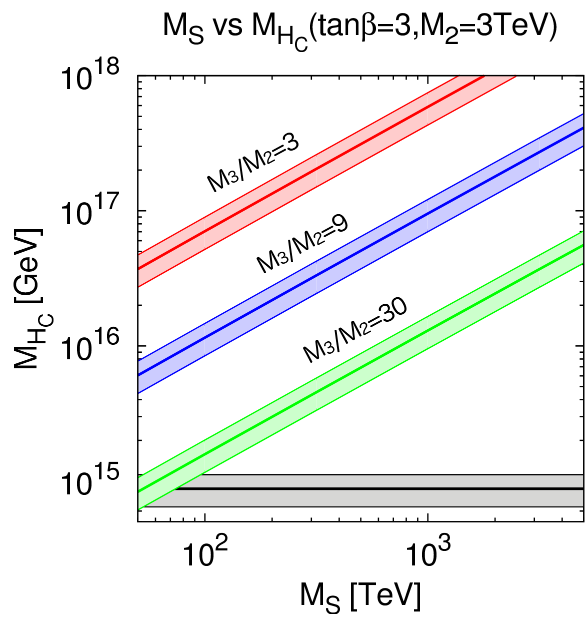


Figure 5: (Left) Gluino mass dependence of the mass of the color-triplet Higgs multiplets. $\tan\beta$ is set to be 3 TeV and the degenerate scalar mass is set to be 10^3 TeV. Error bar coming from strong coupling constant $\alpha_S(m_Z) = 0.1184(7)$ is also shown.

Figure 6: (Right) SUSY breaking scale dependence of color-triplet Higgs boson mass. $\tan\beta$ is set to be 3 TeV and wino mass is set to be 3 TeV. Error bar indicates the input error of strong coupling constant $\alpha_S(m_Z) = 0.1184(7)$.

Next, in Fig. 5, we show the gluino mass dependence of M_{H_C}. In this figure, we set the mass scale of sfermions to be 10^3TeV. A red line and a blue line correspond to the case that the mass of wino is set to be 0.3TeV and 3TeV, respectively. Finally, Fig. 6 shows the SUSY breaking scale dependence of M_{H_C}. In this calculation, we set wino mass to be 3 TeV. A red, a blue and a green region correspond to the case that the mass ratio of gaugino (M_3/M_2) is set to be 3, 9, and 30, respectively.

In each figures, a black region describes the case of low-scale SUSY scenario, which we use the same parameter set as Eq. (4.2.8). By using this parameter set, we find

$$M_{H_C} = (8.77 \pm 2.25) \times 10^{14}\ \text{GeV}. \tag{4.2.9}$$

In low-scale SUSY scenarios, since not only that the breaking scale of SUSY is near TeV but also that the color-triplet Higgs bosons have light mass, proton decay caused by the color-triplet Higgs multiplets have contradicted observational result ($\tau(p \to K^+ + \bar{\nu}) > 5.9 \times 10^{33}$ years) in Super-Kamiokande. However, in high-scale SUSY breaking scenarios, we find that the color-triplet Higgs multiplets have heavy mass compared with low-scale

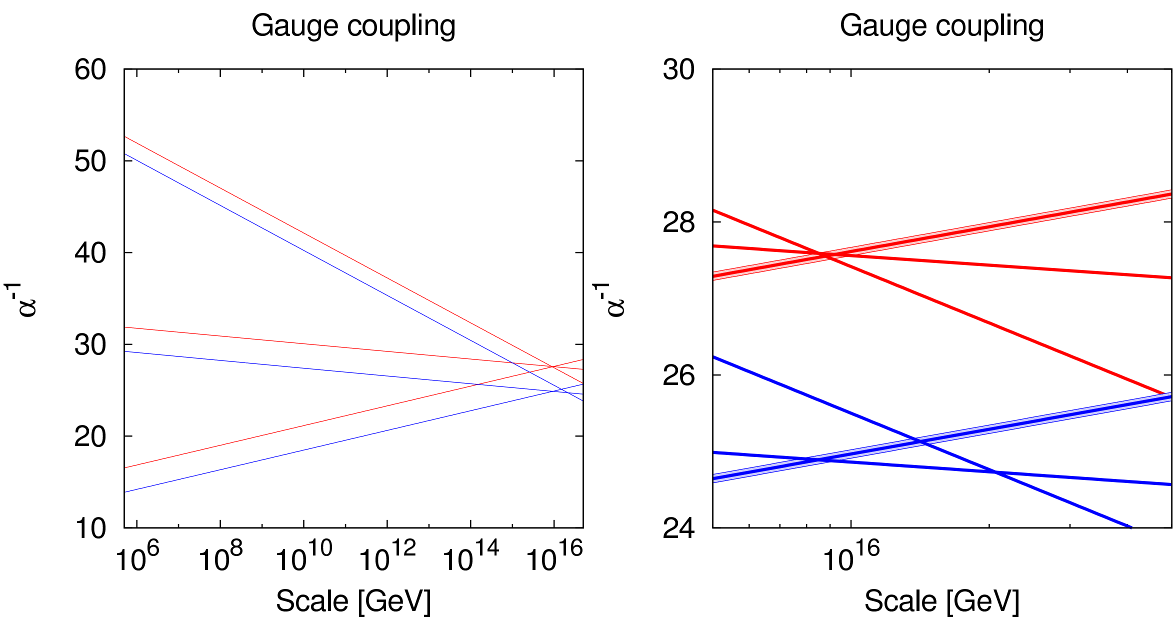


Figure 7: Red lines show the running couplings in the high-scale SUSY($M_S = 10^2$TeV, $M_2 =$ 3TeV, $M_3/M_2 = 9$). Blue lines describe the couplings in the low-scale SUSY($M_S =$ 1TeV, $M_2 = 0.3$TeV, $M_3/M_2 = 3$). In right figure (closed up figure), we include ambiguity of the strong coupling $\alpha_S(m_Z) = 0.1184(7)$.

scenario. We will see whether this dangerous proton decay mode can be consistent with recent experiments in the next section.

In the minimal SUSY $SU(5)$ GUT with low-scale SUSY, it is obvious that there is small mass hierarchy between color-triplet Higgs and the others. In fact, there exists difference about an order of magnitude between Eq. (4.2.8) and Eq. (4.2.9). These figures (Fig. 4, Fig. 5 and Fig. 6) also show us that all of GUT particles can have the same order masses. This also means that threshold corrections at GUT scale can be small in high-scale SUSY. In other words, this means that the gauge coupling unification can be improved without large threshold correction at GUT scale. In Fig. 7, we show that gauge coupling unification in high-scale SUSY scenario can be improved compared with that in low-scale. A horizontal line and a vertical line correspond to a renormalization scale and the inverse squared gauge coupling, α^{-1}, respectively.

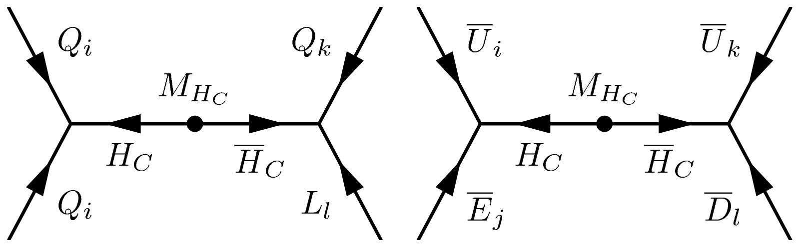


Figure 8: Color-triplet higgs mediated diagram. In the left figure, the LLLL operator are generated. In the right figure, the RRRR operator are generated.

5 Nucleon Decay in Minimal SUSY SU(5) GUT

In GUTs, quarks and leptons are embedded in the same multiplet. For example, in $SU(5)$ GUTs, the right handed up-type quarks, the left-handed quark doublets, and the right-handed electrons are embedded in the anti-symmetric **10** representation fields. The right-handed down quarks and the left-handed lepton doublets are also embedded in the anti-fundamental ($\bar{\mathbf{5}}$) representation fields. Therefore, there are some baryon number (B) violating operators. First, we review how baryon number violating operators are constructed in the SUSY $SU(5)$ GUTs in this section. Second, we show our results for the proton lifetime triggered by dimension-five operators in the high-scale SUSY breaking scenario.

5.1 Construction of the baryon number violating higher dimensional operators

In the minimal SUSY $SU(5)$ GUTs, there are two kinds of fields which couple to both quarks and leptons; the X-type gauge superfields and the color-triplet Higgs multiplets. The X bosons are the $SU(5)$ partners of the standard model gauge fields. The color-triplet Higgs multiplets are the $SU(5)$ partners of two Higgs doublets. The later makes characteristic operators in SUSY GUTs. These give rise to too short lifetime of proton which is inconsistent with the experimental constraints from Super-Kamiokande ($\tau(p \to K^+ + \overline{\nu}) > 5.9 \times 10^{33}$years) [79, 80]. Therefore, these operators must be forbidden or suppressed by some mechanisms in the low-scale SUSY scenario.

5.1.1 Dimension-five operators (Color-triplet Higgs boson exchange)

First, let us consider why these operators give rise to too short partial lifetime of a proton. The color-triplet Higgs multiplets obtain the vector-like mass term $M_{H_C} H_C \overline{H}_C$ after the adjoint Higgs bosons have obtained VEV. The terms including the color-triplet multiplets are given by

$$W_{H_C} = M_{H_C} H_C \overline{H}_C + h^i e^{i\varphi_i} U_i^C E_i^C H_C + \frac{h^i}{2} e^{i\varphi_i} Q_i Q_i H_C + V_{ij}^* f_j Q_i L_j \overline{H}_C + V_{ij}^* f_j U_i^C D_j^C \overline{H}_C. \tag{5.1.1}$$

After we carry out the Gaussian path integral in order to integrate out the color-triplet Higgs multiplets, the superpotential of the dimension-five operators is obtained as:

$$\begin{aligned} W_5 =& -\frac{1}{M_{H_C}} V_{kl}^* f_l h^i e^{i\varphi_i} U_i^C E_i^C Q_k L_l \\ &- \frac{1}{M_{H_C}} \left(V_{kl}^* f_l h^i e^{i\varphi_i} \epsilon^{abc} U_{ia}^C E_i^C U_{kb}^C D_{lc}^C + \frac{h^i}{2} V_{kl}^* f_l e^{i\varphi_i} \epsilon_{abc} (Q_i^b Q_i^c)(Q_k^a L_l) \right). \end{aligned} \tag{5.1.2}$$

When we use mass eigenstates, this is rewritten as:

$$\begin{aligned} W_5 =& -\frac{1}{M_{H_C}} V_{kl}^* f_l h^i e^{i\varphi_i} U_i^C E_i^C Q_k L_l \\ &- \frac{1}{M_{H_C}} \left(V_{kl}^* f_l h^i V_{ij} e^{-i\varphi_k} \epsilon^{abc} U_{ia}^C E_j^C U_{kb}^C D_{lc}^C + \frac{h^i}{2} V_{kl}^* f_l e^{i\varphi_i} \epsilon_{abc} (Q_i^b Q_i^c)(Q_k^a L_l) \right). \end{aligned} \tag{5.1.3}$$

The second and the third terms are related to the nucleon decay. The second term is called “RRRR operator", and the third term is called “LLLL operator". These Wilson coefficients are denoted as follows:

$$C_{LLLL}^{ikl} = -\frac{1}{2M_{H_C}} h^i e^{i\varphi_i} V_{kl}^* f^l, \;\; C_{RRRR}^{ijkl} = -\frac{1}{M_{H_C}} V_{kl}^* f_l h^i V_{ij} e^{-i\varphi_k}. \tag{5.1.4}$$

We obtain the four-fermi interactions for nucleon decay from LLLL operator by intermediating the gaugino or the doublet higgsino as in Fig. 9. Since the neutral gaugino interactions are flavor-diagonal, contributions from these particles can be negligible because of the Yukawa couplings of the 1st and the 2nd generations in the Wilson coefficients of the dimension-five operator. If squarks are not degenerate in mass, the gluino contributions are not negligible due to the strong gauge coupling of the gluino-sfermion-fermion interaction. However, since we carry out the evaluation of proton lifetime by making squarks degenerate in mass M_S, the gluino contributions completely vanish. The higgsino contributions need the Yukawa couplings on the vertices, higgsino-squark-quark and so on. Thus, higgsinos do not generate the dominant contributions. Therefore, the main contribution is obtained from the charged gauginos.

Now, we calculate the triangle diagram amplitude in the left figure of Fig. 9. This contri-

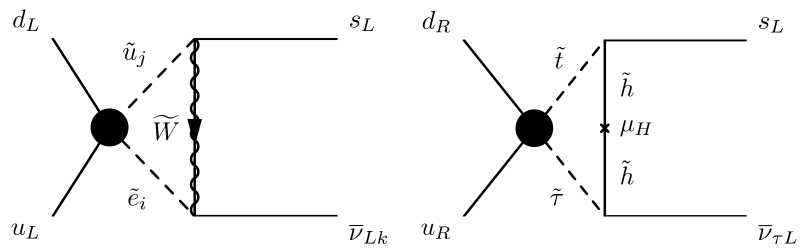


Figure 9: Four-fermi operator from dimension-five operators. In above figures, the black dot denotes the dimension-five operators. In the right figure, charged higgsino-dressed diagram from RRRR operator. In the left figure, charged wino-dressed diagram from LLLL operator.

bution is proportional to the function $f(m_{\tilde{u}}, m_{\tilde{d}}, M_2)$ which is defined as:

$$\begin{aligned}
&(ig_2)^2 \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m_{\tilde{u}}^2} \frac{i}{k^2 - m_{\tilde{d}}^2} \left(\frac{i}{\not{k} - M_{\text{chargino}}} \right)_{11} \\
&\approx \frac{\alpha_2}{4\pi} \frac{M_2}{m_{\tilde{u}}^2 - m_{\tilde{d}}^2} \left(\frac{m_{\tilde{u}}^2}{m_{\tilde{u}}^2 - M_2^2} \ln \frac{m_{\tilde{u}}^2}{M_2^2} - \frac{m_{\tilde{d}}^2}{m_{\tilde{d}}^2 - M_2^2} \ln \frac{m_{\tilde{d}}^2}{M_2^2} \right) \equiv \frac{\alpha_2}{4\pi} f(m_{\tilde{u}}, m_{\tilde{d}}, M_2)
\end{aligned} \tag{5.1.5}$$

where $m_{\tilde{u}}$ and $m_{\tilde{d}}$ are the masses of the sfermions, $\tilde{u}$ and $\tilde{d}$, respectively. In the first line of Eq. (5.1.5), the subscript "11" denotes the (1,1) element of the chargino mass matrix.

$$M_{\text{chargino}} = \begin{pmatrix} M_2 & \sqrt{2} m_W \cos\beta \\ \sqrt{2} m_W \sin\beta & \mu_H \end{pmatrix} \tag{5.1.6}$$

where M_2 and μ_H denote the mass of the charged wino and the charged higgsino, respectively. $\tan\beta$ is the ratio of the VEVs of the two doublet Higgs bosons. The mass term of the charginos is written as:

$$-\frac{1}{2} (\psi^+)^T M_{\text{chargino}} \psi^+, \tag{5.1.7}$$

where the chargino basis is given by $(\psi^+)^T = (\widetilde{W}^+, \tilde{h}^+)$. In general, the integral in Eq. (5.1.5) depends on the inverse matrix of this mass matrix, in other words depends on off-diagonal elements and μ_H. However, since the sparticle masses are much heavier than the electroweak scale in our calculation, the off-diagonal component of this matrix is negligible approximately: that is, the chargino mass matrix is diagonal approximately.

If squarks are degenerate in mass, $M_S^2 \equiv m_{\tilde{u}}^2 = m_{\tilde{d}}^2$, we find that this mass function $f(m_{\tilde{u}}, m_{\tilde{d}}, M_2)$ becomes such as:

$$f(M_S, M_S, M_2) = F(M_2, M_S) \equiv \frac{M_2}{M_S^2 - M_2^2} \left(1 - \frac{M_2^2}{M_S^2 - M_2^2} \ln \frac{M_S^2}{M_2^2} \right). \tag{5.1.8}$$

We choose out two sfermions among four external lines of the dimension-five operators. There are ${}_4C_2 = 6$ pattern diagrams for each LLLL operator included in the superpotential

$$\begin{aligned} W_{\mathrm{LLLL}} &= C^{ikl}_{LLLL}\epsilon_{abc}Q^a_i Q^b_i Q^c_k L_l \\ &= C^{ikl}_{LLLL}\epsilon_{abc}(U^a_i D'^b_i - D'^a_i U^b_i)(U^c_k E_l - D'^c_k N_l). \end{aligned} \tag{5.1.9}$$

Though we obtain the effective operators which are not only associated with the nucleon decay but also are proportional to $(u^a_i u^c_k)(u^b_i \nu_l)$ and so on, these operators vanish due to the antisymmetric tensor ϵ_{abc} included in the Wilson coefficients of the dimension-five operators but these operators are symmetric under exchanging external lines u^a_i, u^b_i. Then, we obtain the four-fermi operators for nucleon decay by combining these results as following form:

$$\begin{aligned} -2\frac{\alpha_2}{4\pi}C^{ikl}_{LLLL}\epsilon_{abc}\Big\{&[f(m_{\tilde{u}_i}, m_{\tilde{d}_i}, M_2) + f(m_{\tilde{\nu}_l}, m_{\tilde{d}_k}, M_2)](d'^a_i u^b_i)(e_l u^c_k) \\ &+ [f(m_{\tilde{e}_l}, m_{\tilde{u}_i}, M_2) + f(m_{\tilde{u}_i}, m_{\tilde{d}_k}, M_2)](d'^a_i \nu_l)(d'^b_i u^c_k) \\ &+ [f(m_{\tilde{u}_k}, m_{\tilde{e}_l}, M_2) + f(m_{\tilde{u}_i}, m_{\tilde{d}_i}, M_2)](u^a_i d'^b_i)(d'^c_k \nu_l) \\ &+ [f(m_{\tilde{d}_i}, m_{\tilde{u}_k}, M_2) + f(m_{\tilde{\nu}_l}, m_{\tilde{d}_i}, M_2)](u^a_i d'^b_k)(u^c_i e_l)\Big\}. \end{aligned} \tag{5.1.10}$$

Now, we also obtain the four-fermi operators from the RRRR operator. We have obtained the RRRR operator which consists of only the right-handed chiral superfields as follows:

$$W_{RRRR} = C^{ijkl}_{RRRR} U^C_i E^C_j U^C_k D^C_l. \tag{5.1.11}$$

Since there is no anti-quark in proton, the four-fermi operators obtained from W^*_{RRRR} contribute proton decay.

$$\begin{aligned} W^*_{RRRR} &= C^{*ijkl}_{RRRR}\epsilon_{abc}(U^{ia}_R E^j_R)(U^{kb}_R D^{lc}_R) \\ &= -\frac{1}{M_{H_C}}\left(V^*_{kl} f^l h^i V_{ij} e^{-i\varphi_k}\right)^* \epsilon_{abc}(U^{ia}_R E^j_R)(U^{kb}_R D^{lc}_R) \end{aligned} \tag{5.1.12}$$

In the similar way as the LLLL operator, the four-fermi operators are induced by mediating sparticles. Of course, the neutralinos and the gluinos do not contribute to nucleon decay as mentioned above. Since there is no left-handed particles in the RRRR operator, sparticles in the triangle diagram are not couple to the charged wino. Thus, the charged higgsino-exchange diagram mainly contributes to nucleon decay. We also obtain the triangle diagram amplitude in the similar way as the case of the LLLL operator. This amplitude is proportional to the function $g(m_{\tilde{u}}, m_{\tilde{d}}, \mu_H)$ which is defined as:

$$\frac{1}{(4\pi)^2} g(m_{\tilde{u}}, m_{\tilde{d}}, \mu_H) \equiv \frac{1}{(4\pi)^2}\frac{\mu_H}{m^2_{\tilde{u}} - m^2_{\tilde{d}}}\left(\frac{m^2_{\tilde{u}}}{m^2_{\tilde{u}} - \mu^2_H}\ln\frac{m^2_{\tilde{u}}}{\mu^2_H} - \frac{m^2_{\tilde{d}}}{m^2_{\tilde{d}} - \mu^2_H}\ln\frac{m^2_{\tilde{d}}}{\mu^2_H}\right). \tag{5.1.13}$$

where μ_H is the mass of the charged higgsino and $m_{\tilde{u}}$ and $m_{\tilde{d}}$ are the masses of the sfermions, $\tilde{u}$ and $\tilde{d}$, respectively. If sfermions are degenerate in mass ($m^2_{\tilde{u}} = m^2_{\tilde{d}} \equiv M^2_S$), this mass dependence is changed as:

$$g(M_S, M_S, \mu_H) = -F(\mu_H, M_S) = -\frac{\mu}{M_S^2 - \mu^2}\left[1 - \frac{\mu^2}{M_S^2 - \mu^2}\ln\frac{M_S^2}{\mu^2}\right]. \tag{5.1.14}$$

This sparticle mass dependence is the same as the mass dependence of the four-fermi operators which are obtained from the LLLL operator. We have the contributions from all of the higgsino-dressed diagrams as follows:

$$\begin{aligned}
&-C^{*ijkl}_{RRRR}\frac{\epsilon_{abc}}{(4\pi)^2} \\
&\times\left\{V^*_{ml}\bar{f}^l\left[\bar{h}^k g(m_{\tilde{u}_k}, m_{\tilde{d}_l}, \mu_H)(u^{ia}_R e^j_R)(u^{mc}_L d'^{kb}_L) + \bar{h}^k g(m_{\tilde{u}_i}, m_{\tilde{d}_l}, \mu_H)(u^{kb}_R e^j_R)(u^{mc}_L d'^{ia}_L)\right]\right. \\
&\left.+\bar{f}^j\left[\bar{h}^k V_{kn} g(m_{\tilde{u}_k}, m_{\tilde{e}_j}, \mu_H)(d^{lc}_R u^{ia}_R)(d^{nb}_L \nu^j_L) + \bar{h}^i V_{in} g(m_{\tilde{u}_i}, m_{\tilde{e}_j}, \mu_H)(d^{lc}_R u^{kb}_R)(d^{na}_L \nu^j_L)\right]\right\}
\end{aligned} \tag{5.1.15}$$

where $\bar{f}^i, \bar{h}^i$ denote Yukawa couplings at the SUSY breaking scale.

Note that the sparticle mass dependence of this function is obtained as

$$F(\mu, M_{\tilde{Q}}) \simeq \begin{cases} \dfrac{\mu}{M^2_{\tilde{Q}}} & M_{\tilde{Q}} \gg \mu \\ \dfrac{1}{2M_{\tilde{Q}}} & M_{\tilde{Q}} = \mu \\ \dfrac{1}{\mu}\ln\dfrac{\mu^2}{M^2_{\tilde{Q}}} & \mu \gg M_{\tilde{Q}}. \end{cases} \tag{5.1.16}$$

This expression shows that the partial proton lifetime via the color-triplet Higgs multiplets is proportional to $M_S^2 M_{H_C}^2$ naively. Thus, it has been pointed out that this partial lifetime would be short in the low-scale SUSY scenario[81, 82].

5.1.2 Dimension-six operators (X-boson exchange)

Nucleon decay is also given rise to by the dimension-six operators. This operator is obtained not only in SUSY GUTs but in non-SUSY GUTs. The dimension-six proton decay operators are generated by exchanging the X-boson. The Kähler potential is given as:

$$\mathcal{K} = \Phi_\alpha\left(e^{-2g_5\mathcal{V}_5}\right)^\alpha{}_\beta\Phi^{*\beta} + \Psi^*_{\alpha\beta}\left(e^{2g_5\mathcal{V}_5}\right)^\alpha{}_\delta\left(e^{2g_5\mathcal{V}_5}\right)^\beta{}_\gamma\Psi^{\gamma\delta} + 2\Sigma^\alpha{}_\beta\left(e^{2g_5\mathcal{V}_5}\right)^\beta{}_\gamma\Sigma^\gamma{}_\delta\left(e^{-2g_5\mathcal{V}_5}\right)^\delta{}_\alpha. \tag{5.1.17}$$

g_5 and $\mathcal{V}_5$ are the gauge coupling constant of $SU(5)$ gauge and the vector superfield including $SU(5)$ gauge bosons, respectively. Σ is the adjoint Higgs which breaks $SU(5)$ gauge symmetry to the standard model gauge symmetry.

After the unified gauge group breaks down to the SM gauge group, we obtain the mass term of the X-boson and the interaction terms with X-boson.

$$\begin{aligned}\mathcal{L} = M_X^2 X^\mu X_\mu^\dagger + \frac{g_5}{\sqrt{2}} \Big[& \epsilon_{rt} \overline{(L^t)^C} \gamma^\mu (X_a^r)_\mu P_R d^a + \epsilon^{ade} \overline{Q^*}_{dr} \gamma^\mu (X_a^r)_\mu P_L (u^C)_e \\ & + \epsilon_{tr} \overline{e^C} \gamma^\mu (X_a^r)_\mu P_L Q^{at} + \text{h.c.} \Big]\end{aligned} \tag{5.1.18}$$

Thus, we obtain the effective Lagrangian by integrating out the X-boson as follows:

$$\begin{aligned}\mathcal{L}_{\text{eff}} = -\frac{g_5^2}{2M_X^2} & \left[\epsilon^{rs} \overline{(d^C)}_a \gamma_\mu P_R L_s^* + \epsilon_{abc} \overline{(u^C)}^b \gamma_\mu P_L Q^{rc} + \epsilon^{rs} \overline{Q^*}_{ra} \gamma_\mu P_L e^C \right] \\ & \times \left[\epsilon_{rt} \overline{(L^t)^C} \gamma^\mu P_R d^a + \epsilon^{ade} \overline{Q^*}_{dr} \gamma^\mu P_L (u^C)_e + \epsilon_{tr} \overline{e^C} \gamma^\mu P_L Q^{at} \right].\end{aligned} \tag{5.1.19}$$

For proton decay, all we have to do is to treat the following operators:

$$\begin{aligned}\mathcal{L}_{\text{eff}}^{\text{p-decay}} = -\frac{g_5^2}{2M_X^2} \epsilon_{abc} \Big[& \overline{(u^C)}_i^b \gamma_\mu P_L u_i^c \overline{(e^C)}_j \gamma^\mu P_R d_j^a + \overline{(u^C)}_i^b \gamma_\mu P_L u_i^c \overline{(e^C)}_j \gamma^\mu P_L d_j^a \\ & - \overline{(u^C)}_i^b \gamma_\mu P_L d_i^c \overline{(e^C)}_j \gamma^\mu P_L u_j^a \Big].\end{aligned} \tag{5.1.20}$$

When we use the mass eigenstate defined as Eq. (3.2.11) and we choose the flavor indices properly, we obtain the effective Lagrangian for the $p \to \pi^0 + e^+$ mode.

$$\begin{aligned}& \mathcal{L}_{\text{eff}}^{p \to \pi^0 + e^+} \\ & = -\frac{g_5^2}{2M_X^2} \epsilon_{abc} e^{i\varphi_1} \left[((\overline{u^C})^b \gamma_\mu P_L u^c)(\overline{e^C} \gamma^\mu P_R d^a) + \left(1 + |V_{ud}|^2\right) ((\overline{u^C})^b \gamma_\mu P_L u^c)(\overline{e^C} \gamma^\mu P_L d^a) \right] \\ & = -\frac{g_5^2}{M_X^2} \epsilon_{abc} e^{i\varphi_1} \left[((\overline{u^C})^a P_R d^b)(\overline{e^C} P_L u^c) + \left(1 + |V_{ud}|^2\right) (\overline{e^C} P_R u^a)((\overline{u^C})^b P_L d^c) \right].\end{aligned} \tag{5.1.21}$$

In the last equality, we use the relations as:

$$\begin{aligned}(\overline{\Psi}^i \gamma^\mu P_L \Psi_j)(\overline{\Psi}^k \gamma_\mu P_L \Psi_l) &= 2(\overline{\Psi}^k P_R \Psi^{iC})(\overline{\Psi^C}_j P_L \Psi_l) \\ (\overline{\Psi}^i \gamma^\mu P_R \Psi_j)(\overline{\Psi}^k \gamma_\mu P_L \Psi_l) &= 2(\overline{\Psi}^i P_R \Psi_l)(\overline{\Psi}^k P_L \Psi_j).\end{aligned} \tag{5.1.22}$$

This expression shows that the proton decay rate by exchanging the X-boson is sufficiently suppressed by the mass of the X-boson.

5.2 Proton lifetime via dimension-five operators

Now we calculate the partial decay rate and the partial lifetime of proton caused by the dimension-five operators numerically. When we calculate these values, we must include the renormalization effects which are renormalization group evolutions and threshold corrections of all couplings at perturbative level which we use. In our calculation for the proton decay rate, we use 1-loop RGEs and tree-level threshold corrections. We only know the low-energy parameters including Yukawa couplings, gauge couplings of the SM, and dimensionful or dimensionless couplings of the Higgs potential. We use the $\overline{\text{DR}}$ scheme for the renormalization since we treat the supersymmetric theories. These renormalization effects are calculated as the following procedure described in Fig. 10.

- We use masses of quarks and leptons at the low-energy scale as input parameters. Since some of these mass parameters are known as $\overline{\text{MS}}$ mass at these mass scale, we can obtain the Yukawa couplings at the electroweak scale by running these masses. Note that we need to transform these masses to $\overline{\text{DR}}$ masses since these masses are determined as $\overline{\text{MS}}$ mass. Since the top quark mass is given as a pole mass, we also transform the top pole mass to the $\overline{\text{DR}}$ mass.

- From the electroweak scale to the GUT scale, these Yukawa couplings and gauge couplings are run by using RGEs. There exist thresholds in the intermediate scales, the gaugino threshold and the SUSY breaking threshold. For each threshold, we change the RGEs and match these coupling appropriately.

- After we construct the Wilson coefficients of the dimension-five operator at the GUT scale, these values evolve in accordance with RGEs for them. Since the dimension-five operators are generated from superpotential, these RGEs are easily obtained due to the non-renormalization theorem [83].

- Under the SUSY breaking scale (the sfermion mass scale), these dimension-five operators are changed into the four-fermi operators. Thus, these Wilson coefficients also have the scale dependence following with RGE evolutions from the SUSY breaking scale to the electroweak scale [84].

- Only QCD corrections for these four-fermi operators must be considered below the electroweak scale. By using these corrections, we obtain the Wilson coefficients of the proton decay operators at 2GeV.

These RGEs are given in Appendix E. Then, in order to obtain the amplitude for the proton decay, we use the hadron matrix elements at 2GeV which are calculated by lattice simulation [85]. The renormalization effects between the electroweak scale and the GUT scale are the so-called short-range renormalization. Below the electroweak scale, the renormalization effects are also called the long-range renormalization.

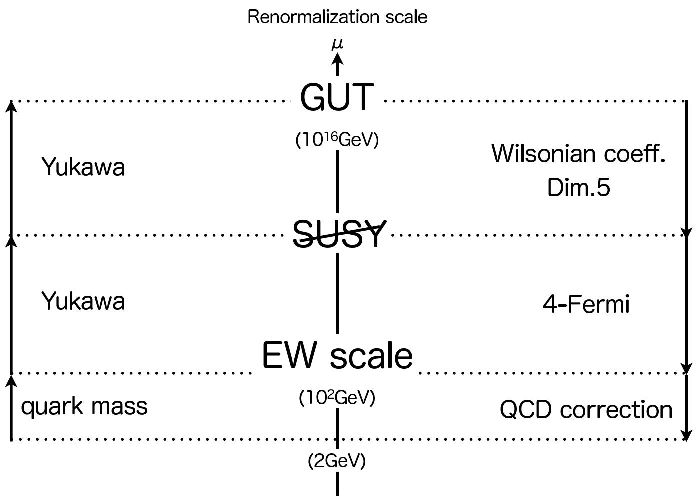


Figure 10: RGEs for calculation of the renormalization factors.

In our calculation, we assume that the squarks, the sleptons, and the heavy Higgs boson are degenerate in mass, M_S. The effective Lagrangian for the decay mode ($p \to K^+ + \bar{\nu}$) is obtained from Eq. (5.1.10),Eq. (5.1.15) at parton level.

$$
\begin{aligned}
\mathcal{L}_{\text{eff}}^{p\to K^+\bar{\nu}} =& -\frac{\alpha_2}{\pi} F(M_2, M_S) \sum_{i,l} C_{LLLL}^{i3l} V_{is} V_{id} \epsilon_{abc} \left[(d_L^b u_L^c)(s_L^a \nu_{lL}) + (s_L^b u_L^c)(d_L^a \nu_{lL}) \right] \\
&+ \frac{F(\mu_H, M_S)}{(4\pi)^2} \bar{h}^3 \bar{f}^3 \epsilon_{abc} \left[C_{RRRR}^{*3311} V_{ts} (d_R^c u_R^a)(s_L^b \nu_{L\tau}) + C_{RRRR}^{*3312} V_{td} (s_R^c u_R^a)(d_L^b \nu_{L\tau}) \right]
\end{aligned}
\tag{5.2.1}
$$

The first line of this Lagrangian is obtained from the LLLL operator and the second line is obtained from the RRRR operator. Since the contribution from the RRRR operator is suppressed by Yukawa couplings, it is enough that the only diagram where the third generation sfermions run in the loop is taken into account as the dominant contribution. In order to obtain the scattering amplitudes for the proton decay ($p \to K^+ + \bar{\nu}$), it is required that the scattering amplitudes at parton level are translated into those at hadron level. The hadron matrix elements of nucleon decay are calculated directly (that is, the three-point functions, which a baryon decay into a pseudo-meson and a lepton, are directly calculated) by lattice simulation [85]*.

$$
\begin{aligned}
\langle K^+|\epsilon_{abc}(u_R^a s_R^b)d_L^c|p\rangle &= -0.054\,\mathrm{GeV}^2,\\
\langle K^+|\epsilon_{abc}(u_L^a s_L^b)d_L^c|p\rangle &= 0.036\,\mathrm{GeV}^2,\\
\langle K^+|\epsilon_{abc}(u_R^a d_R^b)s_L^c|p\rangle &= -0.093\,\mathrm{GeV}^2,\\
\langle K^+|\epsilon_{abc}(u_L^a d_L^b)s_L^c|p\rangle &= 0.111\,\mathrm{GeV}^2.
\end{aligned}
\tag{5.2.2}
$$

The proton decay rate for this mode is given by

$$
\Gamma(p \to K^+\bar{\nu}) = \frac{(m_p^2 - m_K^2)^2}{32\pi m_p^3}\left|\sum_{i=e,\mu\tau}\mathcal{M}(p \to K^+\bar{\nu}_i)\right|^2 . \tag{5.2.3}
$$

where m_p and m_K are the mass of a proton and a charged K meson, respectively, and $\mathcal{M}(p \to K^+\bar{\nu}_i)$ denotes the amplitude for the decay mode: $p \to K^+ + \bar{\nu}_i$. Now let us see the parameter dependence of the partial proton lifetime. When the higgsino is degenerate in sfermion masses, the partial proton lifetime is obtained approximately as:

$$
\tau_p \sim (4.0\times 10^{35}) \times \sin^4 2\beta \left(\frac{0.1}{A_R}\right)^2 \left(\frac{M_S}{10^2\ \mathrm{TeV}}\right)^2 \left(\frac{M_{H_C}}{10^{16}\ \mathrm{GeV}}\right)^2 \ [\mathrm{years}] \tag{5.2.4}
$$

where $\tan\beta$ is the ratio of the VEVs of two doublet Higgs bosons, and M_S and M_{H_C} are the masses of the sfermions and the color-triplet Higgs multiplets, respectively. In addition, A_R denotes the renormalization factor due to the running of the Yukawa coupling and the Wilson coefficient of dimension-five operator and so on. This approximate expression shows that the lifetime is proportional to $M_S^2 M_{H_C}^2$. For $\tan\beta \geq 1$, this value is also proportional to $1/\tan^4\beta$. Thus, in the low-scale SUSY breaking scenario($M_S \sim 1$TeV), this lifetime is around $\tau_p \sim 10^{30}$ years [12]. H. Murayama and A. Pierce pointed out that this partial lifetime was no longer consistent with the experimental bound even if the squarks except the third generation had the superheavy mass [13]*. That is, lifetime of this decay mode is inconsistent with the experimental result ($\tau_p \gtrsim 5.9 \times 10^{33}$ years). On the other hand, this naive analysis tells us that proton lifetime in the high-scale SUSY breaking scenario can be longer than that in the low-scale SUSY breaking scenario.

Now, we show the numerical calculations for the $p \to K^+ + \bar{\nu}$ mode in the high-scale SUSY breaking scenario. The results of the numerical calculations are described in Fig. 11 and Fig. 12.

*Indirectly, the hadron matrix elements in lattice simulation is calculated by using the chiral Lagrangian with baryon [86, 87]. In these lattice simulations, parameters of the chiral Lagrangian are determined by calculating the two-point functions.

*In this decoupling scenario, the stop has mass of several TeV. Thus, the contribution from RRRR operator gives rise to a short lifetime.

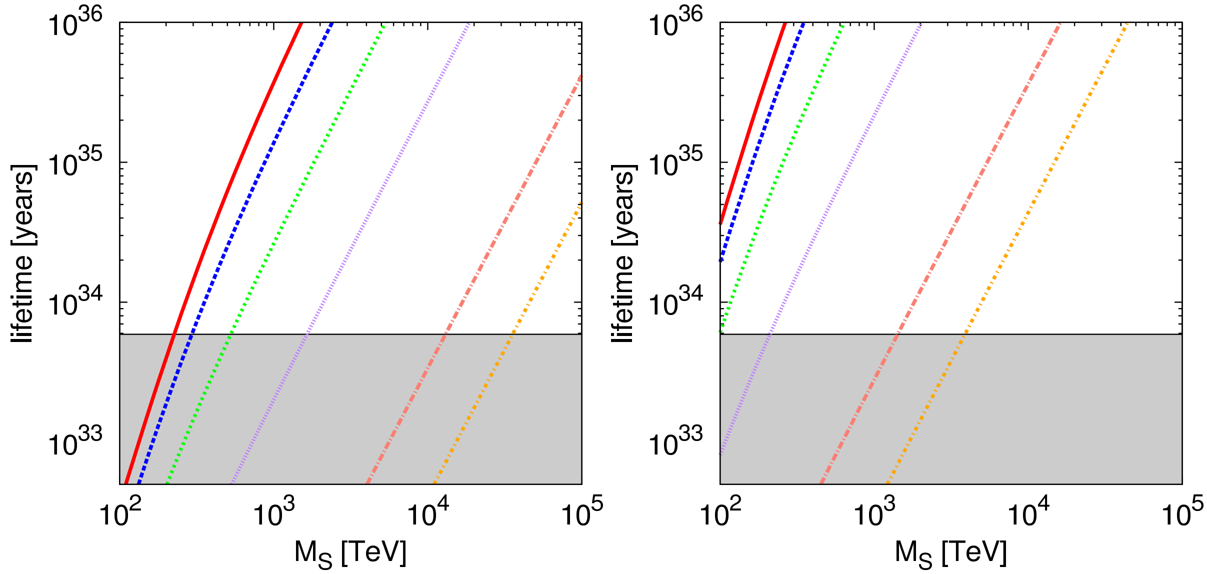


Figure 11: These figures show that proton lifetime can be evaded from the experimental limit of Super-Kamiokande ($\tau > 5.9 \times 10^{33}$years [79, 80]) in the case ($\mu_H = M_S, M_2 = 3$TeV). The gray region is excluded from this limit. From left-top to right-bottom, these lines correspond to $\tan\beta = 2, 3, 5, 10, 30,$ and 50. The mass of the color-triplet Higgs multiplets is set to be 10^{15}GeV(10^{16}GeV) in this left (right) figure.

First, in Fig. 11, we have evaluated the proton lifetime in the case $M_S = \mu_H$, which corresponds to the high-scale SUSY scenario [68, 66], for various $\tan\beta$. In this case, the additional phases which are included in the Yukawa matrices do not affect the result since the higgsino contribution is only dominant. In this calculation, the mass of the color-triplet Higgs bosons is set to be 10^{15} GeV or 10^{16} GeV based on the argument of the constraints on the GUT scale masses from threshold corrections. In Fig. 11, various lines correspond to $\tan\beta = 2, 3, 5, 10, 30,$ and 50 from left-top to right-bottom.

These figures in Fig. 11 show that the proton lifetime of the $p \to K^+ + \bar{\nu}$ mode is consistent with the recent experimental lower bound ($\tau_{p\to K^+ + \bar{\nu}} > 5.9 \times 10^{33}$ years) in the high-scale SUSY breaking scenario even if the SUSY breaking scale is around 100TeV. In particular, in these regions, the observed mass of the SM Higgs boson can be realized. Therefore, the minimal supersymmetric $SU(5)$ grand unified theory is not excluded by this proton lifetime constraint in the high-scale SUSY scenario. Additionally, our results show that the dimension-five proton decay may be detectable by the future experiment (Hyper-Kamiokande) in a broad soft parameter region where the mass of the observed Higgs boson can be realized. In other words, to discover this decay mode ($p \to K^+ + \bar{\nu}$) in future experiments indirectly ensures that the high-scale SUSY breaking scenario may be realized in this world.

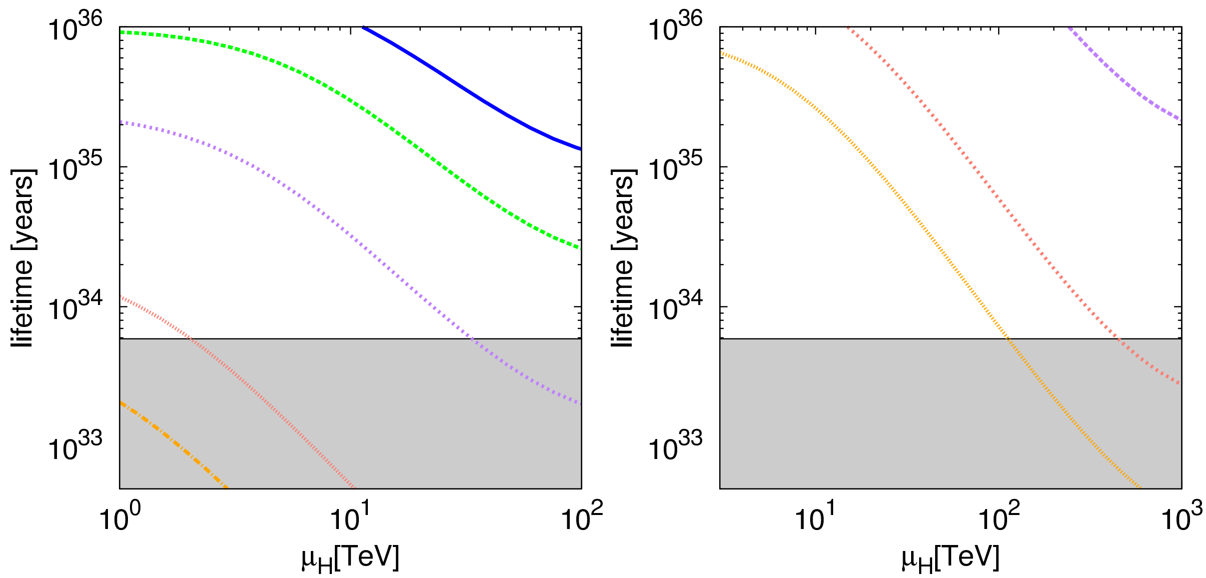


Figure 12: These figures show that proton lifetime can be evaded from the experimental limit of Super-Kamiokande ($\tau > 5.9 \times 10^{33}$years [79, 80]) in the case ($\mu_H < M_S, M_{H_C} = 10^{16}$GeV). The gray region is excluded from this limit. From right-top to left-bottom, these lines correspond to $\tan\beta = 3, 5, 10, 30,$ and $50(10, 30,$ and $50)$ in this right (left) figure. M_2 is set to be 300GeV(3TeV) and M_S is set to be 10^2TeV (10^3TeV) in left (right) figure.

Next, Fig. 12 shows that the dimension-five proton decay is also consistent with experimental result in the case $\mu_H < M_S$, which corresponds to the Split SUSY model [10, 11, 70]. In this scenario, we take the additional phases φ_i so that they yield the maximal amplitudes for this decay mode. In these figures, we set the mass of the color-triplet Higgs multiplets to be 10^{16}GeV. M_2 is set to be 300GeV (3TeV) and M_S is set to be 10^2TeV (10^3TeV) in the left (right) figure. Since the sparticle mass dependence of the amplitude for this proton decay mode is roughly proportional to μ/M_S^2 where μ is mass of charginos, if the mass of chargino is lighter than that of sfermions, the amplitude for this decay mode becomes much smaller. That is, the large mass hierarchy between the bosonic and the fermionic sparticles leads to long proton lifetime.

Now, we show the parameter regions where the observed higgs mass can be realized, the minimal SUSY $SU(5)$ GUT can be excluded by the proton lifetime, and the mass of the color-triplet Higgs is above the Planck scale as the M_S–$\tan\beta$ plot in the minimal SUSY $SU(5)$ GUT with the high-scale SUSY breaking scenario in Fig. 13. In this figure, we set all of soft SUSY breaking parameters as follows: The higgsino mass is set to be degenerate in squark masses ($\mu_H = M_S$). The masses of winos and gluinos are set to be 3 TeV and 10 TeV, respectively. A-terms are set to be 0 since these are generated with the loop suppression through AMSB. The proton decay rate via the dimension-five operators are estimated with the color-triplet

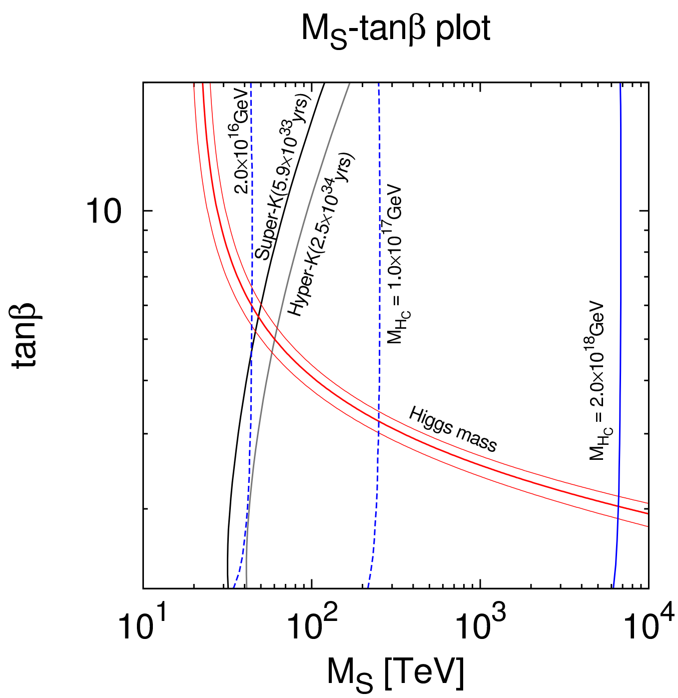


Figure 13: Higgs mass and proton decay in the high-scale SUSY breaking. In this plot, we set wino mass $M_2 = 3$TeV, gluino mass $M_3 = 10$TeV, and A-terms $A = 0$. We use the Higgs mass, the strong coupling, and top pole mass as input parameters. $m_h = 125.9 \pm 0.4$GeV, $\alpha_S(m_Z) = 0.1184(7)$ and $m_t^{\text{pole}} = 173.07 \pm 0.52 \pm 0.72$GeV. Error bar of the region where the SM Higgs mass is realized indicates the input errors of $m_h, \alpha_S(m_Z)$, and m_t^{pole}.

Higgs mass obtained by using the 2-loop RGEs and the 1-loop threshold corrections.

A black solid line shows the current experimental limit by Super-Kamiokande ($\tau(p \to K^+ + \overline{\nu}) > 5.9 \times 10^{33}$years) and a gray solid line displays the sensitivity of the future experiment of Hyper-Kamiokande ($\tau(p \to K^+ + \overline{\nu}) > 2.5 \times 10^{34}$years). Solid and dotted blue lines indicate the mass of the color-triplet Higgs multiplets. Solid line displays the boundary where the mass of color-triplet Higgs boson is equal to the Planck scale $\sim 2 \times 10^{18}$GeV. The dotted blue lines display the boundaries where the mass of the color-triplet Higgs bosons is equal to the GUT scale ($\sim 2 \times 10^{16}$GeV) and 10^{17}GeV. That is, unification of the couplings of the SM gauge interaction can be improved in the region enclosed by dotted lines.

Red lines display the region where the mass of the observed Higgs boson can be realized in the high-scale SUSY breaking scenario. We use $m_h = 125.9 \pm 0.4$GeV, $\alpha_S(m_Z) = 0.1184(7)$ and $m_t^{\text{pole}} = 173.07 \pm 0.52 \pm 0.72$GeV as input parameters. The center line of these lines displays that the mass of the observed Higgs boson can be realized in the high-scale SUSY breaking scenario. We draw this line by using the central value of the Higgs boson mass, top Yukawa coupling, and the strong gauge coupling constant. Ambiguities of these values

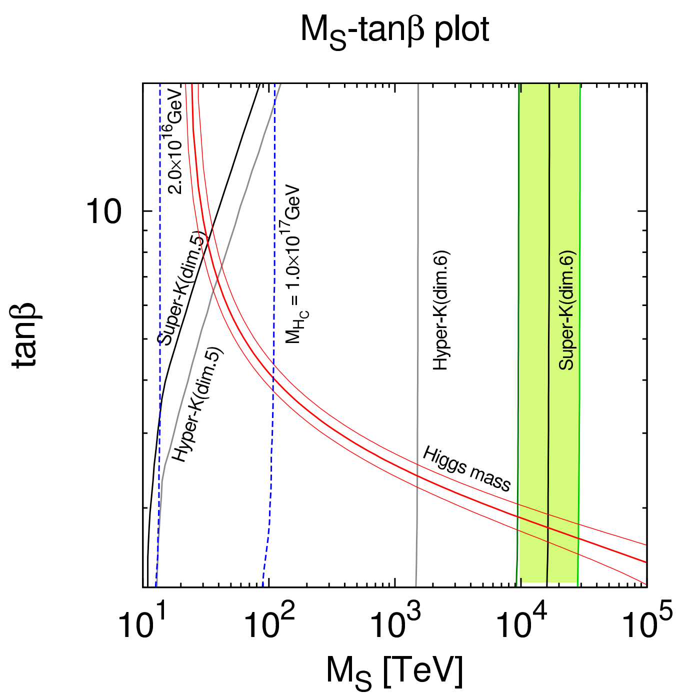


Figure 14: Higgs mass and proton decay in the high-scale SUSY breaking with $M_S = m_{3/2}$. In this scenario, gaugino masses are proportional to $M_S/16\pi^2$. In this plot, we set the ratio of gaugino masses $M_3/M_2 = 3$, and A-terms $A = 0$. We use the Higgs mass, the strong coupling, and top pole mass as input parameters. $m_h = 125.9 \pm 0.4$GeV, $\alpha_S(m_Z) = 0.1184(7)$, and $m_t^{\text{pole}} = 173.07 \pm 0.52 \pm 0.72$GeV. Error bar of the region where the SM Higgs mass is realized indicates the input errors of $m_h, \alpha_S(m_Z)$, and m_t^{pole}.

are shown as error bars. When we draw these lines, we use 2-loop RGEs for the quartic coupling of Higgs boson, the gauge couplings, and Yukawa couplings. Also, we determined $\tan\beta$ at the SUSY breaking scale M_S by using the 1-loop threshold correction for the quartic coupling. Note however that we must determine the Higgs mass by using the effective potential below ~ 10TeV.

Finally, in Fig. 14, we show the case $M_S = m_{3/2}$. In this case, $M_{\text{GUT}} = M_X^{2/3} M_\Sigma^{1/3}$ is changed with M_S since $M_{\text{GUT}} \propto (M_3 M_2)^{-1/9} \propto (m_{3/2})^{-2/9}$. In particular, this case corresponds to such as the pure gravity mediation scenario [66]. If we associate M_X with M_{GUT}, we set a limit on the SUSY breaking scale from above through another decay mode ($p \to \pi^0 + e^+$). The masses of the X-boson and the adjoint Higgs are obtained as:

$$M_X = 5\sqrt{2} g_5 V, \quad M_\Sigma = \frac{5}{2} f V \tag{5.2.5}$$

where g_5 is the gauge coupling of $SU(5)$ and f is the self-coupling constant of the adjoint Higgs, which are defined in Eq. (3.2.7). V is the VEV of the adjoint Higgs. Thus, we have

$$M_{\rm GUT} = 5 g_5^{2/3} f^{1/3} V. \tag{5.2.6}$$

If we assume $f = 2^{3/2} g_5$, then we have $M_{\rm GUT} = M_X$. For this decay mode, the hadron matrix elements are given by lattice simulation [85]:

$$\begin{aligned} \langle \pi^0 | \epsilon_{abc} (u_R^a d_R^b) u_L^c | p \rangle &= -0.103\ {\rm GeV}^2, \\ \langle \pi^0 | \epsilon_{abc} (u_L^a d_L^b) u_L^c | p \rangle &= 0.133\ {\rm GeV}^2. \end{aligned} \tag{5.2.7}$$

Then, the proton decay rate for this mode is given by

$$\Gamma(p \to \pi^0 e^+) = \frac{(m_p^2 - m_{\pi^0}^2)^2}{32\pi m_p^3} \left| \mathcal{M}(p \to \pi^0 e^+) \right|^2, \tag{5.2.8}$$

where m_{π^0} is the mass of the neutral pion and $\mathcal{M}(p \to \pi^0 e^+)$ is the amplitude for the decay $p \to \pi^0 + e^+$.

The lines in the right side of Fig. 14 correspond to the recent lower bound and future sensitivity for the X-boson exchange decay mode($p \to \pi^0 + e^+$). These recent and future lower bounds are 1.4×10^{34} years and 1.0×10^{35} years, respectively [88].

In Fig. 14, we draw the lines corresponding to the case of $f = 2g_5$, $2^{3/2} g_5$, and $4g_5$ for the Super-Kamiokande constraint. A black line in the right side of this figure corresponds to the case $M_{\rm GUT} = M_X$, ($f = 2^{3/2} g_5$). The light green line (the right side boundary) and the dark green line (the left side boundary) are corresponds to the case $f = 2g_5$ and $f = 4g_5$, respectively. This figure shows that the SUSY breaking scale is bounded from above through the decay rate for the mode $p \to \pi^0 + e^+$. However, this constraint depends on the self-coupling of the adjoint Higgs. If this coupling is much small, the SUSY braking scale is not constrained strictly.

The gray line in the right side of Fig. 14 describes the future sensitivity of the Hyper-Kamiokande. When we draw this line, we assume the case $M_{\rm GUT} = M_X$. This figure also shows that the future experiment can investigate the broad region of the SUSY breaking scale since $M_{\rm GUT}$ does not get lower drastically when the gaugino mass becomes larger. Thus, the Hyper-Kamiokande experiment is important in order to bound the SUSY breaking scale in this scenario. Note that this constraint becomes more severe if the gaugino mass-ratio M_3/M_2 is larger than 3. Note also that the gluino direct search in the collider experiment exclude the region $M_S < \mathcal{O}(10{\rm TeV})$ in this figure since the mass of the gluino is related to the squark masses.

6 Conclusion and Discussion

In this thesis, we have revisited the minimal SUSY $SU(5)$ GUT. It is believed that the minimal SUSY $SU(5)$ GUT is excluded due to the inconsistent prediction for the proton decay. First of all, we have evaluated the mass spectrum of the GUT particles, the color-triplet Higgs multiplets, the X-bosons, and the adjoint Higgs bosons in the high-scale SUSY scenario. Then, we also have evaluated the proton lifetime via the color-triplet Higgs boson in the high-scale SUSY breaking scenario.

Grand unification in the high-scale SUSY

We have evaluated the mass spectrum of the GUT particles in the high-scale SUSY scenario. It is revealed that the mass of the color-triplet Higgs multiplets can have the same order of the other GUT particles ($\sim 10^{16}$GeV), which has the mass around $10^{14} \sim 10^{15}$GeV in the low-scale SUSY breaking scenario. This implies that the threshold corrections for gauge couplings at the GUT scale are small, and also the unification of gauge couplings is improved in the high-scale SUSY scenario. We also have found that $M_{\rm GUT} = M_X^{2/3} M_\Sigma^{1/3}$ becomes slightly small in this scenario. Although this does not spoil neither the unification of the GUT-particle mass scale nor improvement of the gauge coupling unification, $M_{\rm GUT}$ affects the proton decay via X-bosons. For example, if the X-boson mass is set to be $M_X = 0.8 \times 10^{16}$GeV, we have the lifetime of this decay mode as $\tau(p \to \pi^0 + e^+) \sim 5 \times 10^{34}$ [years]. For this decay mode, the current observation lower limit is obtained as $\tau_{\rm exp}(p \to \pi^0 + e^+) > 1.4 \times 10^{34}$[years] by Super-Kamiokande experiment. In future experiment, 1.0×10^{35} years partial lifetime can be achieved, which corresponds to eight years running of Hyper-Kamiokande [88]. Thus, this decay mode may be caught by future experiment if the high-scale SUSY breaking scenario is realized in our world.

Proton decay in the minimal SUSY $SU(5)$ GUT with the high-scale SUSY scenario

Next, we also have evaluated the proton decay mode ($p \to K^+ + \overline{\nu}$) caused by the dimension-five operators via the color-triplet Higgs multiplets in the high-scale SUSY breaking scenario. Though the theoretical prediction for this partial lifetime is much smaller than the experimental bounds ($\tau_p > 5.9 \times 10^{33}$ years) in the low-scale SUSY scenario, the prediction in the high-scale SUSY can be evaded from this bound. This is because not only SUSY breaking scale is sufficiently high but the color-triplet Higgs multiplets obtain a heavy mass in the high-scale SUSY breaking scenario. In particular, we also have revealed that the proton lifetime is consistent in a broad region where the observed Higgs boson mass is realized. These results indicate that this decay mode may be discovered in the future experiment, Hyper-Kamiokande. In point of view of the model buildings, the additional symmetries which suppress or prohibit the vector-like mass term $\mu_H H_C \overline{H}_C$ are not needed in the high-scale SUSY breaking scenario.

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A Conventions and Notations

In this appendix, we will give notations, formulae for gamma matrices, and physical constants used in this thesis.

A.1 Conventions and formulae

Conventionally, we use the natural unit.

$$c = \hbar = k_B = 1 \tag{A.1.1}$$

where c is the speed of light ($c = 299792458\mathrm{m/s}$), $\hbar = h/2\pi$ is the (reduced) Planck constant ($\hbar = 6.58211928(15) \times 10^{-22}\mathrm{MeVs}$), and k_B is the Boltzmann constant ($k_B = 8.6173324(78) \times 10^{-5}\mathrm{eVK}^{-1}$). Thus, all of dimensionful constants are evaluated in units of energy.

Conventions

We use the metric tensor in flat spacetime,

$$\eta^{\mu\nu} = \mathrm{diag}(+,-,-,-). \tag{A.1.2}$$

We also use the definition of the totally anti-symmetric tensor as $\epsilon_{0123} = 1$.

The Grassmann derivative and the Grassmann integral measure are defined as:

$$\frac{\partial}{\partial\theta^\alpha}\theta^\beta = \delta_\alpha^\beta, \quad \frac{\partial}{\partial\theta^\dagger_{\dot\alpha}}\theta^\dagger_{\dot\beta} = \delta^{\dot\alpha}_{\dot\beta}, \tag{A.1.3}$$

$$d^2\theta = -\frac{1}{4}d\theta^\alpha d\theta^\beta \epsilon_{\alpha\beta}, \quad d^2\theta^\dagger = -\frac{1}{4}d\theta^\dagger_{\dot\alpha} d\theta^\dagger_{\dot\beta}\epsilon^{\dot\alpha\dot\beta}. \tag{A.1.4}$$

Then, the integral of the products of the Grassmannian coordinate $\theta\theta \equiv \theta^\alpha\theta_\alpha$, $\theta^{\dagger\dot\alpha}\theta^\dagger_{\dot\alpha}$ are obtained as the following form;

$$\int d^2\theta(\theta\theta) = 1, \quad \int d^2\theta^\dagger(\theta^\dagger\theta^\dagger) = 1. \tag{A.1.5}$$

Formulae

For D dimensional γ matrices, we will give the definition of the anti-commutating relation of the gamma matrices and γ^5.

$$\begin{aligned} \{\gamma^\mu, \gamma^\nu\} &= 2\eta^{\mu\nu}, \\ \gamma^5 &= i\gamma^0\gamma^1\gamma^2\gamma^3. \end{aligned} \tag{A.1.6}$$

By using these definitions, we obtain formulae for the gamma matrices as;

$$\begin{aligned}(\gamma^5)^2 = 1, \quad \left\{\gamma^5, \gamma^\nu\right\} &= 0,\\ \gamma^\mu\gamma_\mu &= D,\\ \gamma^\mu\gamma_\nu\gamma_\mu &= -(D-2)\gamma_\nu.\end{aligned} \tag{A.1.7}$$

In the case we consider the supersymmetric theories (that is, we set $D = 4$ below arguments), it is convenient to use two-component (Weyl) spinors. Feynman rules for two-component spinors differ from those of four-component spinors. Thus we need to translate the formulae of the gamma matrices into those in terms of two-component spinors. By using the Chiral (Weyl) representation,

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix} \tag{A.1.8}$$

where $\sigma, \bar{\sigma}$ are the covariant Pauli matrices which are defined as;

$$\sigma^\mu \equiv (1, \sigma^1, \sigma^2, \sigma^3), \quad \overline{\sigma}^\mu \equiv (1, -\sigma^1, -\sigma^2, -\sigma^3). \tag{A.1.9}$$

$\sigma^i \;\; (i = 1, 2, 3)$ are the Pauli matrices defined as

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{A.1.10}$$

Furthermore, we obtain γ_5 in terms of two-component spinors;

$$\gamma^5 = \begin{pmatrix} -\mathbf{1} & 0 \\ 0 & \mathbf{1} \end{pmatrix}. \tag{A.1.11}$$

Formulae for gamma matrices are reduced to formulae for Pauli matrices.

$$\begin{aligned}(\sigma^\mu)_{\alpha\dot{\alpha}}(\overline{\sigma}_\mu)^{\dot{\beta}\beta} &= -2\delta_\alpha^\beta\delta_{\dot{\alpha}}^{\dot{\beta}}\\ (\sigma^\mu)_{\alpha\dot{\alpha}}(\sigma_\mu)_{\beta\dot{\beta}} &= -2\epsilon_{\alpha\beta}\epsilon_{\dot{\beta}\dot{\beta}}\\ (\overline{\sigma}^\mu)^{\dot{\alpha}\alpha}(\overline{\sigma}_\mu)^{\dot{\beta}\beta} &= -2\epsilon^{\alpha\beta}\epsilon^{\dot{\beta}\dot{\beta}}\end{aligned} \tag{A.1.12}$$

$$\begin{aligned}[\sigma^\mu\overline{\sigma}^\nu + \sigma^\nu\overline{\sigma}^\mu]_\alpha^\beta &= 2\eta^{\mu\nu}\delta_\alpha^\beta\\ [\overline{\sigma}^\mu\sigma^\nu + \overline{\sigma}^\nu\sigma^\mu]_{\dot{\alpha}}^{\dot{\beta}} &= 2\eta^{\mu\nu}\delta_{\dot{\alpha}}^{\dot{\beta}}\end{aligned} \tag{A.1.13}$$

A.2 loop integrals

When we evaluate quantum corrections, we need to carry out the calculations of loop momentum integrals. In particular, we use the dimensional regularization for the non-SUSY case or the dimensional reduction for the SUSY case when we regularize the loop integrals. Now, we give the expression of the loop integrals in D-dimensional momentum space:

$$\begin{aligned} \int \frac{d^D l}{(2\pi)^D} \frac{1}{[l^2-\Delta^2]^n} &= \frac{(-1)^n i \Delta^{D-2n}}{(4\pi)^{D/2}} \frac{\Gamma(n-D/2)}{\Gamma(n)}, \\ \int \frac{d^D l}{(2\pi)^D} \frac{l^2}{[l^2-\Delta^2]^n} &= \frac{(-1)^{n-1} i \Delta^{D-2n+2}}{(4\pi)^{D/2}} \frac{\Gamma(n-D/2)}{\Gamma(n)}. \end{aligned} \tag{A.2.1}$$

where Γ is the gamma function defined as;

$$\Gamma(x) \equiv \int_0^\infty dt t^{x-1} e^{-t}. \tag{A.2.2}$$

The expansion of Γ function near origin is given by:

$$\Gamma(x) \sim \frac{1}{x} - \gamma + \mathcal{O}(x) \tag{A.2.3}$$

where $\gamma = 0.5772\ldots$ is the Euler constant.

A.3 Physical constants

We use some physical constants as input parameters when we evaluate the lifetime of proton; the gauge coupling constants for the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, the masses of quarks, leptons, gauge bosons, and the Higgs boson, and the elements of the CKM matrix. In addition, when we estimate the decay rate of proton for each mode, we need to use the masses of hadrons. These values are also summarized in this appendix.

Electroweak parameters

When we evaluate various values by numerical calculations, we use precise values of electroweak parameters as input parameters. In Table 6, we summarize these physical constants. Some of them is evaluated at m_Z renormalization scale in $\overline{\mathrm{MS}}$ scheme.

By using these parameters, we obtain the gauge coupling constants and the VEV of the Higgs boson, and quartic coupling in the Higgs potential. The fine structure constant is related to the electromagnetic charge and α_S is also related to the strong gauge coupling constant: $\alpha_{\mathrm{EM}} = e^2/4\pi$ and $\alpha_S = g_S^2/4\pi$. The electromagnetic coupling is related to the weak gauge coupling g and the hypercharge coupling g';

$$e = \frac{gg'}{\sqrt{g^2+g'^2}} = g\sin\theta_W \tag{A.3.1}$$

Table 6: Electroweak parameters

	name	value
$\frac{1}{\alpha_{\rm EM}(m_Z)}$	Fine structure constant	127.9 ± 0.2
$\sin^2\theta_W(m_Z)$	Weinberg angle	0.2326 ± 0.0008
$\alpha_S(m_Z)$	Strong coupling constant	0.118 ± 0.007
m_Z	Z boson mass	91.1876 ± 0.0021 GeV
m_W	W boson mass	80.385 ± 0.015 GeV
m_h	Higgs boson mass	125.9 ± 0.4GeV

This relation means that we obtain the weak gauge coupling is obtained from the fine structure constant and the Weinberg angle. The masses of the weak gauge bosons are related to the gauge couplings and the VEV of the Higgs boson v:

$$m_W = \frac{1}{2}gv, \;\; m_Z = \frac{1}{2}\sqrt{g^2 + g'^2}v = \frac{m_W}{\cos\theta_W}. \tag{A.3.2}$$

Thus, the VEV of the Higgs boson is given by the weak gauge coupling and the mass of the massive gauge bosons. The scalar potential for the standard model Higgs boson is given by;

$$V(H) = -\mu^2 H^\dagger H + \frac{\lambda}{2}(H^\dagger H)^2. \tag{A.3.3}$$

This potential gives the mass of the physical Higgs boson m_h after expanding the Higgs field around the VEV.

$$m_h^2 = \lambda v^2. \tag{A.3.4}$$

This relation tells us that the quartic coupling of the Higgs potential is obtained from the VEV and the mass of the Higgs boson. It is possible to treat this quartic coupling as known parameter since the Higgs boson is discovered and the mass of the Higgs boson is established at the collider experiments.

We use not only these parameters in Table 6 but also the Cabibbo-Kobayashi-Maskawa (CKM) matrix [26, 27] as input parameters. The mass and weak eigenstate for the left-handed down-type quarks are associated with each other by the CKM matrix.

$$U_{\rm CKM} = \begin{pmatrix} U_{ud} & U_{us} & U_{ub} \\ U_{cd} & U_{cs} & U_{cb} \\ U_{td} & U_{ts} & U_{tb} \end{pmatrix}. \tag{A.3.5}$$

These elements are estimated by some experiments, and the average of them gives the magnitude of CKM elements as follows:

$$U_{\rm CKM} = \begin{pmatrix} 0.97425 \pm 0.00022 & 0.2252 \pm 0.0009 & (4.15 \pm 0.49) \times 10^{-3} \\ 0.230 \pm 0.011 & 1.006 \pm 0.023 & (40.9 \pm 1.1) \times 10^{-3} \\ (8.4 \pm 0.6) \times 10^{-3} & (42.9 \pm 2.6) \times 10^{-3} & 0.89 \pm 0.07 \end{pmatrix}. \tag{A.3.6}$$

This matrix is parametrized by three real parameters "CKM angle" and one phase factor "KM phase"*.

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{A.3.7}$$

where $c_{ij} = \cos\theta_{ij}, s_{ij} = \sin\theta_{ij}$. In our calculation, we used the Wolfenstein parametrization [89].

$$s_{12} \equiv \lambda,\ \ s_{23} \equiv A\lambda^2,\ \ s_{13}e^{i\delta} \equiv \frac{A\lambda^3(\bar{\rho} + i\bar{\eta})\sqrt{1 - A^2\lambda^4}}{\sqrt{1-\lambda^2}[1 - A^2\lambda^4(\bar{\rho} + i\bar{\eta})]}. \tag{A.3.8}$$

These values have been measured by various experiments. The averaged value is [24]

$$\begin{aligned} \lambda = 0.22535 \pm 0.00065,\ \ A = 0.811^{+0.022}_{-0.012}, \\ \bar{\rho} = 0.131^{+0.026}_{-0.013},\ \ \bar{\eta} = 0.345^{+0.013}_{-0.014}. \end{aligned} \tag{A.3.9}$$

Mass parameters

We also use the masses of quarks and leptons as input parameters for Yukawa couplings. For the light quarks (up, down, and strange quarks), masses are estimated in the $\overline{\rm MS}$ scheme which is mass-independent renormalization scheme at $\mu = 2$GeV. For the heavy quark (charm and bottom quarks; except the top quark), masses are estimated in the $\overline{\rm MS}$ scheme at each mass scale. The mass of the top quark is obtained as a pole mass. The mass of the charged leptons (electron, muon, and tau lepton) are estimated by using various ways. The mass of an electron are determined by measuring the ratio m_e/m_A where m_A denotes the mass of a nucleus. The muon's mass is obtained from the muon-electron mass ratio measured of Zeeman transition frequencies in muonium which is a bound state composed of an anti-muon and an electron. The mass of a tau lepton is established by measuring $e^+e^- \to \tau^+\tau^-$ and tau lepton decay. The masses of all of the standard model fermons are summarized in Table 7.

*There are 9 degrees of freedom for a general unitary matrix U because of $UU^\dagger = 1$. Three of them are rotating angles which are real parameters and the remaining 6 parameters are phases. By using phase rotation for 6 quarks, 5 phases are removed since one of six phases rotation corresponds to a flavor-blind phase rotation. This means that the PMNS matrix which is the unitary matrix relating the flavor eigenstate (which is equivalent to the weak eigenstate of charged leptons) to the mass eigenstate in the lepton sector has 3 real angles and 3 phases since neutrinos are neutral fermions which must have no phase rotation

Table 7: The masses of quarks and leptons (μ denotes renormalization scale)

	name	μ	value
m_u	up quark	2 GeV	$2.3^{+0.7}_{-0.5}$ MeV
m_d	down quark	2 GeV	$4.8^{+0.7}_{-0.3}$ MeV
m_e	electron	/	$0.510998928 \pm 0.000000011$ MeV
m_c	charm quark	m_c	1.275 ± 0.025 GeV
m_s	strange quark	2 GeV	95.0 ± 5 MeV
m_μ	muon	/	$105.6583715 \pm 0.0000035$ MeV
m_t	top quark	Pole mass	$173.07 \pm 0.6 \pm 0.8$ GeV
m_b	bottom quark	m_b	4.18 ± 0.03 GeV
m_τ	tau	/	1.776 ± 0.16 GeV

When we evaluate proton decay rates for each mode, we need to use hadron masses which are described in Table 8. Since we use the mass of proton, neutron, pions, and charged K meson, we summarize only these values in Table 8.

Table 8: The masses of Hadrons

	name	value
m_p	proton	938.27 MeV
m_n	neutron	939.57 MeV
m_{π^0}	neutral pion	134.98 MeV
$m_{\pi^\pm}$	charged pion	139.57 MeV
m_K	charged K meson	493.677 ± 0.016 MeV

For the hadrons consisted of only up quarks and down quarks (proton, neutron, and pions), their masses are measured much precisely though we do not write down these expressions explicitly.

B Properties of SUSY

In this appendix, we will give some properties of supersymmetric theories. In particular, the non-renormalization theorem for superpotentials and gauge kinetic terms is impotent for our analysis since we use the dimension-five operator which is included in a superpotential. In the $\mathcal{N}=1$ supersymmetric theories, there is no well-established renormalization scheme though we use dimensional reduction as the renormalization scheme in the SUSY theories.

B.1 SUSY algebra

In general, the algebra is schematically constructed as follows:

$$\begin{aligned} \{Q, Q'\} &= X \\ [X, X'] &= X'' \\ [Q, X'] &= Q'' \end{aligned} \tag{B.1.1}$$

where $Q, Q', \ldots$ are the fermionic generators and $X, X' \ldots$ are the bosonic generators. As for the bosonic generators, the spin-0, -1, and -2 generators are allowed due to the Coleman-Mandula theorem. These generators are decomposed into the irreducible representation of the homogeneous Lorentz group. Thus, the spin-3/2 or the higher fermionic operators are forbidden. The most general supersymmetric algebra for four dimensional spacetime is given by [90].

$$\begin{aligned} &\left[P_\mu, P_\nu\right] = 0 \\ &[P_\mu, M_{\rho\sigma}] = i(\eta_{\mu\rho}P_\sigma - \eta_{\mu\sigma}P_\rho) \\ &\left[P_\mu, Q_\alpha^L\right] = \left[P_\mu, \overline{Q}_{\dot{\alpha}L}\right] = 0 \\ &\left[P_\mu, B_l\right] = \left[P_\mu, a^{l,\langle LM\rangle}B_l\right] = 0 \\ &\left\{Q_\alpha^L, \overline{Q}_{\dot{\beta}M}\right\} = 2\sigma^\mu_{\alpha\dot{\beta}}P_\mu\delta^L_M \\ &\left\{Q_\alpha^L, Q_\beta^M\right\} = \epsilon_{\alpha\beta}X^{\langle LM\rangle} \\ &\left\{\overline{Q}_{\dot{\alpha}L}, \overline{Q}_{\dot{\beta}M}\right\} = \epsilon_{\dot{\alpha}\dot{\beta}}X^\dagger_{\langle LM\rangle} \\ &\left[X^{\langle LM\rangle}, \overline{Q}_{\dot{\alpha}K}\right] = \left[X^{\langle LM\rangle}, Q_\alpha^K\right] = 0 \\ &\left[X^{\langle LM\rangle}, X^{\langle KN\rangle}\right] = \left[X^{\langle LM\rangle}, B_l\right] = 0 \\ &[B_l, B_m] = iC_{lm}^{\ \ k}B_k \\ &\left[Q_\alpha^L, B_l\right] = S^L_{l\ M}Q^M_\alpha \\ &[\overline{Q}_{\dot{\alpha}L}, B_l] = -S^{*l\ M}_L\overline{Q}_{\dot{\alpha}M}. \end{aligned} \tag{B.1.2}$$

where $L, M, \dots$ are the indices of the generators of supersymmetry. X and $X^\dagger$ are defined as

$$X^{\langle LM \rangle} = a^{l,\langle LM \rangle} B_l,\ X^\dagger_{\langle LM \rangle} = a^*_{l,\langle LM \rangle} B^l. \tag{B.1.3}$$

$P_\mu, M_{\mu\nu}$ are generators of translation and Lorentz rotation, respectively. $Q, \overline{Q}$ are grassmann odd generators. B_l compose compact Lie algebra. And $\langle LM \rangle$ denotes anti-symmetric indices. Especially, this algebra is simplified when we limit the argument to $\mathcal{N} = 1$.

$$\begin{aligned}
&[P_\mu, P_\nu] = 0\\
&[P_\mu, M_{\rho\sigma}] = i(\eta_{\mu\rho}P_\sigma - \eta_{\mu\sigma}P_\rho)\\
&[P_\mu, Q_\alpha] = [P_\mu, \overline{Q}_{\dot\alpha}] = 0\\
&[P_\mu, B_l] = 0\\
&\left\{Q_\alpha, \overline{Q}_{\dot\beta}\right\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu\\
&\{Q_\alpha, Q_\beta\} = \left\{\overline{Q}_{\dot\alpha}, \overline{Q}_{\dot\beta}\right\} = 0\\
&[B_l, B_m] = iC_{lm}^{\ \ k} B_k\\
&[Q_\alpha, B_l] = [\overline{Q}_{\dot\alpha}, B_l] = 0.
\end{aligned} \tag{B.1.4}$$

B.2 Spinor relations

Gamma matrices in chiral representation are given by

$$\gamma^\mu \equiv \begin{pmatrix} 0 & \sigma^\mu_{\alpha\dot\beta} \\ \overline{\sigma}^{\mu\dot\alpha\beta} & 0 \end{pmatrix}, \quad \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -\delta_\alpha^\beta & 0 \\ 0 & \delta^{\dot\alpha}_{\dot\beta} \end{pmatrix}. \tag{B.2.1}$$

Also, we define the anti-symmetric tensor as

$$\Gamma^{\mu\nu} \equiv \frac{i}{4}[\gamma^\mu, \gamma^\nu],\ \sigma^{\mu\nu} \equiv \frac{i}{4}(\sigma^\mu\overline{\sigma}^\nu - \sigma^\nu\overline{\sigma}^\mu),\quad \overline{\sigma}^{\mu\nu} \equiv \frac{i}{4}(\overline{\sigma}^\mu\sigma^\nu - \overline{\sigma}^\nu\sigma^\mu). \tag{B.2.2}$$

These tensor are related as:

$$\Gamma^{\mu\nu} \equiv \begin{pmatrix} \sigma^{\mu\nu} & 0 \\ 0 & \overline{\sigma}^{\mu\nu} \end{pmatrix} \tag{B.2.3}$$

Chiral projection operators are defined as:

$$\begin{cases} P_L \equiv \dfrac{1}{2}(1-\gamma_5) = \begin{pmatrix} \delta_\alpha^\beta & 0 \\ 0 & 0 \end{pmatrix} \\ P_R \equiv \dfrac{1}{2}(1+\gamma_5) = \begin{pmatrix} 0 & 0 \\ 0 & \delta^{\dot\alpha}_{\dot\beta} \end{pmatrix} \end{cases}. \tag{B.2.4}$$

Now, we divide Dirac spinor $\Psi(x)$ into two mass-degenerate, opposite charged Weyl spinors $\chi_\alpha(x), \eta_\alpha(x)$ as:

$$\Psi(x) = \begin{pmatrix} \chi_\alpha(x) \\ \eta^{\dagger\dot{\alpha}}(x) \end{pmatrix}. \tag{B.2.5}$$

The Hermitian conjugate of Weyl-spinor is defined as:

$$(\psi_\alpha)^\dagger = (\psi^\dagger)_{\dot{\alpha}} \equiv \psi^\dagger_{\dot{\alpha}}, \quad (\psi^{\dagger\dot{\alpha}})^\dagger = \psi^\alpha. \tag{B.2.6}$$

These relation means that the Hermite conjugate of the left-handed Weyl spinor is the right-handed Weyl spinor, and vice versa. Then, we have

$$\begin{aligned} \overline{\Psi}(x) &= \Psi^\dagger(x) A \\ &= (\chi^\dagger_{\dot{\alpha}},\ \eta^\alpha) \begin{pmatrix} 0 & \delta^{\dot{\alpha}}_{\ \dot{\beta}} \\ \delta^\beta_\alpha & 0 \end{pmatrix} = (\eta^\beta,\ \chi^\dagger_{\dot{\beta}}) \end{aligned} \tag{B.2.7}$$

and

$$\begin{aligned} \Psi^C(x) &= C\overline{\Psi}^T(x) \\ &= \begin{pmatrix} \epsilon_{\alpha\beta} & 0 \\ 0 & \epsilon^{\dot{\alpha}\dot{\beta}} \end{pmatrix} \begin{pmatrix} \eta^\beta \\ \chi^\dagger_{\dot{\beta}} \end{pmatrix} = \begin{pmatrix} \eta_\alpha \\ \chi^{\dagger\dot{\alpha}} \end{pmatrix}. \end{aligned} \tag{B.2.8}$$

Then, we also have

$$\Psi_L(x) = P_L\Psi(x) = \begin{pmatrix} \chi_\alpha(x) \\ 0 \end{pmatrix}, \quad \Psi_R(x) = P_R\Psi(x) = \begin{pmatrix} 0 \\ \eta^{\dagger\dot{\alpha}}(x) \end{pmatrix}. \tag{B.2.9}$$

We are able to divide all of the Dirac bilinear forms into the bilinear forms in terms of the Weyl spinors:

$$\begin{aligned} \overline{\Psi^i} P_L \Psi_j = \eta^i \chi_j, &\quad \overline{\Psi^i} P_R \Psi_j = \chi^{\dagger i} \eta^\dagger_j \\ \overline{\Psi^i} P_L \Psi^C_j = \eta^i \eta_j, &\quad \overline{\Psi^i} P_R \Psi^C_j = \chi^{\dagger i} \chi^\dagger_j \\ \overline{\Psi^{iC}} P_L \Psi_j = \chi^i \chi_j, &\quad \overline{\Psi^{iC}} P_R \Psi_j = \eta^{\dagger i} \eta^\dagger_j \end{aligned} \tag{B.2.10}$$

$$\begin{aligned} \overline{\Psi^i} \gamma^\mu P_L \Psi_j &= \chi^{\dagger i} \overline{\sigma}^\mu \chi_j \\ \overline{\Psi^i} \gamma^\mu P_R \Psi_j &= \eta^i \sigma^\mu \eta^\dagger_j \end{aligned} \tag{B.2.11}$$

$$\begin{aligned} \overline{\Psi^i} \Gamma^{\mu\nu} P_L \Psi_j &= \eta^i \sigma^{\mu\nu} \chi_j \\ \overline{\Psi^i} \Gamma^{\mu\nu} P_R \Psi_j &= \chi^{\dagger i} \overline{\sigma}^{\mu\nu} \eta^\dagger_j \end{aligned} \tag{B.2.12}$$

Thus, we have the transformation between the bilinear forms of the Dirac spinor and the Weyl spinors:

$$\begin{aligned}
\overline{\Psi^i}\Psi_j &= \eta^i\chi_j + \chi^{\dagger i}\eta_j^\dagger, \\
\overline{\Psi^i}\gamma_5\Psi_j &= -\eta^i\chi_j + \chi^{\dagger i}\eta_j^\dagger, \\
\overline{\Psi^i}\gamma^\mu\Psi_j &= \chi^{\dagger i}\overline{\sigma}^\mu\chi_j + \eta^i\sigma^\mu\eta_j^\dagger, \\
\overline{\Psi^i}\gamma^\mu\gamma_5\Psi_j &= -\chi^{\dagger i}\overline{\sigma}^\mu\chi_j + \eta^i\sigma^\mu\eta_j^\dagger, \\
\overline{\Psi^i}\Gamma^{\mu\nu}\Psi_j &= \eta^i\sigma^{\mu\nu}\chi_j + \chi^{\dagger i}\overline{\sigma}^{\mu\nu}\eta_j^\dagger, \\
\overline{\Psi^i}\Gamma^{\mu\nu}\gamma_5\Psi_j &= -\eta^i\sigma^{\mu\nu}\chi_j + \chi^{\dagger i}\overline{\sigma}^{\mu\nu}\eta_j^\dagger.
\end{aligned} \tag{B.2.13}$$

By using the formulae for the Pauli matrices, we have

$$\begin{aligned}
(\chi^{\dagger i}\overline{\sigma}^\mu\chi_j)(\eta^k\sigma_\mu\eta_l^\dagger) &= \chi_{\dot\alpha}^{\dagger i}(\overline{\sigma}^\mu)^{\dot\alpha\alpha}\chi_{j\alpha}\eta^{k\beta}(\sigma_\mu)_{\beta\dot\beta}(\eta_l^\dagger)^{\dot\beta} \\
&= 2(\chi_{\dot\alpha}^{\dagger i}\eta_l^{\dagger\dot\alpha})(\chi_{j\alpha}\eta^{k\alpha}).
\end{aligned} \tag{B.2.14}$$

Similarly we obtain

$$\begin{aligned}
(\chi^{\dagger i}\overline{\sigma}^\mu\chi_j)(\chi^{k\dagger}\overline{\sigma}_\mu\chi_l) &= 2(\epsilon^{\dot\alpha\dot\beta}\chi_{\dot\alpha}^{i\dagger}\chi_{\dot\beta}^{\dagger k})(\epsilon^{\alpha\beta}\chi_{j\alpha}\chi_{l\beta}) \\
(\eta^i\sigma^\mu\eta_j^\dagger)(\eta^k\sigma_\mu\eta_l^\dagger) &= 2(\epsilon_{\dot\alpha\dot\beta}(\eta_j^\dagger)^{\dot\alpha}(\eta_l^\dagger)^{\dot\beta})(\epsilon_{\alpha\beta}\eta^{i\alpha}\eta_{k\beta}).
\end{aligned} \tag{B.2.15}$$

From these relations, we obtain the Dirac spinor relations, which we have used in the calculation of the four-fermi operators generated by the X-boson mediation.

$$\begin{aligned}
(\overline{\Psi}^i\gamma^\mu P_L\Psi_j)(\overline{\Psi}^k\gamma_\mu P_L\Psi_l) &= 2(\overline{\Psi}^k P_R\Psi^{iC})(\overline{\Psi^C}_j P_L\Psi_l) \\
&= 2(\overline{\Psi}^i P_R\Psi^{kC})(\overline{\Psi^C}_l P_L\Psi_j) \\
(\overline{\Psi}^i\gamma^\mu P_R\Psi_j)(\overline{\Psi}^k\gamma_\mu P_L\Psi_l) &= 2(\overline{\Psi}^i P_R\Psi_l)(\overline{\Psi}^k P_L\Psi_j).
\end{aligned} \tag{B.2.16}$$

B.3 Regularization

When we evaluate quantum (radiative) corrections, some of them diverge due to the infinite momentum integrals. Since the S matrix, however, is finite, these divergence must be regularized. It seems that classical symmetries are usually preserved in the effective Lagrangian except some symmetries; for instance the scale (conformal) invariance and the global $U(1)$ axial symmetry (the Adler-Bardeen theorem [91]).

If the regularization method breaks these symmetries, we obtain the effective theory without these symmetries. For example, the Pauli-Villars regularization can not preserve the non-abelian gauge symmetries [92]. In 1972, G. 't Hooft and M. J. G. Veltman published the method which preserves manifestly the non-abelian gauge symmetries [93], the so-called dimensional regularization (DREG). Now, it is well-known as the gauge-invariant regularization scheme.

The modified minimal subtraction ($\overline{\text{MS}}$ scheme) with using the DREG is the following procedure. We carry out the analytic continuation of the dimension of the loop momentum; $4 \to D$ dimension. We set the dimension of the momentum integral, $D = 4 - \epsilon$, and then, we subtract the $1/\epsilon$ term by the re-definition of the operator normalization.

Though this procedure preserve the gauge invariance, the SUSY invariance is not preserved in the SUSY Yang-Mills (SYM) theories. In the SYM, the bosonic degrees of freedom (gauge bosons) are equal to the fermionic ones (gauginos). In the $4-\epsilon$-dimensional theories, the number of the gauge bosons is less than that of the gauginos.

In 1979, W. Siegel modified the dimensional regularization by the compactification or the dimensional reductions [74]. Thus, the number of fields does not change while we carry out the momentum integral in D-dimensional space. However, there remain the ambiguity with dimensional reduction (DRED) associated with treating the Levi-Civita tensor $\epsilon^{\mu\nu\rho\sigma}$.

Now, we introduce DRED by using a concrete model which includes a Yang-Mills multiplet and a multiplet of spin-1/2. The bare Lagrangian is given as:

$$\mathcal{L} = -\frac{1}{4}\left(F^a_{0\mu\nu}\right)^2 - \frac{1}{2\alpha}(\partial^\mu A_{0\mu})^2 + c_0^{a\dagger}\partial^\mu (D_\mu c_0)^a + i\overline{\psi}_0^\alpha \gamma^\mu (D_\mu \psi_0)^\alpha \tag{B.3.1}$$

where we define as:

$$\begin{aligned} F^a_{0\mu\nu} &= \partial_\mu A^a_{0\nu} - \partial_\nu A^a_{0\mu} + g f^{abc} W^b_\mu W^c_\nu, \\ (D_\mu c_0)^a &= \partial_\mu c_0^a + g f^{abc} A^b_{0\mu} c^c_0, \\ (D_\mu \psi_0)^\alpha &= \partial_\mu \psi_0^\alpha - i g A^a_{0\mu} (T^a)^{\alpha\beta} \psi_0^\beta. \end{aligned} \tag{B.3.2}$$

In DRED, we assume that all of the fields depend on the D-dimensional coordinates but not four-dimensional coordinates. $\mu, \nu, \ldots$ denote the four-dimensional indices and $i, j, \ldots$ denote D-dimensional indices. Then, we divide the Lagrangian into two parts; one includes only the D-dimensional gauge fields, the other includes only the remnant fields as follows:

$$A^a_\mu(x^j) = \left\{ A^a_i(x^j), A^a_\sigma(x^j) \right\}, \tag{B.3.3}$$

also, $\partial_\mu = \{\partial_i, \partial_\sigma\}$, and the divided Lagrangians are obtained as follows:

$$\begin{aligned} \mathcal{L}^D &= -\frac{1}{4}\left(F^a_{0ij}\right)^2 - \frac{1}{2\alpha}(\partial^i A_{0i})^2 + c_0^{a\dagger}\partial^i (D_i c_0)^a + i\overline{\psi}_0^\alpha \gamma^i (D_i \psi_0)^\alpha \\ \mathcal{L}^\epsilon &= \frac{1}{2\alpha}(D^i A_{0\sigma})^2 - g\overline{\psi}_0^\alpha \gamma^\sigma T^a A^a_{0\sigma}\psi - \frac{g^2}{4} f^{abc} f^{ade} A^b_{0\sigma} A^c_{0\sigma'} A^d_{0\sigma} A^e_{0\sigma'}. \end{aligned} \tag{B.3.4}$$

The gauge transformation of these fields is given:

$$\begin{aligned}\delta W_i^a &= \partial_i \Lambda^a + g f^{abc} W_i^b \Lambda^c \\ \delta W_\sigma^a &= g f^{abc} W_\sigma^b \Lambda^c \\ \delta \psi^\alpha &= ig (T^a \psi)^\alpha \Lambda^a\end{aligned} \tag{B.3.5}$$

This means that the W_σ^a behaves as the scalar field belonging to the adjoint representation; the so-called ϵ-scalar. In the $\overline{\text{MS}}$ scheme (that is, in the DREG), $\mathcal{L}^\epsilon$ is neglected.

The difference between the $\overline{\text{MS}}$ scheme and the $\overline{\text{DR}}$ scheme is the product of the gamma matrices. $\gamma^\mu \gamma_\mu$ is D in the $\overline{\text{MS}}$ scheme, on the other hand, is four in the $\overline{\text{DR}}$ scheme. Due to this property, the SUSY Ward identity is preserved in the $\overline{\text{DR}}$ scheme [75].

On the other hand, in the $\overline{\text{DR}}$ scheme, there are ambiguities in the calculation of the Levi-Civita tensor. For simplicity, we consider the two-dimensional case. In $D < 2$ dimension, we define the gamma matrices, the metric tensor and the ϵ-tensor with hat.

$$\begin{aligned} g^{\mu\nu} g_{\mu\nu} = 4, \;\; \hat{g}^{\mu\nu} \hat{g}_{\mu\nu} = g^{ij} g_{ij} = D \\ \hat{g}^{\mu\nu} g_\nu{}^\lambda = \hat{g}^{\mu\lambda}, \;\; \hat{g}^{\mu\nu} \gamma_\nu = \hat{\gamma}^\mu \end{aligned} \tag{B.3.6}$$

$$\hat{\epsilon}^{\mu\nu} \equiv \hat{g}^{\mu\rho} \hat{g}^{\nu\sigma} \epsilon_{\rho\sigma} \tag{B.3.7}$$

A tensor defined as follows:

$$A^{\mu\nu} \equiv \hat{\epsilon}^{\mu\nu} \hat{\epsilon}^{\rho\sigma} \hat{\epsilon}_{\rho\sigma} \tag{B.3.8}$$

includes an ambiguities in the two different calculations. On the one hand, we obtain a result by applying the formula for $\hat{\epsilon}^{\mu\nu}\hat{\epsilon}^{\rho\sigma}$ to this value

$$\begin{aligned}\hat{\epsilon}^{\mu\nu} \hat{\epsilon}^{\rho\sigma} \hat{\epsilon}_{\rho\sigma} &= (\hat{g}^{\mu\rho} \hat{g}^{\nu\sigma} - \hat{g}^{\mu\sigma} \hat{g}^{\nu\rho}) \hat{\epsilon}_{\rho\sigma} \\ &= 2 \hat{\epsilon}^{\mu\nu}.\end{aligned} \tag{B.3.9}$$

On the other hand, we also obtain another result by applying the formula for $\hat{\epsilon}^{\rho\sigma}\hat{\epsilon}_{\rho\sigma}$ to this value

$$\begin{aligned}\hat{\epsilon}^{\mu\nu} \hat{\epsilon}^{\rho\sigma} \hat{\epsilon}_{\rho\sigma} &= \hat{\epsilon}^{\mu\nu} \hat{g}^{\rho\xi} \hat{g}^{\sigma\eta} \hat{g}_{\rho\alpha} \hat{g}_{\sigma\beta} \epsilon_{\xi\eta} \epsilon^{\alpha\beta} \\ &= (D^2 - D) \hat{\epsilon}^{\mu\nu}.\end{aligned} \tag{B.3.10}$$

As a result, we obtain

$$(D+1)(D-2)\hat{\epsilon}^{\mu\nu} = 0. \tag{B.3.11}$$

This means that this equation is only satisfied in the case $D = -1, 2$, or $\hat{g}^{\mu\nu} = 0$ since in the 2-dimensional theories $\epsilon^{\mu\nu}$ is not zero. Thus, we find that the calculation of the gamma matrices includes mathematically inconsistency.

In Ref. [77], they have shown the formulation of the DRED in a mathematically consistent way. In DRED, the D-dimensional space must be the subset of the 4-dimensional space in order to define the ϵ-scalar and the metric $\hat{g}^{\mu\nu}$ projecting to the D-dimensional space. It seems, however, that the construction of such D-dimensional space can not be realized since the 4-dimensional space is a finite vector space.

This complexity can be solved partially by embedding this four-dimensional space in the infinite vector space which is called "quasi-four-dimensional" space (Q4S). The operators which live on Q4S behave as having the appropriate properties which the operators living on the 4-dimension space. For this way, we can construct the DRED mathematically consist way. On the other hand, the Fierz identity is no longer valid since the Dirac algebra is extended to the infinite dimensional one.

This inconsistency (mathematically inconsistency or no validity of the Fierz identity) leads that there is a non-vanishing variation of Lagrangian under the SUSY transformation. To see this fact, we perform the SUSY transformation of the SUSY gauge theory in the Wess-Zumino gauge.

$$\mathcal{L}_S = -\frac{1}{4} F^{a\mu\nu} F^a_{\mu\nu} + \frac{i}{2} \overline{\lambda}^a \gamma^\mu (D_\mu \lambda)^a + \frac{1}{2} D^2, \tag{B.3.12}$$

where $F^{\mu\nu}$, λ, and D are a field strength tensor for the gauge field, a gaugino, and an auxiliary field. The SUSY transformation for these fields are given in terms of the 4-component spinors;

$$\begin{cases} \delta A^a_\mu = -\dfrac{1}{\sqrt{2}} \bar{\epsilon} \gamma_\mu \lambda^a, \\ \delta \lambda^a = \dfrac{i}{2\sqrt{2}} F^a_{\mu\nu} \gamma^\mu \gamma^\nu \epsilon + \dfrac{1}{\sqrt{2}} D^a \gamma^5 \epsilon, \\ \delta D^a = \dfrac{i}{\sqrt{2}} \bar{\epsilon} \gamma_\mu \gamma_5 (D^\mu \lambda)^a. \end{cases} \tag{B.3.13}$$

We obtain the SUSY transformation of the Lagrangian;

$$\delta \mathcal{L}_S = \frac{i}{\sqrt{2}} \left[\frac{1}{2} g f^{abc} (\overline{\lambda}^a \gamma^\mu \lambda^b)(\bar{\epsilon} \gamma_\mu \lambda^c) + \partial_\mu \left\{ \frac{i}{4} \overline{\lambda}^a \gamma^\mu \gamma^\rho \gamma^\sigma \epsilon F^a_{\rho\sigma} + \frac{1}{2} \overline{\lambda}^a \gamma^\mu \gamma_5 \epsilon D^a \right\} \right]. \tag{B.3.14}$$

In four-dimensional theories, we can easily check that the first term is vanished by using the Fierz identity. The insertion of $\mathcal{L}_S$ leads the breaking of SUSY in the regularization at higher loop contributions.

In $\mathcal{N} = 2$ and $\mathcal{N} = 4$ supersymmetric theories, the structures of the divergence are mild [75]; there is only an one-loop divergence in $\mathcal{N} = 2$ SUSY theories and no divergence in

$\mathcal{N} = 4$ SUSY theories. Although these properties are attractive, the matter fields are no longer chiral fields; that is, the matter fields are a pair of a certain representation T and its complex representation T^* or belong to an adjoint representation since there are two or more SUSY charges. Thus, for constructing the realistic model, it seems that we do not use these properties.

B.4 Non-renormalization theorem

In the supersymmetric theories, there is a spatial property called the non-renormalization theorem [48]. Thanks to this theorem, there is no vertex correction for the interactions which are generated from superpotential in perturbative theories. We consider the case that there is the global symmetry

$$G \equiv U(N_f)_L \times U(N_f)_R \times U(1)_R \tag{B.4.1}$$

in the free field theories classically. N_f is the number of flavor, which characterizes all of fields as some species. L and R denotes left-handed and right-handed chiral superfields. The $U(1)_R$ is the abelian R symmetry.

Interaction terms in general break these global symmetry explicitly. We treat the coupling constants for these interactions as the spurion superfields whose scalar components obtain the VEVs by the spontaneous symmetry breaking*. Then, the effective Lagrangian should be invariant under these global symmetry.

B.4.1 Non-renormalization theorem for the Wess-Zumino model

Now let consider the Wess-Zumino(WZ) model as a concrete example. At tree-level, the superpotential of the WZ model is given as

$$W_{\text{tree}} = m\phi^2 + \lambda\phi^3. \tag{B.4.2}$$

In this model, there is an only left-chiral superfield ϕ. Thus, the global symmetry is

$$U(1) \times U(1)_R, \tag{B.4.3}$$

if there is no interaction terms. We assign the $U(1)$ charges to all of fields not only ϕ but also m and λ since we treat the couplings as spurion field.

$$\begin{cases} \phi & : (g_\phi, r_\phi) \\ m & : (-2g_\phi, 0) \\ \lambda & : (-3g_\phi, -r_\phi) \end{cases} \tag{B.4.4}$$

*The VEV of a scalar component of a spurion chiral superfield does not break supersymmetry since the scalar field appears with derivative in the SUSY transformation.

where we define as $(U, R) \equiv (U(1)\text{ charge}, U(1)_R\text{ charge})$. The R-charge assignment is defined so that the R-charge of the superpotential is equal 2. On the other hand, the superpotential is $U(1)$ neutral since the $U(1)$ symmetry is exact symmetry in the limit $m = 0, \lambda = 0$.

Since the effective superpotential has $U(1) \times U(1)_R$ symmetry, the effective superpotential should have the form as

$$W_{\text{eff}} = m\phi^2 f\left(\frac{\lambda\phi}{m}\right). \tag{B.4.5}$$

Only $\lambda\phi/m$ is neutral under both of $U(1)$ and $U(1)_R$. $f(x)$ is the holomorphic function. Thus, in general, the interaction term should have the form as follows:

$$W_{\text{eff}} = \sum_n \lambda^{n-2} m^{-n+3} \phi^n. \tag{B.4.6}$$

Furthermore, we consider the theories that are consistent in the massless limit ($m \to 0$) and the weak coupling limit ($\lambda = 0$). In the massless limit $m = 0$, $n \leq 3$ is required. In the weak coupling limit $\lambda = 0$, $n \geq 2$ is also required. Thus, since $n = 2$ and 3 are only allowed, the effective superpotential should have the form as:

$$W_{\text{eff}} = m\phi^2 + \lambda\phi^3 = W_{\text{tree}}. \tag{B.4.7}$$

This theorem implies that there is no additional effective operators and no vertex corrections in the effective superpotential at not only all of the perturbation levels but also non-perturbation effects.*

B.4.2 Non-renormalization theorem for SUSY gauge theories

Next we consider the non-renormalization theorem for the SUSY gauge theories perturbatively. A gauge kinetic term with the holomorphic gauge coupling is given by

$$S_{\text{SYM}} = \int d^4x d^2\theta \frac{\tau^a}{16} \mathcal{W}^{a\alpha} \mathcal{W}^a_\alpha + (\text{h.c.}). \tag{B.4.8}$$

The holomorphic gauge coupling is defined as follows:

$$\tau^a = \frac{1}{g_a^2} - i\frac{\Theta^a}{8\pi^2} \tag{B.4.9}$$

where g_a is the gauge coupling. Θ^a is the Θ-angle which breaks CP and characterizes the non-perturbative effects. Indeed, the imaginary part of the holomorphic gauge coupling generates the total derivative term

$$\frac{\Theta^a}{32\pi^2} \int d^4x \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} = n\Theta^a, \tag{B.4.10}$$

* It is also made sure that this non-renormalization theorem is correct at all of the perturbation levels by using the supergraph in [76].

where n is the winding number of the gauge field. Thus, when we calculate perturbative corrections, Θ^a is not appear in any amplitudes. When we define the intrinsic scale $|\Lambda|$ as the scale where the Landau pole for the gauge coupling exists $g^{-1}(|\Lambda|) = 0$, we obtain this at scale μ;

$$\frac{1}{g_a^2(\mu)} = -\frac{b}{8\pi^2} \ln \frac{|\Lambda|}{\mu}, \tag{B.4.11}$$

where for an $\mathcal{N} = 1$ SUSY $SU(N)$ gauge theory with F flavors, $b = 3N - F$. We also write the 1-loop holomorphic coupling as

$$\tau^a_{1-\text{loop}}(\mu) = -\frac{1}{8\pi^2} \ln \left[\left(\frac{|\Lambda|}{\mu} \right)^b e^{i\Theta^a} \right]. \tag{B.4.12}$$

The holomorphic intrinsic scale is defined as $\Lambda = |\Lambda| e^{i\Theta^a/b}$. Since Θ-angle does not affect the perturbation theory, there is a translation symmetry of this as:

$$\Theta^a \to \Theta^a + \alpha^a \quad (\text{for } \alpha^a \in \mathbb{R}). \tag{B.4.13}$$

On the other hands, the quantum corrections by the gauge coupling g_a must have the form in terms of the holomorphic coupling as:

$$g_a^2 = \frac{2}{\tau + \tau^\dagger}. \tag{B.4.14}$$

Therefore, since the superpotential must be a holomorphic function of τ^a, the quantum corrections which are proportional to a power of g_a^2 are not allowed in terms of the holomorphic gauge coupling. As a result, the non-renormalization theorem for the holomorphic gauge coupling states that the holomorphic gauge coupling is only allowed to have a shifting degree of freedom in perturbative theories.

If we take into account the non-perturbative effects, the action has no continuous Θ-angle translation. However, a discrete rotation is only allowed since this term is proportional to an integer n called the winding number as mentioned above. So, the action is invariant under the Θ^a-angle translation as

$$\Theta^a \to \Theta^a + 2\pi. \tag{B.4.15}$$

In terms of the holomorphic intrinsic scale, we find the action is invariant under the transformation defined as

$$\Lambda \to \Lambda' = e^{2\pi i/b} \Lambda. \tag{B.4.16}$$

This means that the non-perturbative contributions are proportional to Λ^{bn} for invariant under the discrete rotation. In general, we obtain the exact holomorphic gauge coupling;

$$\tau^a(\mu) = -\frac{1}{8\pi^2} \ln \left(\frac{\Lambda}{\mu}\right)^b + \sum_{n=1}^{\infty} a_n \left(\frac{\Lambda}{\mu}\right)^{bn}. \tag{B.4.17}$$

where a_n are the coefficients of the instanton effects. The first term explains that the holomorphic gauge coupling is exhausted at 1-loop level, and the second term describes the non-perturbative effects on the holomorphic coupling (which are called the n-instanton effects).

However, we know the beta functions for the gauge couplings by direct calculations beyond 1-loop corrections (for example, the explicit expression is given by [94]). This confusion was solved by V.A. Novikov, M.A. Shifman, A.I. Vainshtein, and V.I. Zakharov. Thus, this beta-function is called the NSVZ β function [95]. N. Arkani-Hamed and H. Murayama revealed that the physical gauge coupling is affected by the rescaling anomaly of the vector supermultiplet and the matter multiplet though the holomorphic coupling is exhausted at 1-loop level [96].

C Supersymmetry Breaking

In this appendix, we explain the details of the anomaly mediated SUSY breaking (AMSB) mechanism. These arguments are based on [60, 97, 98].

C.1 Coupling superfields

The soft SUSY breaking parameters are obtained by treating the renormalized coupling constants as superfields [50, 51, 52]. Now let us consider the simple case, the Wess-Zumino model:

$$\mathcal{L} = \int d^4\theta Z \Phi^\dagger \Phi + \int d^2\theta \left(\frac{1}{2} M \Phi^2 + \frac{1}{6} \lambda \Phi^3 \right) + \text{h.c.} \tag{C.1.1}$$

The soft SUSY breaking are obtained from the VEVs of the θ-dependent (but x-independent) components of Z, M, and λ which are extended to the superfields. Z is the real superfield, and M and λ are the chiral superfields. We treat the bare couplings as the superfields including quantum level by regularize the theory with keeping SUSY preserving. If these superfields have the θ-dependent non-zero VEV, we obtain the soft SUSY breaking terms. If we consider the renormalized coupling, the scalar component of Z is unity.

$$\begin{aligned} Z &\to 1 + (B\theta^2 + \text{h.c.}) + C\theta^2\theta^{\dagger 2} \\ M &\to M + F_M \theta^2 \\ \lambda &\to \lambda + F_\lambda \theta^2 \end{aligned} \tag{C.1.2}$$

then, we divide the Lagrangian into supersymmetric one and soft terms as follows:

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{\text{SUSY}} + \mathcal{L}_{\text{soft}} \\ \mathcal{L}_{\text{soft}} &= -(|B|^2 - C)\phi^\dagger \phi - \left[\frac{1}{2} (2BM - F_M) \phi^2 + \frac{1}{6} (3B\lambda - F_\lambda) \phi^2 + \text{h.c.} \right]. \end{aligned} \tag{C.1.3}$$

These soft parameters are denoted as:

$$\begin{aligned} m^2 &= |B|^2 - C = -[\ln Z]_{\theta^2\theta^{\dagger 2}} \\ A_M &= -(2BM - F_M) = -[\hat{M}]_{\theta^2} \\ A_\lambda &= -(3B\lambda - F_\lambda) = -[\hat{\lambda}]_{\theta^2}. \end{aligned} \tag{C.1.4}$$

In the last equalities, we defined the renormalized (physical) couplings as

$$\hat{\lambda} \equiv \frac{\lambda}{\zeta^{3/2}}, \quad \hat{M} \equiv \frac{M}{\zeta} \tag{C.1.5}$$

where ζ is the chiral superfield component of Z. Actually, the coupling Z absorbs the divergence as the wave function renormalization of Φ and $\Phi^\dagger$, Z must be divided into the chiral superfield and anti-chiral superfield.

$$\begin{aligned} Z &= (\zeta\zeta^\dagger)^{1/2} + (C - |B^2|)\theta^2\theta^{\dagger 2}, \\ \zeta &= 1 + 2B\theta^2. \end{aligned} \tag{C.1.6}$$

C.2 Anomaly Mediated Supersymmetry Breaking

We consider the case that there is no gauge singlet field in the hidden sector. We assume that the dynamical SUSY breaking in the hidden sector is given rise to at the energy scale μ_{SUSY}. The scalar potential of the supergravity (SUGRA) is obtained as:

$$V = \left|\frac{\partial W}{\partial z}\right|^2 - \frac{3}{M_{Pl}^2}|W|^2 + \text{D-terms} + \ldots \tag{C.2.1}$$

where z is the superfield which has SUSY breaking F-term. In this potential, we do not include the multiplets which have the VEVs at Planck scale. The leading contribution of the Kähler potential is given by

$$\mathcal{K} = Z^\dagger Z + \mathcal{O}\left(\frac{Z^3}{M_{Pl}}\right). \tag{C.2.2}$$

There is no linear term in Kähler potential since there is no gauge singlet field.

If a gauge singlet field exists in the SUSY breaking sector, the gaugino masses are generated by the higher-dimensional operator as:

$$\int d^2\theta \frac{Z}{M_{Pl}} \mathrm{tr}\mathcal{W}^\alpha \mathcal{W}_\alpha + (\mathrm{h.c.}). \tag{C.2.3}$$

Thus, the typical mass scale of the gauginos is $\mu_{\mathrm{SUSY}}^2/M_{Pl}$. On the other hand, in the case that no the gauge singlet field exists in the hidden sector, the typical mass scale of the gauginos, which are generated from the more higher-dimensional operators, are less than $\mu_{\mathrm{SUSY}}^3/M_{Pl}^2$. However, since the mass scale of the sfermions is proportional to $\mu_{\mathrm{SUSY}}^2/M_{Pl}$, the SUSY breaking scale is assumed to be $\mu_{\mathrm{SUSY}} \sim 10^{10}$GeV for the several TeV superpartners. The gauginos has the mass of several keV in the theory which has no gauge singlet field in the hidden sector.

It implies that there is a serious problem in the model of the dynamical SUSY breaking in the hidden sector.

Now, we will see that the mass of gauginos and the A-terms are generated with an order of $\mathcal{O}(1/M_{Pl})$. The effective theory in the visible sector is obtained by integrating out the massive particles in the hidden sector with regularizing appropriately the theory. The

Table 9: The Weyl weight for each operators

	superfields	Weyl weight	R-charge
Q	matter (chiral)	+1	r_Q
$Q^\dagger$	matter (anti-chiral)	+1	$-r_Q$
V	gauge (vector)	0	0
$\mathcal{W}^a$	field strength chiral superfield	+3/2	+1
D_α	chiral covariant derivative	+1/2	-1
$D^\dagger_{\dot\alpha}$	anti-chiral covariant derivative	+1/2	1
$\Box$	d'Alembertian	2	0
ϕ	compensator (chiral)	+1	+2/3
$\phi^\dagger$	compensator (anti-chiral)	+1	-2/3

contributions from the massive SUGRA particles are negligible until we consider the higher corrections an order of $\mathcal{O}(M_{Pl}^{-2})$, since the effects from exchanging the SUGRA particles are generated with $\mathcal{O}(M_{Pl}^{-2})$.

By using the superconformal calculus formulation of SUGRA, we obtain the gaugino mass and A-terms. First, we construct the action which is invariant under the local superconformal transformation. Then, this superconformal symmetry is explicitly broken down to local super-Poincaré symmetry. All of the fields are assigned a Weyl weight, which is a scaling dimension and is equal to the mass dimension of superfields, the conformal invariance is broken by the scalar component of the compensator chiral superfields ϕ. This compensator has Weyl weight +1. The Weyl weight and the R-charge for every field are shown in Table 9. The Weyl weight and R-charge of Lagrangian are

$$\mathrm{d}(\mathcal{L}) = 4, \quad R(\mathcal{L}) = 0, \tag{C.2.4}$$

where $\mathrm{d}(\Phi)$ denotes the Weyl weight of the superfield Φ and $R(\Phi)$ means the R-charge of Φ. Since Lagrangian is composed of the Kähler potential ($\mathcal{K}$) and the superpotential (W)

$$\mathcal{L} = \int d^4\theta\, \mathcal{K} + \left(\int d^2\theta\, W + \text{h.c.} \right), \tag{C.2.5}$$

these charges are assigned as follows:

$$\begin{aligned} \mathrm{d}(\mathcal{K}) = 2, \quad R(\mathcal{K}) = 0 \\ \mathrm{d}(W) = 3, \quad R(W) = 2. \end{aligned} \tag{C.2.6}$$

For constructing the superconformal action, it is convenient that we set the Weyl weight and R-charge of every field to be 0. That is, all of superfields are combined with the compensator appropriately. Then, the compensator must couple to the Kähler potential and the superpotential since the action is not conformally invariant. The R-charge of the compensator is defined in order to give the proper R-charge for the superpotential. That is,

$$\int d^2\theta\ \phi^3 + \text{h.c.} \tag{C.2.7}$$

is R-invariant.

The Weyl invariant action has the form as:

$$\begin{aligned} S &= \int d^4x\ \mathcal{L} \\ \mathcal{L} &= \int d^4\theta\ \phi\phi^\dagger \mathcal{K}\left(\frac{Q}{\phi}, \frac{Q^\dagger}{\phi^\dagger}, \frac{\mathcal{W}^a}{\phi^{3/2}}, \frac{\mathcal{W}^\dagger_{\dot{\alpha}}}{\phi^{\dagger 3/2}}, V; \frac{\phi^{1/2}}{\phi^\dagger} D_\alpha, \frac{\phi^{\dagger 1/2}}{\phi} D^{\dagger\dot{\alpha}}\right) \\ &\quad + \left(\int d^2\theta\ \phi^3 W\left(\frac{Q}{\phi}, \frac{\mathcal{W}^a}{\phi^{3/2}}\right) + \text{h.c.}\right). \end{aligned} \tag{C.2.8}$$

This action is rewritten in terms of the Kähler potentials Ω with the Weyl weight $\dim\Omega$ and the superpotentials Ξ with the Weyl weight $\dim\Xi$

$$S = \int d^4x \left[\int d^4\theta\ \phi\phi^\dagger \sum_\Omega (\phi\phi^\dagger)^{-\dim\Omega/2}\Omega + \left(\int d^2\theta\ \phi^3 \sum_\Xi \phi^{-\dim\Xi}\Xi + \text{h.c.}\right)\right]. \tag{C.2.9}$$

When we calculate the gaugino mass and the A-terms, we use the dimensional reduction (DRED) as the regularization method. Thus, we consider the Weyl invariant action in D-dimensional Minkowski spacetime. We have

$$S = \int d^Dx \left[\int d^4\theta\ (\phi\phi^\dagger)^{\frac{D-2}{2}} \sum_\Omega (\phi\phi^\dagger)^{-\dim\Omega/2}\Omega + \left(\int d^2\theta\ \phi^{D-1} \sum_\Xi \phi^{-\dim\Xi}\Xi + \text{h.c.}\right)\right]. \tag{C.2.10}$$

The scalar VEVs of the compensator breaks the superconformal symmetry into the super-Poincaré symmetry. On the other hand, the VEVs of higher component of the compensator breaks supersymmetry. When the supersymmetry is broken, the compensator has the form as*:

$$\phi = 1 + m_{3/2}\theta^2 \tag{C.2.11}$$

Then, we have

$$\begin{aligned} S = \int d^D x \Bigg[\int d^4\theta \sum_{\Omega} \left(1 + \frac{1}{2}(D - 2 - \dim\Omega) m_{3/2}(\theta^2 + \bar{\theta}^{\dagger 2})\right) \Omega \\ + \left(\int d^2\theta \sum_{\Xi} \left(1 + (D - 1 - \dim\Xi) m_{3/2}\theta^2\right) \Xi + \text{h.c.} \right) + \mathcal{O}(m_{3/2}^2) \Bigg]. \end{aligned} \tag{C.2.12}$$

The Weyl weight for the fundamental superfields in D dimension is defined as equivalent to the mass dimension for these fields. For example, the Weyl weight for the vector superfields is zero, and then, in the gauge sector, we have

$$\Xi = \frac{1}{4g_0^2}\mathrm{tr}\mathcal{W}^\alpha\mathcal{W}_\alpha \quad : \dim\Xi = 3 \tag{C.2.13}$$

where g_0 is the bare gauge coupling. At tree level, we obtain

$$\mathcal{L}_{\text{gauge}} = \int d^2\theta \, \left(1 - \epsilon m_{3/2}\theta^2\right) \frac{1}{4g_0^2}\mathrm{tr}\mathcal{W}^\alpha\mathcal{W}_\alpha. \tag{C.2.14}$$

This expression means that the bare gaugino mass is proportional to $-\epsilon m_{3/2}$. Combining this bare mass with $1/\epsilon$ terms in the bare gauge coupling, we have

$$\mathcal{L}_{\text{gauge}} = \int d^2\theta \, \frac{1}{4g^2(\mu)} \left(1 - \epsilon m_{3/2} \cdot \frac{\beta(g^2)}{\epsilon g^2}\theta^2\right) \mathrm{tr}\mathcal{W}^\alpha\mathcal{W}_\alpha \tag{C.2.15}$$

then, the mass of the gaugino is as:

$$M_\lambda(\mu) = -\frac{\beta(g^2)}{2g^2} m_{3/2}. \tag{C.2.16}$$

In this mechanism, we are also able to obtain the A-terms and the sfermion mass. First, we consider the dimension-n superpotential given as:

$$\Xi = \lambda\Phi_1 \cdots \Phi_n. \tag{C.2.17}$$

*It is known that the F-component of the compensator chiral superfield is equivalent to the gravitino mass at the leading order by using the equation of motion of the auxiliary field [97].

The corresponding soft SUSY breaking term (A_n-terms) in the SUGRA background is given by Eq. (C.2.12) as $(n-3)m_{3/2}\lambda\phi_1\cdots\phi_n$ at tree-level. That is, the A-terms are not induced at tree-level, and the other soft masses are the same order as the gravitino mass.

Then, we incorporate the quantum corrections from the kinetic terms of these fields in the 1PI effective action.

$$\begin{aligned}&\sum_r \int d^4\theta \Phi_r^\dagger Z_r\left[(\Box/\phi\phi^\dagger)\right]\Phi_r + \mathcal{O}(m_{3/2}^2)\\&=\sum_r \int d^4\theta \Phi_r^\dagger Z_r\left[\Box(1-(m_{3/2}\theta^2+\text{h.c.}))\right]\Phi_r + \mathcal{O}(m_{3/2}^2)\end{aligned} \tag{C.2.18}$$

The renormalized A_n-terms receive the quantum corrections from $Z_r(\Box=\mu^2)$.

$$A_n = \left(n-3-\frac{1}{2}\sum_{r=1}^{n}\gamma_r(\mu)\right)m_{3/2}\lambda, \quad \gamma_r(\mu)\equiv\frac{\mathrm{d}\ln Z_r}{\mathrm{d}\ln\mu}. \tag{C.2.19}$$

Thus, the scalar trilinear coupling are induced with the 1-loop suppression:

$$A_3(\mu) = -\frac{1}{2}\sum_{r=1}^{3}\gamma_r(\mu)m_{3/2}\lambda(\mu). \tag{C.2.20}$$

When this renormalized operator relates to the bare operators as:

$$\begin{aligned}\lambda_0\Phi_1^0\Phi_3^0\Phi_3^0 &= \lambda(\mu)\Phi_1\Phi_3\Phi_3\\&=\lambda(\mu)Z_1^{-1/2}Z_2^{-1/2}Z_3^{-1/2}\Phi_1^0\Phi_3^0\Phi_3^0\end{aligned} \tag{C.2.21}$$

and then, we have

$$\begin{aligned}\frac{\mathrm{d}\ln\lambda(\mu)}{\mathrm{d}\ln\mu} &= \frac{1}{2}\sum_r\frac{\mathrm{d}\ln Z_r}{\mathrm{d}\ln\mu}\\&=-\frac{1}{2}\sum_r\gamma_r(\mu).\end{aligned} \tag{C.2.22}$$

Thus, the A-terms are obtained as:

$$A_3(\mu) = m_{3/2}\frac{\mathrm{d}\lambda(\mu)}{\mathrm{d}\ln\mu} = \beta(\lambda)m_{3/2}. \tag{C.2.23}$$

The A-terms are also induced with loop suppression.

The scalar soft mass terms are also induced from the wave function renormalization. As mentioned above, the relation between the scalar soft mass and $Z_r(\Box = \mu^2)$ is given*:

$$\begin{aligned} m_r^2 &= -[\ln Z_r]_{\theta^2\theta^{\dagger 2}} \\ &= -\frac{1}{4} m_{3/2}^2 \frac{\mathrm{d}}{\mathrm{d}\ln\mu}\left(\frac{\mathrm{d}Z_r}{\mathrm{d}\ln\mu}\right). \end{aligned} \tag{C.2.25}$$

Then, by using the anomalous dimensions and the beta functions, we have:

$$m_r^2 = \frac{1}{4} m_{3/2}^2 \left[\beta(g_a)\frac{\partial}{\partial g_a} + \beta(y^{ijk})\frac{\partial}{\partial y^{ijk}} + \cdots\right]\gamma_r \tag{C.2.26}$$

where g_a and y^{ijk} are the gauge couplings and the Yukawa couplings, respectively. In the AMSB, the superpartner spectrum is proportional to $m_{3/2}$ with the 1-loop suppression. Also, these expression shows that there is a phenomenologically important result. We can neglect the Yukawa couplings since they for the light SM model particles are too small. The anomalous dimension at the 1-loop level is

$$\begin{aligned} \gamma_r &= \frac{1}{16\pi^2}\left(\frac{1}{2} y^{rst} y^*_{rst} - \sum_a (2g_a^2 C^a)\right) \quad \text{(no sum for } r\text{)} \\ &\approx -\frac{1}{2\pi}\sum_a \alpha_a C^a \end{aligned} \tag{C.2.27}$$

where C^a is the Casimir invariant defined as $C^a\delta^i_j \equiv \sum_A (T^A T^A)^i_j$, especially $C^a = (N^2 - 1)/2N$ for $SU(N)$. Thus, we have the form for the soft mass:

$$m_r^2 = -\sum_a \frac{C^a \alpha_a}{4\pi g_a}\beta(g_a) m_{3/2}^2. \tag{C.2.28}$$

For the asymptotically free gauge symmetry, these soft masses are positive values since $\beta(g) < 0$. Unfortunately, however, the standard model includes the gauge groups which become strong at high-energy scale, $SU(2)_L$ and $U(1)_Y$, the sleptons are tachyonic scalar particles. If these particle are tachyonic, our Universe changes into the charge breaking vacuum.

* We use the general formula:

$$F(a + b\theta^2 + c\theta^{\dagger 2}) = F(a) + (b\theta^2 + c\theta^{\dagger 2})F'(a) + bc\theta^2\theta^{\dagger 2}F''(a) \tag{C.2.24}$$

where a, b, and c are independent from θ and $\theta^\dagger$ coordinates, and the prime denotes the differential of $F(a)$ with respect to a.

D The Effective Gauge Theory Approach to GUT

In this appendix, we review how to construct the effective gauge theory following [99, 100]. The unified gauge group G breaks into Standard Model (SM) gauge group at very high energy (a typical scale $M_{\text{GUT}} \sim 10^{16}$GeV).

$$G \to SU(3)_C \times SU(2)_L \times U(1)_Y$$

M_{GUT} is estimated by the development of the β functions of these gauge couplings. That is why the massive particles are decoupled, and do not affect the effective theory at low-energy scale (this fact is ensured by the so-called Appelquist-Carrazzone theorem) [101]. Since we use $\overline{\text{MS}}$ scheme (in SUSY case, we use the $\overline{\text{DR}}$ scheme [74] which is the supersymmetric invariant scheme), β functions are mass-independent. In order to use perturbation at low-energy, we construct the effective gauge theory (EGT) by integrating out heavy particles. When we integrate out these heavy particles, we should consider "boundary conditions" known as “threshold corrections”. These are the conditions for coupling constants to match the couplings between the full theory above the energy where the massive particles are integrated out and the effective theory after these particles are integrated out.

In our analysis, there are four or more scales where massive particles are integrated out.

1. GUT scale (M_{GUT}) : $G \to SU(3)_C \times SU(2)_L \times U(1)_Y$
   In grand unified theories, many fields obtain the mass terms after the adjoint Higgs acquired VEVs. For example, the X-boson, the adjoint Higgs bosons, and the color-triplet Higgs boson have the heavy masses in $SU(5)$ GUT.

2. SUSY scale (M_{SUSY}) and other mass scales of massive particles
   If supersymmetry is softly broken, many soft mass terms are introduced. Below this mass scale, the effective theory includes the standard model particles and remnants which are not integrated out at the SUSY breaking scale. For instance, in the high-scale SUSY scenario, there is a large mass difference between scalar fermions and gauginos.

3. Weak scale (M_W)
   We integrate out W-, Z-bosons, ghosts, and the Higgs boson which are massive after the Higgs boson has non-zero VEV. Below this scale, we construct effective quantum chromodynamics (QCD), and quantum electrodynamics (QED) including only quarks, leptons and electromagnetic gauge boson, namely photon.

4. Massive fermion mass scales
   In effective QCD and QED, all of charged fermions obtain masses. Then, under these mass scales, these heavy fermions do not appear in external lines and are integrated out. Therefore, the number of flavors of quarks is decreased as the energy scale becomes lower.

D.1 How to construct threshold correction for gauge couplings

The threshold correction is the boundary condition between gauge coupling constants of the full theory and the effective theory (for example GUT and SM). Now, we consider the case that the simple gauge group G is spontaneously broken down to $\prod_i G_i \times U(1)$. Namely the full theory corresponds to the gauge theory based on unified gauge group G and the effective theory corresponds to the gauge theories based on $\prod_i G_i \times U(1)$ which include non-renormalizable interactions. The relation between the indices of these generators is described as:

$$\{\alpha\} = \{A\} + \sum_i \{a_i\} \tag{D.1.1}$$

where α, A, and a_i mean the generators of G , broken symmetry and unbroken residual symmetry G_i, respectively.

We choose R_ξ gauge when we construct the effective Lagrangian since this gauge make us to introduce invariant terms under G_i gauge transformation and to vanish interaction terms such as the two-point interactions between different fields.

The Lagrangian of the full theory which is invariant under G gauge transformation is given by

$$\mathcal{L} = -\frac{1}{4} F_\alpha^{\mu\nu} F_{\alpha\mu\nu} + \overline{\psi}_i (i\not{D} - M_F)\psi_i + \frac{1}{2}|D_\mu \phi_i|^2 - V(\phi) + \mathcal{L}_Y + \mathcal{L}_{GF} + \mathcal{L}_{FP}. \tag{D.1.2}$$

We decompose this gauge kinetic term to local gauge G_i invariant part and massive gauge boson part.

$$\begin{aligned}
-\frac{1}{4} F_\alpha^{\mu\nu} F_{\alpha\mu\nu} =& -\frac{1}{4}\left[\partial^\mu A_\alpha^\nu - \partial^\nu A_\alpha^\mu - g C_{\alpha\beta\gamma} A_\beta^\mu A_\gamma^\nu\right]\left[\partial_\mu A_{\alpha\nu} - \partial_\nu A_{\alpha\mu} - g C_{\alpha\beta\gamma} A_{\beta\mu} A_{\gamma\nu}\right] \\
=& -\frac{1}{4}\left[\tilde{F}_A^{\mu\nu} - g\sum_l \left(C_{ABc_l} A_B^\mu A_{c_l}^\nu + C_{Ab_lC} A_{b_l}^\mu A_C^\nu\right)\right]^2 \\
& -\frac{1}{4}\sum_l \left[\tilde{F}_{a_l}^{\mu\nu} - g C_{a_lBC} A_B^\mu A_C^\nu\right]^2.
\end{aligned} \tag{D.1.3}$$

Here, $\tilde{F}$ denotes

$$\begin{aligned}
\tilde{F}_A^{\mu\nu} &= \partial^\mu A_A^\nu - \partial^\nu A_A^\mu - g C_{ABC} A_B^\mu A_C^\nu, \\
\tilde{F}_{a_l}^{\mu\nu} &= \partial^\mu A_{a_l}^\nu - \partial^\nu A_{a_l}^\mu - g C_{a_lb_lc_l} A_{b_l}^\mu A_{c_l}^\nu.
\end{aligned} \tag{D.1.4}$$

where $A, B, \ldots$ are the indices of the generators of the broken symmetry and $a_l, b_l \ldots$ are those of the generators of the residual G_l symmetries. Note that there is no term which has coefficients $C_{Ab_lc_l}$ since the generators of the unbroken symmetry have the closed structure of algebras. Therefore, anti-commutators of generators of residual symmetries which are given by

$$\left\{T^{b_l}, T^{c_l}\right\} = iC_{b_l c_l a_l} T^{a_l} \tag{D.1.5}$$

are only permitted. In other words, $iC_{b_l c_l A}$ are exactly zero since T^A are not included in algebras of the generators of residual symmetries.

Now we consider gauge transformations of broken gauge symmetries. The infinitesimal gauge transformation for the massive gauge boson $A_{A\mu}$ and scalar fields ϕ_i are given by

$$\begin{aligned} A_{A\mu} &\to A_{A\mu} - \frac{1}{g}\partial_\mu \alpha_A \\ \phi_j &\to (1 + i\alpha_A t_s^A)_{ji}\phi_i. \end{aligned} \tag{D.1.6}$$

The kinetic term of scalar field is obtained as follows:

$$\begin{aligned} |D_\mu \phi|^2 = \partial^\mu \phi_i' \partial_\mu \phi_i' - igA^{\alpha\mu}\left[(\lambda_j + \phi_j')(t_s^\alpha)_{ji}\partial_\mu \phi_i' - \partial_\mu \phi_j'(t_s^\alpha)_{ji}(\lambda_i + \phi_i')\right] \\ + g^2 A^{\alpha\mu} A_\mu^\alpha (\lambda_j + \phi_j')(t_s^\alpha t_s^\alpha)_{ji}(\lambda_i + \phi_i') \end{aligned} \tag{D.1.7}$$

where λ_i is the VEV of the scalar field ϕ_i and ϕ_i' is the quantum fluctuation around the VEV, $\phi = \lambda + \phi'$. Since two-point interactions between the scalar boson and the massive gauge boson are generated from the second term, we must avoid these terms by using R_ξ gauge. A gauge fixing function and a gauge fixing term are given by:

$$\begin{aligned} f_{Ax} &= \frac{1}{\sqrt{\xi}}(D_\mu^a A_A^\mu + ig\xi\lambda_i (t_s^A)_{ij}\phi_j'), \\ \mathcal{L}_{GF} &= -\frac{1}{2} f_{Ax}^2. \end{aligned} \tag{D.1.8}$$

Since this gauge fixing term is also invariant under residual gauge symmetries, the derivative term in f_{Ax} must transform covariantly under the residual gauge symmetry, $D_\mu^a A_A^\mu$. The gauge transformation of gauge fixing function is obtained as:

$$\begin{aligned} f_{Ax} \to \frac{1}{\sqrt{\xi}} &\left[\partial^\mu \left(A_{A\mu} - \frac{1}{g}\partial_\mu \alpha_A\right)\right. \\ &\left. + gC_{ABa}\left(A_B^\mu - \frac{1}{g}\partial^\mu \alpha_B\right) A_{a\mu} + ig\xi\lambda_i (t_s^A)_{ij}\left[(1 + i\alpha_A t_s^A)_{jk}\phi_k - \lambda_j\right]\right]. \end{aligned} \tag{D.1.9}$$

The last term is obtained from the transformation rule of ϕ but not ϕ'. Thus, we obtain the functional derivative of this gauge fixing function with respect to the gauge parameter α_B as

$$\frac{\delta f_{Ax}}{\delta \alpha_B} = \frac{1}{\sqrt{\xi}}\left[-\frac{\delta_{AB}}{g}\partial_\mu \partial^\mu + C_{ABa}[\partial^\mu A_{a\mu} + A_{a\mu}\partial^\mu] - g\xi\delta_{AB}\lambda_i (t_s^A t_s^A)_{ij}\phi_j\right], \tag{D.1.10}$$

and the Fadeev-Popov ghost Lagrangian as

$$\det\left(\frac{\delta f_{Ax}}{\delta\alpha_B}\right) = \int \mathcal{D}c\mathcal{D}\bar{c}\exp\Big[i\int d^4x\bar{c}_A\Big(-\delta_{AB}\partial^2 + gC_{ABa}[\partial^\mu A_{a\mu} + A_{a\mu}\partial^\mu] \\ -\delta_{AB}g^2\xi\lambda_i[(t_s^A)^2]_{ij}(\lambda_j+\phi_j')\Big)c_B\Big] \tag{D.1.11}$$

This Lagrangian implies that this ghost field has mass, $m_A^2 = g^2\xi\lambda_i[(t_s^A)^2]_{ij}\lambda_j$, which is the same mass as the mass of the heavy gauge bosons. The effective Lagrangian which is manifestly invariant under residual gauge symmetries G_i is written as

$$\tilde{\mathcal{L}}_i = -\frac{1}{4}\tilde{F}^{\mu\nu}_{a_l}\tilde{F}_{a_l\mu\nu} + \frac{1}{4}l_i\tilde{F}^{\mu\nu}_{a_l}\tilde{F}_{a_l\mu\nu} + \cdots \tag{D.1.12}$$

where the second term of this Lagrangian is obtained by integrating out heavy particles. Because of gauge G_i invariance, these effects must be proportional to the residual gauge kinetic term.

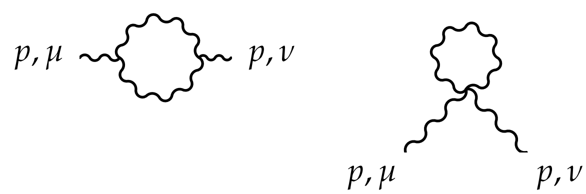


Figure 15: (Left) This diagram generate both of the external momentum dependent and independent amplitude. (Right) This diagram generate only the external momentum independent amplitude.

D.2 Feynman rule of the Effective Gauge Theory

When we construct effective gauge theories by integrating out massive gauge bosons, we need Feynman rules of the effective gauge theories. Therefore, let us decompose unified gauge invariant terms into residual gauge invariant terms and interaction terms of massive gauge bosons, ghosts, and residual gauge bosons. Gauge interaction terms with massive gauge bosons are obtained from the GUT invariant gauge kinetic term $-F_{\mu\nu}F^{\mu\nu}/4$.

$$\frac{g}{2}\tilde{F}_{A\mu\nu}\sum_l\left(C_{ABc_l}A^\mu_B A^\nu_{c_l} + C_{Ab_lC}A^\mu_{b_l}A^\nu_C\right) - \frac{1}{4}g^2\left[\sum_l\left(C_{ABc_l}A^\mu_B A^\nu_{c_l} + C_{Ab_lC}A^\mu_{b_l}A^\nu_C\right)\right]^2 \\ + \frac{1}{2}g\sum_i\tilde{F}^{\mu\nu}_{a_i}C_{a_iBC}A_{B\mu}A_{C\nu} \tag{D.2.1}$$

Gauge three-point interaction terms are decomposed as follows:

$$
\begin{aligned}
&g(\partial_\mu A_{A\nu} - \partial_\nu A_{A\mu}) \sum_l C_{ABc_l} A^\mu_B A^\nu_{c_l} + \frac{1}{2} g \sum_i (\partial_\mu A_{a_i\nu} - \partial_\nu A_{a_i\mu}) C_{a_iBC} A^\mu_B A^\nu_C \\
&= g \sum_i C_{ABc_i} \left(\partial_\mu A_{A\nu} A^\mu_B A^\nu_{c_i} + \partial_\nu A_{B\mu} A^\mu_A A^\nu_{c_i} + \partial_\mu A_{c_i\nu} A^\mu_A A^\nu_B \right)
\end{aligned}
\tag{D.2.2}
$$

where we use the property of the structure constant C_{ABc_i}, which is antisymmetric under changing indices as $B \leftrightarrow C$. There is the other contribution from the gauge fixing term. This contribution has the following form:

$$
-\frac{g}{\xi} C_{ABc_l} \partial^\mu A_{A\mu} A^\nu_B A_{c_l\nu}. \tag{D.2.3}
$$

Therefore, the three-point interactions including massive gauge bosons and massless gauge bosons are obtained as follows:

$$
\mathcal{L}_{ABi} = \sum_i \left[g C_{ABc_l} \left(\partial_\mu A_{A\nu} A^\mu_B A^\nu_{c_i} + \partial_\nu A_{B\mu} A^\mu_A A^\nu_{c_i} + \partial_\mu A_{c_i\nu} A^\mu_A A^\nu_B \right) - \frac{g}{\xi} C_{ABc_i} \partial^\mu A_{A\mu} A^\nu_B A_{c_i\nu} \right]. \tag{D.2.4}
$$

Then, we consider the three-point vertex which includes ghosts and massless gaugebosons. This interaction term is given by

$$
\mathcal{L}_{\bar{c}cA} = g C_{ABa} \bar{c}_A \partial^\mu A_{a\mu} c_B. \tag{D.2.5}
$$

Here, since all fields go into the vertex, anti-ghost has momentum with opposite sign to arrow of this external line.

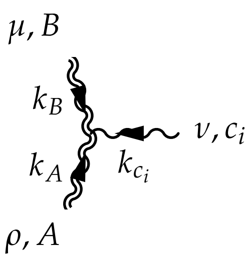
Next, we consider the interaction term of massive scalar and massless gauge boson. This corresponding Lagrangian is given by

$$
\mathcal{L}_{\phi\phi A} = -i A^{a\mu} \left[\phi'_j (t^a_s)_{ji} \partial_\mu \phi'_i - \partial_\mu \phi'_j (t^a_s)_{ji} \phi'_i \right]. \tag{D.2.6}
$$

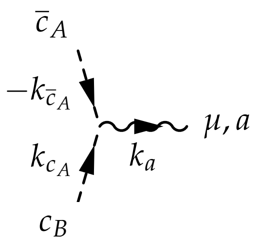
Note that "heavy scalar" means either the Nambu-Goldstone boson or the physical heavy scalar boson. We also obtain the Lagrangian for interaction of massive fermions and massless gauge boson as the following form.

$$
\mathcal{L}_{\psi\psi A} = -ig \overline{\psi}_m \gamma^\mu i A_{a\mu} (t^a_F)_{mn} \psi_n \tag{D.2.7}
$$

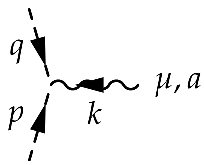
Therefore, the Feynman rules which describe the interaction between residual massless gauge boson and massive gauge bosons, scalars and fermions are obtained as follows:



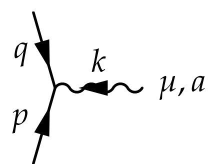
$$
= g C_{ABc_i} \left[(k_A - k_{c_i} + \frac{1}{\xi} k_B)_\mu g_{\rho\nu} + (k_{c_i} - k_B - \frac{1}{\xi} k_A)_\rho g_{\mu\nu} + (k_B - k_A)_\nu g_{\rho\mu} \right] \tag{D.2.8}
$$



$$= -i\left(-igC_{ABa}k_a^\mu\right) = gC_{ABa}(k_{\bar{c}} - k_c)^\mu \tag{D.2.9}$$



$$= -i(-ig)(-ip + iq)_\mu (t_s^a)_{ji} = ig(p - q)_\mu (t_s^a)_{ji} \tag{D.2.10}$$



$$= -ig\gamma^\mu (t_F^a)_{mn} \tag{D.2.11}$$

D.3 One-loop calculation for threshold correction

In this subsection, we show details for the calculation of threshold corrections for gauge couplings. As mentioned above, the effects of integrating out the massive particles are included as threshold corrections. To calculate these effects of heavy particles, it is needed to match amplitudes which are calculated in full theory and effective theory. For instance, all we have to obtain threshold corrections of gauge couplings is to calculate two-point function of gauge fields thanks to gauge symmetry (that is, Ward-Takahashi identity). Now, let

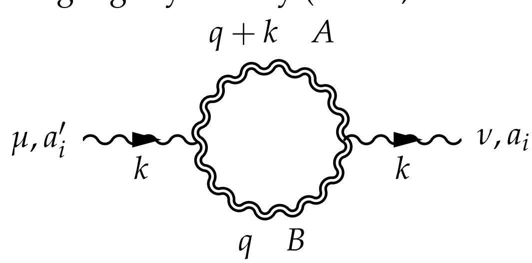


Figure 16: heavy gauge loop

us calculate heavy gauge 1-loop correction. In Fig. D.3, the doubly-waving lines explain the massive gauge bosons, and waving lines describe massless gauge boson. Fig. D.3 gives the contribution to the two-point gauge vertex as:

$$-\frac{1}{2}g^2 C_{BAa_i'}C_{BAa_i}\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q+k)^2-M_{V_A}^2}\frac{1}{q^2-M_{V_B}^2}$$
$$\times g^{\eta\sigma}g^{\xi\rho}\left\{2k_\rho g_{\mu\sigma}-2k_\sigma g_{\mu\rho}+(2q+k)_\mu g_{\rho\sigma}\right\}\left\{-2k_\xi g_{\nu\eta}+2k_\eta g_{\nu\xi}-(2q+k)_\nu g_{\xi\eta}\right\}. \tag{D.3.1}$$

where $1/2$ is a symmetry factor. By using the Feynman parameter, we find that this integral will change as the following form:

$$g^2 C_{BAa_i'}C_{BAa_i}\int_0^1 dx\int\frac{d^dq}{(2\pi)^d}\frac{1}{[(q+xk)^2-\Delta^2]^2}\left\{8(k^2g_{\mu\nu}-k_\mu k_\nu)+d'(2q+k)_\mu(2q+k)_\nu\right\}. \tag{D.3.2}$$

Here, d' means

$$d'=\begin{cases}4 & (\overline{\mathrm{DR}}\text{ scheme})\\ 4-\epsilon & (\overline{\mathrm{MS}}\text{ scheme})\end{cases} \tag{D.3.3}$$

We also define $\Delta^2\equiv x(x-1)k^2+xM_{V_B}^2+(1-x)M_{V_A}^2=x(x-1)k^2+M_{V_A}^2$. The second equality is justified since the 1-loop amplitude is symmetric under exchange A with B. We set loop momentum to be $P_\mu=q_\mu+xk_\mu$. By using loop integral formulae, we find this integral to be

$$\frac{ig^2}{(4\pi)^{d/2}}C_{BAa_i'}C_{BAa_i}\Gamma(2-d/2)$$
$$\times\int_0^1 dx\Delta^{d-4}\left\{\frac{4d}{d-2}g_{\mu\nu}[x(x-1)k^2+M_{V_A}^2]+8(k^2g_{\mu\nu}-k_\mu k_\nu)+d'(1-2x)^2k_\mu k_\nu\right\}. \tag{D.3.4}$$

Since we have set $d=4-\epsilon$, the gauge coupling constant has a mass dimension $[\text{mass}]=\epsilon/2$. ,we introduce parameter μ which has mass dimension, $g=\mu^{\epsilon/2}\tilde{g}$

$$\frac{i}{(4\pi)^2}\tilde{g}^2C_{BAa_i'}C_{BAa_i}\int_0^1 dx\left(\frac{2}{\epsilon}+\ln 4\pi-\gamma\right)\left(f_{\mu\nu}(k,x)+8g_{\mu\nu}M_{V_A}^2\right)$$
$$+\frac{i}{(4\pi)^2}\tilde{g}^2C_{BAa_i'}C_{BAa_i}\int_0^1 dx\ln\frac{\mu^2}{\Delta^2}\left(f_{\mu\nu}(k,x)+8g_{\mu\nu}M_{V_A}^2\right)$$
$$+2\frac{i}{(4\pi)^2}\tilde{g}^2C_{BAa_i'}C_{BAa_i}\int_0^1 dx\left[2[x(x-1)k^2+M_{V_A}^2]g_{\mu\nu}-(1-2x)^2k_\mu k_\nu\right] \tag{D.3.5}$$

where $f_{\mu\nu}(x,k)$ is the function has the form

$$f_{\mu\nu}(x,k)=8g_{\mu\nu}x(x-1)k^2+8(k^2g_{\mu\nu}-k_\mu k_\nu)+4(1-2x)^2k_\mu k_\nu. \tag{D.3.6}$$

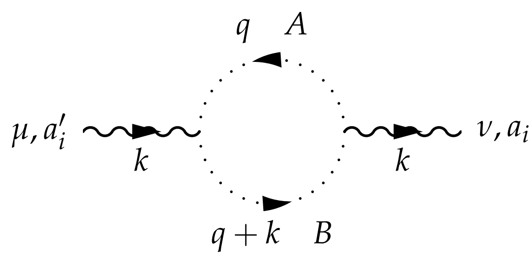


Figure 17: Massive ghost loop

And, integral of this function $f_{\mu\nu}$ is as below;

$$\int_0^1 dx f_{\mu\nu}(x,k) = \frac{20}{3}(k^2 g_{\mu\nu} - k_\mu k_\nu). \tag{D.3.7}$$

And expanding the term $\ln \Delta^2$ as below;

$$\ln \Delta^2 = \ln M_{V_A}^2 + \ln \left(1 + \frac{x(x-1)k^2}{M_{V_A}^2}\right) \sim \ln M_{V_A}^2 + \frac{x(x-1)k^2}{M_{V_A}^2} \tag{D.3.8}$$

Since this diagram has the gauge (G_i) invariance, the gauge variant term will vanish with the gauge-gauge-massive gauge-massive gauge interaction.

$$\begin{aligned} &= i(k^2 g_{\mu\nu} - k_\mu k_\nu)\tilde{g}^2 C_{BAa_i'} C_{BAa_i} \frac{1}{48\pi^2} \cdot 20 \left[\frac{1}{\epsilon} + \frac{1}{2}\ln 4\pi - \frac{1}{2}\gamma + \ln \frac{\mu}{M_{V_A}}\right] \\ &\qquad + i\tilde{g}^2 C_{BAa_i'} C_{BAa_i} \frac{1}{48\pi^2} \left(k^2 g_{\mu\nu} - k_\mu k_\nu\right) + \mathcal{O}(k^4, \epsilon, M_{V_A}^2) \end{aligned} \tag{D.3.9}$$

Similarly if we set $d' = 4$, we will find the contribution from the integral of Eq. (D.3.4) as follows:

$$i(k^2 g_{\mu\nu} - k_\mu k_\nu)\tilde{g}^2 C_{BAa_i'} C_{BAa_i} \frac{1}{48\pi^2} \cdot 20 \left[\frac{1}{\epsilon} + \frac{1}{2}\ln 4\pi - \frac{1}{2}\gamma + \ln \frac{\mu}{M_{V_A}}\right] + \mathcal{O}(k^4, \epsilon, M_{V_A}^2). \tag{D.3.10}$$

Next, we consider the contribution from the massive ghost loop. The internal momentum and the generator indices are assigned as Fig. 17.

$$-g^2 C_{BAa_i'} C_{BAa_i} \int \frac{d^d q}{(2\pi)^d} (-k-2q)_\mu \frac{i}{(q+k)^2 - M_{V_A}^2} \frac{i}{q^2 - M_{V_B}^2} (2q+k)_\nu \tag{D.3.11}$$

where (-1) is multiplied because of the ghost loop. Then, we carry out this integral we find this value will be as:

$$-g^2 C_{BAa'_i} C_{BAa_i} \int_0^1 dx \frac{i\Delta^{d-4}}{(4\pi)^{d/2}} \left[g^{\mu\nu} \frac{4d/d'}{d-2} (k^2 x(x-1) + M_{V_A}^2) + (1-2x)^2 k^\mu k^\nu \right] \Gamma(2-d/2) \tag{D.3.12}$$

In the $\overline{\mathrm{MS}}$ scheme, we set $d = d' = 4 - \epsilon$ and $g^2 = \mu^\epsilon \tilde{g}^2$,

$$\begin{aligned} &-\frac{i\tilde{g}^2}{(4\pi)^2} C_{BAa'_i} C_{BAa_i} \int_0^1 dx \left\{ \frac{2}{\epsilon} \left[2g^{\mu\nu}(k^2 x(x-1) + M_{V_A}^2) + (1-2x)^2 k^\mu k^\nu \right] \right. \\ &\left. + \left(\ln \frac{4\pi}{\Delta^2} - \gamma + \ln \mu^2 \right) \left[2g^{\mu\nu}(k^2 x(x-1) + M_{V_A}^2) + (1-2x)^2 k^\mu k^\nu \right] + 2g^{\mu\nu}(k^2 x(x-1) + M_{V_A}^2) \right\} \end{aligned} \tag{D.3.13}$$

Note that the terms which are not gauge invariant cancel out with the amplitude generated from the four-point interaction as in the right figure of . Since we need the leading term of k, we expand the logarithm "$\ln \Delta^2$" as before. Finally, this contribution from the ghost loop is obtained as the following form in the $\overline{\mathrm{MS}}$ scheme:

$$\frac{2i\tilde{g}^2}{48\pi^2} C_{BAa'_i} C_{BAa_i} \left(k^2 g^{\mu\nu} - k^\mu k^\nu \right) \left(\frac{1}{\epsilon} + \frac{1}{2} \ln 4\pi - \frac{1}{2}\gamma + \ln \frac{\mu}{M_{V_A}} \right) + \mathcal{O}(k^4, \epsilon, M_{V_A}^2) \tag{D.3.14}$$

In the $\overline{\mathrm{DR}}$ scheme, we set $d = 4 - \epsilon, d' = 4$, and $g^2 = \mu^\epsilon \tilde{g}^2$. We obtain the contribution from the ghost loop in the $\overline{\mathrm{DR}}$ scheme by means of the same procedure as before:

$$\frac{2i\tilde{g}^2}{48\pi^2} C_{BAa'_i} C_{BAa_i} \left(k^2 g^{\mu\nu} - k^\mu k^\nu \right) \left(\frac{1}{\epsilon} + \frac{1}{2} \ln 4\pi - \frac{1}{2}\gamma + \ln \frac{\mu}{M_{V_A}} \right) + \mathcal{O}(k^4, \epsilon, M_{V_A}^2) \tag{D.3.15}$$

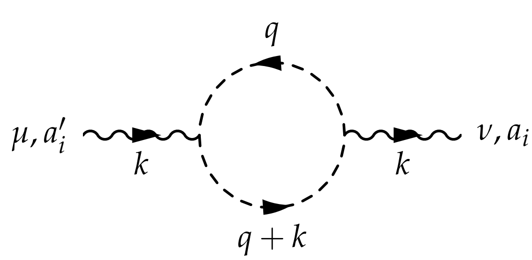


Figure 18: scalar loop

We also obtain the scalar amplitude with the similar procedure which we used for the ghost one. So, the contribution from Fig. 18 is obtained as:

$$= -\frac{i\tilde{g}^2}{48\pi^2} (t_S)_{ij} (t_S)_{ij} \left(k^2 g^{\mu\nu} - k^\mu k^\nu \right) \left(\frac{1}{\epsilon} + \frac{1}{2} \ln 4\pi - \frac{1}{2}\gamma + \ln \frac{\mu}{M_{V_A}} \right) + \mathcal{O}(k^4, \epsilon, M_{V_A}^2) \tag{D.3.16}$$

In particular, in case of Nambu-Goldstone (NG) boson, we replace the generator with that of adjoint representation

$$C_{a'AB} = [t_S^{a'}]_{AB}. \tag{D.3.17}$$

Then, we also obtain the contribution from NG boson is obtained as:

$$= -\frac{i\tilde{g}^2}{48\pi^2} C_{a_i' AB} C_{a_i AB} \left(k^2 g^{\mu\nu} - k^\mu k^\nu\right) \left(\frac{1}{\epsilon} + \frac{1}{2}\ln 4\pi - \frac{1}{2}\gamma + \ln\frac{\mu}{M_{V_A}}\right) + \mathcal{O}(k^4, \epsilon, M_{V_A}^2) \tag{D.3.18}$$

Combining the informations above, we obtain the two-point correlation function which includes the contributions of the massive gauge boson, the massive ghost, the massive NG boson, the massive Dirac fermion and massive real scalar. What we need to do is to treat only external momentum dependent terms since the momentum independent contributions are cancelled with the amplitude generated from the four-point interaction of massive gauge bosons and massless gauge bosons.

In the $\overline{\text{MS}}$ scheme, this contribution is obtained as follows:

$$\begin{aligned} -l_{\overline{\text{MS}}} \equiv \frac{i\tilde{g}^2}{48\pi^2} \Bigg\{ & \frac{1}{\epsilon'} \left[20 \mathrm{Tr} T_{V_A} T_{V_A} + 2 \mathrm{Tr} T_{V_{\text{ghost}}} T_{V_{\text{ghost}}} - \mathrm{Tr} T_{V_{\text{NG}}} T_{V_{\text{NG}}} \right] + \mathrm{Tr} T_{V_A} T_{V_A} \\ & + \left[20 \mathrm{Tr} T_{V_A} T_{V_A} \ln\frac{\mu}{M_{V_A}} + 2 \mathrm{Tr} T_{V_{\text{ghost}}} T_{V_{\text{ghost}}} \ln\frac{\mu}{M_{\text{ghost}}} - \mathrm{Tr} T_{V_{\text{NG}}} T_{V_{\text{NG}}} \ln\frac{\mu}{M_{\text{NG}}} \right] \Bigg\} \end{aligned} \tag{D.3.19}$$

where we set

$$\frac{1}{\epsilon'} \equiv \frac{1}{\epsilon} - \frac{1}{2}\gamma + \ln\sqrt{4\pi}. \tag{D.3.20}$$

In the $\overline{\text{DR}}$ scheme, this contribution is also obtained as follows:

$$\begin{aligned} -l_{\overline{\text{DR}}} \equiv \frac{i\tilde{g}^2}{48\pi^2} \Bigg\{ & \frac{1}{\epsilon'} \left[20 \mathrm{Tr} T_{V_A} T_{V_A} + 2 \mathrm{Tr} T_{V_{\text{ghost}}} T_{V_{\text{ghost}}} - \mathrm{Tr} T_{V_{\text{NG}}} T_{V_{\text{NG}}} \right] \\ & + \left[20 \mathrm{Tr} T_{V_A} T_{V_A} \ln\frac{\mu}{M_{V_A}} + 2 \mathrm{Tr} T_{V_{\text{ghost}}} T_{V_{\text{ghost}}} \ln\frac{\mu}{M_{\text{ghost}}} - \mathrm{Tr} T_{V_{\text{NG}}} T_{V_{\text{NG}}} \ln\frac{\mu}{M_{\text{NG}}} \right] \Bigg\} \end{aligned} \tag{D.3.21}$$

Note that the difference between the $\overline{\text{DR}}$ scheme and the $\overline{\text{MS}}$ scheme is whether the constant term which is proportional to $\mathrm{Tr} T_{V_A} T_{V_A}$ exist or not. In the effective theory after integrating out the massive particles, it is needed to match an amplitude with one in full theory.

Therefore, the effective kinetic terms for gauge fields must be formed as the following form:

$$-\sum_i \frac{1}{4}(1-l_i)\tilde{F}^{\mu\nu}_{a_i}\tilde{F}_{a_i\mu\nu}. \tag{D.3.22}$$

In order to obtain canonical kinetic terms, we have to redefine the gauge fields and gauge couplings in effective theory as:

$$\begin{aligned} A'_{a_i\mu} &\equiv \sqrt{1-l_i}A_{a_i\mu} \\ g_i &\equiv g/\sqrt{1-l_i} \end{aligned} \tag{D.3.23}$$

where $A_{a_i\mu}$ and g are gauge fields and gauge coupling in full theory, respectively. We obtain the relation between bare couplings in full theory and effective theory. We also find easily the renormalized couplings in each theories.

$$\begin{aligned} g\mu^{\epsilon/2} &= g(\mu) - b_G g^3(\mu)/\epsilon' + \dots \\ g_i\mu^{\epsilon/2} &= g_i(\mu) - b_i g_i^3(\mu)/\epsilon' + \dots \end{aligned} \tag{D.3.24}$$

To understand the relation between the renormalized couplings at 1-loop level, Eq. (D.3.21) and Eq. (D.3.19) are divided into the finite part λ_i and the divergent part λ'_i

$$l_i = g_i^2(\lambda_i(\mu) + \lambda'_i(\mu)/\epsilon'). \tag{D.3.25}$$

By using Eq. (D.3.24), we obtain this relation as:

$$\alpha_i^{-1}(\mu) = \alpha^{-1}(\mu) - 4\pi\lambda_i(\mu) \tag{D.3.26}$$

where $\alpha_i(\mu) = g_i^2(\mu)/4\pi, \alpha(\mu) = g^2(\mu)/4\pi$. Thus, the threshold correction in the $\overline{\text{MS}}$ scheme is

$$\lambda_i^{\overline{\text{MS}}}(\mu) = \frac{1}{48\pi^2}\left[-\text{Tr}\left(t_{iV}^2\right) - 21\text{Tr}\left(t_{iV}^2 \ln\frac{M_V}{\mu}\right) + 8\text{Tr}\left(t_{iF}^2\ln\frac{M_F}{\mu}\right) + \text{Tr}\left(t_{iS}^2\ln\frac{M_S}{\mu}\right)\right]. \tag{D.3.27}$$

What is more, the threshold correction in the $\overline{\text{DR}}$ scheme is

$$\lambda_i^{\overline{\text{DR}}}(\mu) = \frac{1}{48\pi^2}\left[-21\text{Tr}\left(t_{iV}^2 \ln\frac{M_V}{\mu}\right) + 8\text{Tr}\left(t_{iF}^2\ln\frac{M_F}{\mu}\right) + \text{Tr}\left(t_{iS}^2\ln\frac{M_S}{\mu}\right)\right]. \tag{D.3.28}$$

We also obtain the relation between the gauge couplings in the $\overline{\text{MS}}$ scheme and the $\overline{\text{DR}}$ scheme. This relation is able to be defined as the following form since the difference between the $\overline{\text{MS}}$ and the $\overline{\text{DR}}$ gauge couplings is only the constant terms.

$$\frac{1}{\alpha_i^{\overline{\text{DR}}}(\mu)} = \frac{1}{\alpha_i^{\overline{\text{MS}}}(\mu)} - \frac{C_i}{12\pi}. \tag{D.3.29}$$

Similarly, for the unified gauge couplings, the relation is defined as

$$\frac{1}{\alpha_G^{\overline{\mathrm{DR}}}(\mu)} = \frac{1}{\alpha_G^{\overline{\mathrm{MS}}}(\mu)} - \frac{C_G}{12\pi}. \tag{D.3.30}$$

Since the gauge fields belong to adjoint representation of gauge group, the threshold corrections in each scheme are related by

$$\lambda_G^{\overline{\mathrm{DR}}} = \lambda_G^{\overline{\mathrm{MS}}} - \frac{1}{48\pi^2}\mathrm{Tr}t_{GV}^2, \quad \lambda_i^{\overline{\mathrm{DR}}} = \lambda_i^{\overline{\mathrm{MS}}} - \frac{1}{48\pi^2}\mathrm{Tr}t_{G_iV}^2. \tag{D.3.31}$$

First equation means $C_G = \mathrm{Tr}t_{GV}^2$. For instance, if the unified gauge group is $SU(N)$, $C_G = N$. The relation between residual gauge couplings and the unified coupling is obtained above,

$$\frac{1}{\alpha_i^{\overline{\mathrm{DR}}}(\mu)} = \frac{1}{\alpha_G^{\overline{\mathrm{DR}}}(\mu)} - 4\pi\lambda_i^{\overline{\mathrm{DR}}}(\mu). \tag{D.3.32}$$

The relation of gauge couplings in $\overline{\mathrm{MS}}$ scheme is also obtained by using Eq. (D.3.29), Eq. (D.3.30) and Eq. (D.3.31).

$$\frac{1}{\alpha_i^{\overline{\mathrm{MS}}}(\mu)} = \frac{1}{\alpha_G^{\overline{\mathrm{MS}}}(\mu)} + \frac{1}{12\pi}(C_i - C_G) + \frac{1}{12\pi}\mathrm{Tr}t_{G_iV}^2 - 4\pi\lambda_i^{\overline{\mathrm{MS}}}(\mu). \tag{D.3.33}$$

In $\overline{\mathrm{MS}}$ scheme, however, the relation between the couplings is also determined as Eq. (D.3.32). Therefore, C_i and C_G should be satisfied the equation

$$\frac{1}{12\pi}(C_i - C_G) + \frac{1}{12\pi}\mathrm{Tr}t_{G_iV}^2 = 0 \tag{D.3.34}$$

For example, let us consider the case that the unified gauge group is SU(5) and the residual gauge group is the standard model one, $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. In this case, $C_G = 5$. In addition, massive gauge fields which are integrated out are transformed under the standard model gauge as $(\mathbf{3}, \mathbf{2}, 5/6)$. Therefore, for $i = 1, 2, 3$, we have

$$\begin{aligned}
&(C_1 - 5) + \left(\frac{5}{6}\right)^2 \times 2 \times 2 \times 3 \times \frac{3}{5} = 0 \\
&(C_2 - 5) + \frac{1}{2} \times 2 \times 3 = 0 \\
&(C_3 - 5) + \frac{1}{2} \times 2 \times 2 = 0
\end{aligned} \tag{D.3.35}$$

Therefore, the relation between gauge couplings in $\overline{\text{MS}}$ and $\overline{\text{DR}}$ scheme is obtained as follows:

$$
\begin{aligned}
\frac{1}{\alpha_S^{\overline{\text{DR}}}(\mu)} &= \frac{1}{\alpha_S^{\overline{\text{MS}}}(\mu)} - \frac{1}{4\pi} \\
\frac{1}{\alpha_2^{\overline{\text{DR}}}(\mu)} &= \frac{1}{\alpha_2^{\overline{\text{MS}}}(\mu)} - \frac{1}{6\pi} \\
\frac{1}{\alpha_1^{\overline{\text{DR}}}(\mu)} &= \frac{1}{\alpha_1^{\overline{\text{MS}}}(\mu)}
\end{aligned}
\tag{D.3.36}
$$

E Renormalization Group Equations

In this appendix, we give the renormalization effects for some scales. At first, the renormalization group equations (RGEs) for the gauge couplings and Yukawa couplings are displayed in SUSY case and non-SUSY case at 2-loop level. Next, we will give the short-distance renormalization effects of the Wilsonian coefficients including those of Yukawa couplings; "short-distance" means the high-energy region between the GUT scale and the electroweak scale. Finally, the QCD corrections (which is the renormalization effects between the electroweak scale and 2 GeV) are given.

E.1 RGEs for gauge couplings and Yukawa couplings

To understand the behavior of the couplings including quantum corrections at high-energy scale, we need to run these couplings into this scale. By using RGEs, we find these running couplings at high-energy scale. In general, 1-loop renormalization group equations for gauge couplings are given by

$$\beta_g = \frac{\mathrm{d}g}{\mathrm{d}\ln\mu} = \frac{g^3}{16\pi^2}\left[-\frac{11}{3}t_2(V) + \sum_{\text{Dirac Fermions}}\frac{4}{3}t_2(F) + \sum_{\text{complex scalars}}\frac{1}{3}t_2(S)\right] \tag{E.1.1}$$

where a non-abelian gauge boson couples to Dirac Fermions and complex scalars. t_2 denotes the Casimir invariant defined as $\mathrm{Tr}[T^A T^B] \equiv t_2\delta^{AB}$. In the supersymmetric notation, Weyl fermions including standard model fermion and gauginos couple to gauge boson. Therefore, in the supersymmetric theories, this 1-loop RGEs are modified as follows:

$$\beta_g = \frac{\mathrm{d}g}{\mathrm{d}\ln\mu} = \frac{g^3}{16\pi^2}\left[-3t_2(V) + \sum_{\Phi} t_2(\Phi)\right] \tag{E.1.2}$$

because there are Weyl fermions as gaugino and the complex scalars as superpartners for the standard model fermions. Note that, in the $\mathcal{N}=1$ supersymmetric theories, holomorphic gauge couplings (these are not physical couplings) are exactly determined by this 1-loop RGEs because of the non-renormalization theorem [95].

In our analysis, we have used the RGEs at 2-loop level. The RGEs for the gauge couplings at 2-loop level are as follows [94, 102]:

$$\frac{\mathrm{d}g_i}{\mathrm{d}\ln\mu} = \frac{g_i}{16\pi^2}\left[b_i g_i^2 + \frac{1}{16\pi^2}\left(\sum_j b_{ij} g_i^2 g_j^2 - \sum_{j=U,D,E} a_{ij} g_i^2 \mathrm{Tr}[\mathbf{Y}_j \mathbf{Y}_j^\dagger]\right)\right]. \tag{E.1.3}$$

$\mathbf{Y}_U = \mathbf{U}, \mathbf{Y}_D = \mathbf{D}, \mathbf{Y}_E = \mathbf{E}$ are the Yukawa couplings matrices in the SM. Up to the gaugino threshold, the particle content includes only the SM particles. Thus, the corresponding coefficients are given as;

$$b_{ij} = \begin{pmatrix} 199/50 & 27/10 & 44/5 \\ 9/10 & 35/6 & 12 \\ 11/10 & 9/2 & -26 \end{pmatrix}, b_i = (41/10, -19/6, -7), a_{ij} = \begin{pmatrix} 17/10 & 1/2 & 3/2 \\ 3/2 & 3/2 & 1/2 \\ 2 & 2 & 0 \end{pmatrix}. \tag{E.1.4}$$

Above the gaugino threshold, we should consider contributions from gauginos. These are calculated as gauginos are the fermions belonging to the adjoint representation in $SU(N)$.

$$\frac{g^3}{(4\pi)^2}\left(-\frac{4}{3}\cdot\frac{1}{2}N\right) - \frac{g^5}{(4\pi)^4}\left[-\frac{1}{2}\left(4N + \frac{20}{3}N\right)\right] N = \begin{cases} \dfrac{g^3}{(4\pi)^2}\cdot\dfrac{4}{3} + \dfrac{g^5}{(4\pi)^4}\cdot\dfrac{64}{3} & (SU(2)) \\ \dfrac{g^3}{(4\pi)^2}\cdot 2 + \dfrac{g^5}{(4\pi)^4}\cdot 48 & (SU(3)) \end{cases} \tag{E.1.5}$$

From the gaugino threshold, the coefficients in the RGEs can be changed into

$$\begin{aligned} b_{ij} &= \begin{pmatrix} 199/50 & 27/10 & 44/5 \\ 9/10 & 35/6 & 12 \\ 11/10 & 9/2 & -26 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 64/3 & 0 \\ 0 & 0 & 48 \end{pmatrix} \\ b_i &= (41/10, -19/6, -7) + (0, 4/3, 2). \end{aligned} \tag{E.1.6}$$

In addition, in the Standard Model region, we have the 1-loop RGEs for the Yukawa matrices as follows*.

$$\begin{aligned} \frac{d\mathbf{U}}{d\ln\mu} &= \frac{1}{16\pi^2}\left[-\sum_i c_i^{\mathrm{SM}} g_i^2 + \frac{3}{2}\mathbf{U}\mathbf{U}^\dagger - \frac{3}{2}\mathbf{D}\mathbf{D}^\dagger + Y_2(S)\right]\mathbf{U} \\ \frac{d\mathbf{D}}{d\ln\mu} &= \frac{1}{16\pi^2}\left[-\sum_i c_i'^{\mathrm{SM}} g_i^2 + \frac{3}{2}\mathbf{D}\mathbf{D}^\dagger - \frac{3}{2}\mathbf{U}\mathbf{U}^\dagger + Y_2(S)\right]\mathbf{D} \\ \frac{d\mathbf{E}}{d\ln\mu} &= \frac{1}{16\pi^2}\left[-\sum_i c_i''^{\mathrm{SM}} g_i^2 + \frac{3}{2}\mathbf{E}\mathbf{E}^\dagger + Y_2(S)\right]\mathbf{E} \end{aligned} \tag{E.1.7}$$

In these RGEs, the coefficients of the square of the gauge couplings are calculated as;

$$c_i^{\mathrm{SM}} = \left(\frac{17}{20}, \frac{9}{4}, 8\right), \quad c_i'^{\mathrm{SM}} = \left(\frac{1}{4}, \frac{9}{4}, 8\right), \quad c_i''^{\mathrm{SM}} = \left(\frac{9}{4}, \frac{9}{4}, 0\right). \tag{E.1.8}$$

*In our calculation, we need the RGEs for the gauge couplings at 2-loop level. It is sufficient to take into account the Yukawa couplings at 1-loop level since these are not appeared until we treat the 2-loop level RGEs for the gauge couplings.

The scalar function Y_2 is the trace of the squared Yukawa coupling matrices;

$$Y_2(S) = \mathrm{Tr}\left[3\mathbf{U}\mathbf{U}^\dagger + 3\mathbf{D}\mathbf{D}^\dagger + \mathbf{E}\mathbf{E}^\dagger\right]. \tag{E.1.9}$$

When we estimate the mass of the Higgs boson in the high-scale SUSY scenario, we use the RGE for the quartic coupling. This RGE is given by,

$$\frac{\mathrm{d}\lambda}{\mathrm{d}\ln\mu} = \frac{1}{16\pi^2}\left[12\lambda^2 - \left(\frac{9}{5}g_1^2 + g_2^2\right)\lambda + \frac{9}{4}\left(\frac{3}{25}g_1^4 + \frac{2}{5}g_1^2g_2^2 + g_2^2\right) + 4Y_2(S)\lambda - 4H(S)\right], \tag{E.1.10}$$

where

$$\begin{aligned} Y_2(S) &\equiv \mathrm{Tr}\left[3\mathbf{U}\mathbf{U}^\dagger + 3\mathbf{D}\mathbf{D}^\dagger + \mathbf{E}\mathbf{E}^\dagger\right], \\ H(S) &\equiv \mathrm{Tr}\left[3(\mathbf{U}\mathbf{U}^\dagger)^2 + 3(\mathbf{D}\mathbf{D}^\dagger)^2 + (\mathbf{E}\mathbf{E}^\dagger)^2\right]. \end{aligned} \tag{E.1.11}$$

Finally, above the SUSY breaking scale, the particle content includes the SM particles, the extra Higgs doublet, and these superpartners. In this region, the RGEs for the gauge coupling are obtained as

$$\frac{\mathrm{d}g_i}{\mathrm{d}\ln\mu} = \frac{g_i}{16\pi^2}\left[b_ig_i^2 + \frac{1}{16\pi^2}\left(\sum_j b_{ij}g_i^2g_j^2 - \sum_{j=\tilde{U},\tilde{D},\tilde{E}} a_{ij}g_i^2\mathrm{Tr}[\widetilde{\mathbf{Y}}_j\widetilde{\mathbf{Y}}_j^\dagger]\right)\right]. \tag{E.1.12}$$

And the corresponding coefficients is calculated as

$$b_{ij} = \begin{pmatrix} 199/25 & 27/5 & 88/5 \\ 9/5 & 25 & 24 \\ 11/5 & 9 & 14 \end{pmatrix}, b_i = (33/5, 1, -3), a_{ij} = \begin{pmatrix} 26/5 & 14/5 & 18/5 \\ 6 & 6 & 2 \\ 4 & 4 & 0 \end{pmatrix}. \tag{E.1.13}$$

The boundary condition is what relates Y_j in the SM and $\tilde{Y}_j$ in the MSSM at the SUSY breaking scale.

$$\begin{aligned} \widetilde{U}(M_S) &= \frac{1}{\sin\beta}U(M_S) \\ \widetilde{Y}_j(M_S) &= \frac{1}{\cos\beta}Y_j(M_S) \qquad (j = D, E) \end{aligned} \tag{E.1.14}$$

where M_S is SUSY breaking scale. In the MSSM region, we have the 1-loop RGEs for the

Yukawa matrices as follows.

$$\begin{aligned}
\frac{\mathrm{d}\widetilde{\mathbf{U}}}{\mathrm{d}\ln\mu} &= \frac{1}{16\pi^2}\left[-\sum_i c_i^{\mathrm{MSSM}} g_i^2 + 3\widetilde{\mathbf{U}}\widetilde{\mathbf{U}}^\dagger + \widetilde{\mathbf{D}}\widetilde{\mathbf{D}}^\dagger + \mathrm{Tr}(3\widetilde{\mathbf{U}}\widetilde{\mathbf{U}}^\dagger)\right]\widetilde{\mathbf{U}} \\
\frac{\mathrm{d}\widetilde{\mathbf{D}}}{\mathrm{d}\ln\mu} &= \frac{1}{16\pi^2}\left[-\sum_i c_i'^{\mathrm{MSSM}} g_i^2 + 3\widetilde{\mathbf{D}}\widetilde{\mathbf{D}}^\dagger + \widetilde{\mathbf{U}}\widetilde{\mathbf{U}}^\dagger + \mathrm{Tr}(3\widetilde{\mathbf{D}}\widetilde{\mathbf{D}}^\dagger + \widetilde{\mathbf{E}}\widetilde{\mathbf{E}}^\dagger)\right]\widetilde{\mathbf{D}} \\
\frac{\mathrm{d}\widetilde{\mathbf{E}}}{\mathrm{d}\ln\mu} &= \frac{1}{16\pi^2}\left[-\sum_i c_i''^{\mathrm{MSSM}} g_i^2 + 3\widetilde{\mathbf{E}}\widetilde{\mathbf{E}}^\dagger + \mathrm{Tr}(3\widetilde{\mathbf{D}}\widetilde{\mathbf{D}}^\dagger + \widetilde{\mathbf{E}}\widetilde{\mathbf{E}}^\dagger)\right]\widetilde{\mathbf{E}}
\end{aligned} \tag{E.1.15}$$

In these RGEs, the coefficients of the square of the gauge couplings are calculated as;

$$c_i^{\mathrm{MSSM}} = \left(\frac{13}{15}, 3, \frac{16}{3}\right), \quad c_i'^{\mathrm{MSSM}} = \left(\frac{7}{15}, 3, \frac{16}{3}\right), \quad c_i''^{\mathrm{MSSM}} = \left(\frac{9}{5}, 3, 0\right). \tag{E.1.16}$$

E.2 RGEs for Wilson coefficients as short-distance renormalization effects

In this subsection, we introduce the RGEs for the Wilson coefficients of the dimension-five operators and the four-fermi operators.

First, in order to calculate RGEs for Wilson coefficients of dimension-five operators, we need to only calculate the anomalous dimensions of each external fields because of non-renormalization theorem. First, since the RGEs for the Yukawa couplings are obtained by using the same procedure, we derive them. For the superpotential defined as

$$W = \frac{1}{3!} y^{ijk}\Phi_i\Phi_j\Phi_k, \tag{E.2.1}$$

the β function for this coupling is given by

$$\frac{\mathrm{d}y^{ijk}}{\mathrm{d}\ln\mu} = \gamma^i_n y^{njk} + \gamma^j_n y^{ink} + \gamma^k_n y^{ijn}. \tag{E.2.2}$$

where γ^i_j is the anomalous dimension for chiral superfield Φ_i. The Yukawa terms in MSSM are given by

$$W_{\mathrm{Yukawa}} = Y^u_{ij} Q_i U^C_j H_f + Y^d_{ij} Q_i D^C_j \overline{H}_f + Y^e_{ij} L_i E^C_j \overline{H}_f. \tag{E.2.3}$$

These terms give rise to anomalous dimensions for each external fields $\gamma_i = -\mathrm{d}\ln Z_i/\mathrm{d}\ln\mu$

where Z_i is the wave-function renormalization. For quark chiral superfields, we have

$$\begin{aligned}
(\gamma_Q)_i^n &= \frac{1}{16\pi^2}\left[Y_{ij}^u(Y^{u\dagger})^{jn} + Y_{ij}^d(Y^{d\dagger})^{jn} - \left(\frac{8}{3}g_3^2 + \frac{3}{2}g_2^2 + \frac{1}{30}g_1^2\right)\delta_i^n\right],\\
(\gamma_U)_i^n &= \frac{1}{16\pi^2}\left[2Y_{ij}^u(Y^{u\dagger})^{jn} - \left(\frac{8}{3}g_3^2 + \frac{8}{15}g_1^2\right)\delta_i^n\right],\\
(\gamma_D)_i^n &= \frac{1}{16\pi^2}\left[2Y_{ij}^d(Y^{d\dagger})^{jn} - \left(\frac{8}{3}g_3^2 + \frac{2}{15}g_1^2\right)\delta_i^n\right].
\end{aligned} \tag{E.2.4}$$

where g_1, g_2 and g_3 are gauge couplings for $U(1)_Y, SU(2)_L$ and $SU(3)_C$, respectively. $i, j, \cdots$ denote the flavor indices. Then, for lepton chiral superfields, we have anomalous dimensions for these external line:

$$\begin{aligned}
(\gamma_L)_i^n &= \frac{1}{16\pi^2}\left[Y_{ij}^e(Y^{e\dagger})^{jn} - \left(\frac{3}{2}g_2^2 + \frac{3}{10}g_1^2\right)\delta_i^n\right],\\
(\gamma_E)_i^n &= \frac{1}{16\pi^2}\left[2Y_{ij}^e(Y^{e\dagger})^{jn} - \frac{6}{5}g_1^2\delta_i^n\right].
\end{aligned} \tag{E.2.5}$$

For these anomalous dimensions, $i, j, \cdots$ are the flavor indices. Finally, for Higgs chiral superfields, we have anomalous dimensions for these external line

$$\begin{aligned}
(\gamma_{H_f}) &= \frac{1}{16\pi^2}\left[3Y_{ij}^u(Y^{u\dagger})^{ji} - \left(\frac{3}{2}g_2^2 + \frac{3}{10}g_1^2\right)\right],\\
(\gamma_{\overline{H}_f}) &= \frac{1}{16\pi^2}\left[3Y_{ij}^d(Y^{d\dagger})^{ji} + Y_{ij}^e(Y^{e\dagger})^{jn} - \left(\frac{3}{2}g_2^2 + \frac{3}{10}g_1^2\right)\right]
\end{aligned} \tag{E.2.6}$$

The RGEs of Yukawa couplings are given by combining these contributions.

$$\begin{aligned}
\frac{\mathrm{d}Y_{ij}^u}{\mathrm{d}\ln\mu} &= \frac{1}{16\pi^2}\left[3(Y^uY^{u\dagger})_i^n + Y_{ik}^d(Y^{d\dagger})^{kn} + \left(3\mathrm{Tr}Y^uY^{u\dagger} - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2\right)\delta_i^n\right]Y_{nj}^u\\
\frac{\mathrm{d}Y_{ij}^d}{\mathrm{d}\ln\mu} &= \frac{1}{16\pi^2}\left[(Y^uY^{u\dagger})_i^n + 3(Y^dY^{d\dagger})_i^n + \left(3\mathrm{Tr}Y^dY^{d\dagger} + \mathrm{Tr}Y^eY^{e\dagger} - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2\right)\delta_i^n\right]Y_{nj}^d\\
\frac{\mathrm{d}Y_{ij}^e}{\mathrm{d}\ln\mu} &= \frac{1}{16\pi^2}\left[2(Y^eY^{e\dagger})_i^n + \left(3\mathrm{Tr}Y^dY^{d\dagger} + \mathrm{Tr}Y^eY^{e\dagger} - 3g_2^2 - \frac{9}{5}g_1^2\right)\right]Y_{nj}^e
\end{aligned} \tag{E.2.7}$$

These RGEs are consistent with the results as mentioned above.

Next, let us consider the RGEs for the Wilson coefficients of dimension-five operators. For dimension-five operators, the superpotential is given by

$$W_5 = C_{iijk}^L(Q_iQ_i)(Q_jL_k) + C_{ijkl}^R U_i^C E_j^C U_k^C D_l^C. \tag{E.2.8}$$

RGEs for these Wilson coefficients are easily obtained due to non-renormalization theorem.

$$
\begin{aligned}
\frac{\mathrm{d}C^L_{iijk}}{\mathrm{d}\ln\mu} &= \gamma^n_i C^L_{nijk} + \gamma^n_i C^L_{injk} + \gamma^n_j C^L_{iink} + \gamma^n_k C^L_{iijn} \\
\frac{\mathrm{d}C^R_{ijkl}}{\mathrm{d}\ln\mu} &= \gamma^n_i C^R_{njkl} + \gamma^n_j C^R_{inkl} + \gamma^n_k C^R_{ijnl} + \gamma^n_l C^R_{ijkn}
\end{aligned} \tag{E.2.9}
$$

where γ^i_n denotes, of course, anomalous dimension of chiral superfield. This RGE for LLLL operator is obtained as:

$$
\begin{aligned}
\frac{\mathrm{d}C^L_{iijk}}{\mathrm{d}\ln\mu} = \frac{1}{16\pi^2}\Big[2\left((Y^uY^{u\dagger})^i_i + (Y^dY^{d\dagger})^i_i\right) \\
+ (Y^uY^{u\dagger})^j_j + (Y^dY^{d\dagger})^j_j + (Y^eY^{e\dagger})^k_k - 8g_3^2 - 6g_2^2 - \frac{2}{5}g_1^2\Big] C^L_{iijk}.
\end{aligned} \tag{E.2.10}
$$

The RGE for the RRRR operators is obtained as the following form:

$$
\frac{\mathrm{d}C^R_{ijkl}}{\mathrm{d}\ln\mu} = \frac{1}{16\pi^2}\left[2(Y^uY^{u\dagger})^i_i + 2(Y^eY^{e\dagger})^j_j + 2(Y^uY^{u\dagger})^k_k + 2(Y^dY^{d\dagger})^l_l - 8g_3^2 - \frac{12}{5}g_1^2\right] C^R_{ijkl}. \tag{E.2.11}
$$

Next, we introduce the RGEs for the four-fermi operators based on [84]. The effective operators for nucleon decay are

$$
\begin{aligned}
\mathcal{O}^{(1)}_{ijkl} &= \epsilon_{abc}\epsilon_{rs}(d^a_{iR}u^b_{jR})(q^{cr}_{kL}l^s_{lL}), \\
\mathcal{O}^{(2)}_{ijkl} &= \epsilon_{abc}\epsilon_{rs}(q^{ar}_{iL}q^{bs}_{jL})(u^c_{kR}e_{lR}), \\
\mathcal{O}^{(3)}_{ijkl} &= \epsilon_{abc}\epsilon_{ru}\epsilon_{st}(q^{ar}_{iL}q^{bs}_{jL})(q^{ct}_{kL}l^u_{lL}), \\
\mathcal{O}^{(4)}_{ijkl} &= \epsilon_{abc}(d^a_{iR}u^b_{jR})(u^c_{kR}e_{lR}).
\end{aligned} \tag{E.2.12}
$$

For these operators, we obtain the relation between the bare and renormalized operators

$$
\begin{aligned}
\mathcal{O}^{(1)}_{0\ ijkl} &= \left[1 + \frac{1}{4\pi\epsilon}\left(2\alpha_S + \frac{9}{4}\alpha_2 + \frac{11}{20}\alpha_1\right)\right]\mathcal{O}^{(1)}_{ijkl} \\
\mathcal{O}^{(2)}_{0\ ijkl} &= \left[1 + \frac{1}{4\pi\epsilon}\left(2\alpha_S + \frac{9}{4}\alpha_2 + \frac{23}{20}\alpha_1\right)\right]\mathcal{O}^{(2)}_{ijkl} \\
\mathcal{O}^{(3)}_{0\ ijkl} &= \left[1 + \frac{1}{4\pi\epsilon}\left(2\alpha_S + \frac{3}{2}\alpha_2 + \frac{1}{10}\alpha_1\right)\right]\mathcal{O}^{(3)}_{ijkl} + \frac{2\alpha_2}{4\pi\epsilon}\left(\mathcal{O}^{(3)}_{jikl} + \mathcal{O}^{(3)}_{kjil} + \mathcal{O}^{(3)}_{ikjl}\right) \\
\mathcal{O}^{(4)}_{0\ ijkl} &= \left[1 + \frac{1}{4\pi\epsilon}\left(2\alpha_S + \frac{3}{5}\alpha_1\right)\right]\mathcal{O}^{(4)}_{ijkl} + \frac{2\alpha_1}{4\pi\epsilon}\mathcal{O}^{(4)}_{ikjl}
\end{aligned} \tag{E.2.13}
$$

where α_1 is the unified $U(1)$ gauge coupling.

Now we consider the effective Lagrangian for our calculations. For the $p \to K^+ + \bar{\nu}$ mode, we consider the operators:

$$\begin{aligned}
C^{(1)}_{1133}(\mu)\mathcal{O}_1(\mu) &\equiv C^{(1)}_{1133}(\mu)\epsilon_{abc}\epsilon_{rs}(d^a_{1R}u^b_{1R})(q^{cr}_{3L}l^s_{3L}),\\
C^{(1)}_{2133}(\mu)\mathcal{O}_2(\mu) &\equiv C^{(1)}_{2133}(\mu)\epsilon_{abc}\epsilon_{rs}(d^a_{2R}u^b_{1R})(q^{cr}_{3L}l^s_{3L}),\\
C^{(3)}_{221l}(\mu)\mathcal{O}_3(\mu) &\equiv C^{(3)}_{221l}(\mu)\epsilon_{abc}\epsilon_{rs}\epsilon_{tu}(q^{ar}_{2L}q^{bs}_{2L})(q^{ct}_{1L}l^u_{lL}),\\
C^{(3)}_{221l}(\mu)\mathcal{O}_4(\mu) &\equiv C^{(3)}_{221l}(\mu)\epsilon_{abc}\epsilon_{rs}\epsilon_{tu}(q^{ar}_{3L}q^{bs}_{3L})(q^{ct}_{1L}l^u_{lL}).
\end{aligned} \tag{E.2.14}$$

Then, we obtain the ratio of the Wilson coefficients between the SUSY braking scale M_S and the electroweak scale m_Z:

$$\begin{aligned}
\frac{C^{(1)}_{a133}(M_S)}{C^{(1)}_{a133}(m_Z)} &= \left(\frac{\alpha_S(m_Z)}{\alpha_S(M_S)}\right)^{-2/b_3}\left(\frac{\alpha_2(m_Z)}{\alpha_2(M_S)}\right)^{-9/4b_2}\left(\frac{\alpha_1(m_Z)}{\alpha_1(M_S)}\right)^{-11/20b_1}\\
\frac{C^{(3)}_{aabc}(M_S)}{C^{(3)}_{aabc}(m_Z)} &= \left(\frac{\alpha_S(m_Z)}{\alpha_S(M_S)}\right)^{-2/b_3}\left(\frac{\alpha_2(m_Z)}{\alpha_2(M_S)}\right)^{-15/2b_2}\left(\frac{\alpha_1(m_Z)}{\alpha_1(M_S)}\right)^{-1/10b_1}.
\end{aligned} \tag{E.2.15}$$

For the $p \to \pi^0 + e^+$ mode, we consider the operators:

$$\begin{aligned}
C^{(1)}_{1111}(\mu)\mathcal{O}^{(1)}_{ijkl} &= \epsilon_{abc}\epsilon_{rs}(d^a_{1R}u^b_{1R})(q^{cr}_{1L}l^s_{1L}),\\
C^{(2)}_{1111}(\mu)\mathcal{O}^{(2)}_{1111} &= \epsilon_{abc}\epsilon_{rs}(q^{ar}_{1L}q^{bs}_{1L})(u^c_{1R}e_{1R}).
\end{aligned} \tag{E.2.16}$$

Then, we obtain the ratio of the Wilson coefficients between the SUSY braking scale M_S and the electroweak scale m_Z:

$$\begin{aligned}
\frac{C^{(1)}_{1111}(M_S)}{C^{(1)}_{1111}(m_Z)} &= \left(\frac{\alpha_S(m_Z)}{\alpha_S(M_S)}\right)^{-2/b_3}\left(\frac{\alpha_2(m_Z)}{\alpha_2(M_S)}\right)^{-9/4b_2}\left(\frac{\alpha_1(m_Z)}{\alpha_1(M_S)}\right)^{-11/20b_1}\\
\frac{C^{(2)}_{1111}(M_S)}{C^{(2)}_{1111}(m_Z)} &= \left(\frac{\alpha_S(m_Z)}{\alpha_S(M_S)}\right)^{-2/b_3}\left(\frac{\alpha_2(m_Z)}{\alpha_2(M_S)}\right)^{-9/4b_2}\left(\frac{\alpha_1(m_Z)}{\alpha_1(M_S)}\right)^{-23/12b_1}.
\end{aligned} \tag{E.2.17}$$

E.3 QCD corrections as long-distance renormalization effects

Below the electroweak scale, all we have to do is to consider the QCD corrections for Yukawa couplings and Wilson coefficients. First, we introduce the QCD correction for Yukawa interaction. For the external Fermion line, divergent term is obtained from gluon 1-loop diagram as the following form:

$$\sum_A (igT^A)^2 \int \frac{d^Dk}{(2\pi)^D}\, \frac{i}{(\not{p}+\not{k})}\gamma^\nu \frac{-ig_{\nu\mu}}{k^2}\gamma^\mu = \frac{ig^2}{16\pi^2}\frac{2}{\epsilon}\frac{4}{3}\not{p} + (\text{finite.}) \tag{E.3.1}$$

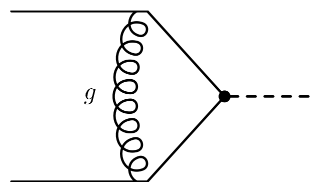


Figure 19: QCD corrections for Yukawa interaction

where p is external momentum of Fermion. Thus, this divergence is absorbed by the wave-function renormalization defined as

$$\begin{aligned}\overline{\Psi}_0\Psi_0 &\to Z_\Psi\overline{\Psi}\Psi \\ &= \overline{\Psi}\Psi + (Z_\Psi - 1)\overline{\Psi}\Psi,\end{aligned} \tag{E.3.2}$$

where the second term plays a role as a counter term absorbing divergence. Then, we have to absorb the divergent contribution Eq. (E.3.1) by Z_Ψ

$$Z_\Psi = 1 - \frac{g^2}{16\pi^2}\frac{2}{\epsilon}\frac{4}{3} + \mathcal{O}(g^4). \tag{E.3.3}$$

Similarly, the gluon 1-loop correction for the Yukawa coupling is given by

$$\sum_A (igT^A)^2 iy \int \frac{d^Dk}{(2\pi)^D}\frac{i}{\not{k}}\gamma^\nu\frac{-ig_{\nu\mu}}{k^2}\gamma^\mu\frac{i}{\not{k}} = \frac{ig^2y}{16\pi^2}\frac{2}{\epsilon}\frac{16}{3} + (\text{finite.}). \tag{E.3.4}$$

where y is Yukawa coupling constant. We make this divergence absorbed by using renormalization factor of this composite operator and wave-function renormalization.

$$y_0\Phi_0\overline{\Psi}_0\Psi_0 \to y\Phi\overline{\Psi}\Psi + \left(Z_\Psi Z_y - 1\right) y\Phi\overline{\Psi}\Psi \tag{E.3.5}$$

where Φ is the color-singlet scalar field and $y_0 = Z_y\mu^{\epsilon/2}y$. Since the divergent term must be absorbed by $Z_\Psi Z_y$, we obtain Z_y as

$$Z_y = 1 - \frac{g^2}{16\pi^2}\frac{8}{\epsilon} \tag{E.3.6}$$

and we also obtain anomalous dimension by using $g^2 = \mu^{-\epsilon}Z_g^{-2}g_0^2$

$$\frac{\mathrm{d}y}{\mathrm{d}\ln\mu} = -\frac{\mathrm{d}\ln Z_y}{\mathrm{d}\ln\mu}y = -\frac{8}{16\pi^2}g^2y, \quad \gamma_y = \frac{8}{16\pi^2}g^2. \tag{E.3.7}$$

Since β function for QCD coupling is given by

$$\beta(g) \equiv \frac{\mathrm{d}g}{\mathrm{d}\ln\mu} = \frac{1}{16\pi^2}b_S g^3, \tag{E.3.8}$$

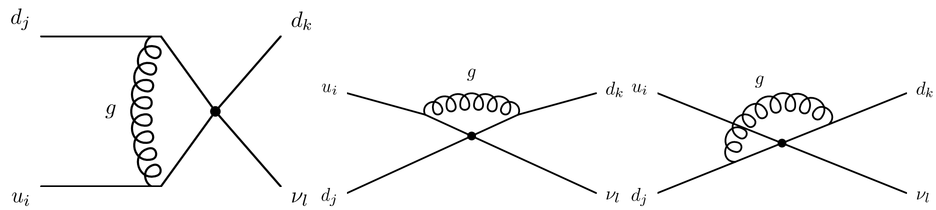


Figure 20: QCD corrections for four-Fermi operator

we treat a Yukawa coupling y as the function of g as follows:

$$\frac{\mathrm{d}y}{\mathrm{d}g} = -\frac{8y}{b_S g} \quad \Rightarrow \quad \ln\frac{y(\mu)}{y(\mu_0)} = \ln\left[\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}\right]^{-4/b_S} \tag{E.3.9}$$

where b_S depends on the number of flavor N_q and is determined as

$$b_S = -\left(11 - \frac{2}{3}N_q\right). \tag{E.3.10}$$

Thus, we can find easily the relation between Yukawa coupling at m_Z scale and that at 2GeV scale

$$y(m_Z) = \left[\frac{\alpha_S(m_Z)}{\alpha_S(m_b)}\right]^{12/23}\left[\frac{\alpha_S(m_b)}{\alpha_S(2\mathrm{GeV})}\right]^{12/25} y(2\mathrm{GeV}) \tag{E.3.11}$$

Similarly, we can obtain the relation between the Wilson coefficient of four-Fermi operator at m_Z and at 2GeV. Now, we consider the case that the bare four-Fermi operator is defined as

$$\lambda_0 \Psi^0_{q_a}\Psi^0_{q_b}\Psi^0_{q_c}\Psi^0_{l_d} \to \lambda\Psi_{q_a}\Psi_{q_b}\Psi_{q_c}\Psi_{l_d} + \left(Z_\Psi^{3/2}Z_\lambda - 1\right)\lambda\Psi_{q_a}\Psi_{q_b}\Psi_{q_c}\Psi_{l_d} \tag{E.3.12}$$

where Ψ_q's correspond to quarks and Ψ_l corresponds to lepton. From QCD correction diagrams for four-Fermi operator described in Fig. 20, we obtain the divergence term as follows:

$$\left(Z_\Psi^{3/2}Z_\lambda - 1\right) = -\frac{4g^2}{16\pi^2}\frac{2}{\epsilon} \tag{E.3.13}$$

We have the anomalous dimension for four-Fermi operator from Z_λ

$$Z_\lambda = 1 - \frac{4g^2}{16\pi^2}\frac{1}{\epsilon}, \quad \gamma_\lambda = \frac{\mathrm{d}\ln Z_\lambda}{\mathrm{d}\ln\mu} = \frac{4g^2}{16\pi^2} \tag{E.3.14}$$

that is, we have the relation between the gauge coupling and the coupling of the four-fermi operator;

$$\frac{\mathrm{d}\lambda}{\mathrm{d}g} = -\frac{4\lambda}{b_S g} \quad \Rightarrow \quad \ln\frac{\lambda(\mu)}{\lambda(\mu_0)} = \ln\left[\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}\right]^{-2/b_S}. \tag{E.3.15}$$

Therefore, we obtain the long-range effect for the four-fermi operator as

$$\lambda(2\mathrm{GeV}) = \left[\frac{\alpha_S(m_Z)}{\alpha_S(m_b)}\right]^{-6/23}\left[\frac{\alpha_S(m_b)}{\alpha_S(2\mathrm{GeV})}\right]^{-6/25}\lambda(m_Z). \tag{E.3.16}$$

References

[1] J. Hisano, T. Kuwahara, and N. Nagata, "Grand Unification in High-scale Supersymmetry," *Phys.Lett.* **B723** (2013) 324–329, arXiv:1304.0343 [hep-ph].

[2] J. Hisano, D. Kobayashi, T. Kuwahara, and N. Nagata, "Decoupling Can Revive Minimal Supersymmetric SU(5)," *JHEP* **1307** (2013) 038, arXiv:1304.3651 [hep-ph].

[3] **CMS Collaboration** Collaboration, S. Chatrchyan *et al.*, "Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC," *Phys.Lett.* **B716** (2012) 30–61, arXiv:1207.7235 [hep-ex].

[4] **ATLAS Collaboration** Collaboration, G. Aad *et al.*, "Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC," *Phys.Lett.* **B716** (2012) 1–29, arXiv:1207.7214 [hep-ex].

[5] S. Glashow, "Partial Symmetries of Weak Interactions," *Nucl.Phys.* **22** (1961) 579–588.

[6] S. Weinberg, "A Model of Leptons," *Phys.Rev.Lett.* **19** (1967) 1264–1266.

[7] A. Salam, "Weak and Electromagnetic Interactions," *Conf.Proc.* **C680519** (1968) 367–377.

[8] U. Amaldi, W. de Boer, and H. Furstenau, "Comparison of grand unified theories with electroweak and strong coupling constants measured at LEP," *Phys.Lett.* **B260** (1991) 447–455.

[9] G. F. Giudice and A. Strumia, "Probing High-Scale and Split Supersymmetry with Higgs Mass Measurements," *Nucl.Phys.* **B858** (2012) 63–83, arXiv:1108.6077 [hep-ph].

[10] G. Giudice and A. Romanino, "Split supersymmetry," *Nucl.Phys.* **B699** (2004) 65–89, arXiv:hep-ph/0406088 [hep-ph].

[11] N. Arkani-Hamed, A. Gupta, D. E. Kaplan, N. Weiner, and T. Zorawski, "Simply Unnatural Supersymmetry," arXiv:1212.6971 [hep-ph].

[12] T. Goto and T. Nihei, "Effect of RRRR dimension five operator on the proton decay in the minimal SU(5) SUGRA GUT model," *Phys.Rev.* **D59** (1999) 115009, arXiv:hep-ph/9808255 [hep-ph].

[13] H. Murayama and A. Pierce, "Not even decoupling can save minimal supersymmetric SU(5)," *Phys.Rev.* **D65** (2002) 055009, arXiv:hep-ph/0108104 [hep-ph].

[14] J. Hisano, T. Moroi, K. Tobe, and T. Yanagida, "Suppression of proton decay in the missing partner model for supersymmetric SU(5) GUT," *Phys.Lett.* **B342** (1995) 138–144, arXiv:hep-ph/9406417 [hep-ph].

[15] L. J. Hall and Y. Nomura, "A Complete theory of grand unification in five-dimensions," *Phys.Rev.* **D66** (2002) 075004, arXiv:hep-ph/0205067 [hep-ph].

[16] C. Froggatt and H. B. Nielsen, "Hierarchy of Quark Masses, Cabibbo Angles and CP Violation," *Nucl.Phys.* **B147** (1979) 277.

[17] R. Harnik, D. T. Larson, H. Murayama, and M. Thormeier, "Probing the Planck scale with proton decay," *Nucl.Phys.* **B706** (2005) 372–390, arXiv:hep-ph/0404260 [hep-ph].

[18] Y. Nambu and G. Jona-Lasinio, "Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. 1.," *Phys.Rev.* **122** (1961) 345–358.

[19] Y. Nambu and G. Jona-Lasinio, "Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. 2.," *Phys.Rev.* **124** (1961) 246–254.

[20] F. Englert and R. Brout, "Broken Symmetry and the Mass of Gauge Vector Mesons," *Phys.Rev.Lett.* **13** (1964) 321–323.

[21] P. W. Higgs, "Broken symmetries, massless particles and gauge fields," *Phys.Lett.* **12** (1964) 132–133.

[22] P. W. Higgs, "Broken Symmetries and the Masses of Gauge Bosons," *Phys.Rev.Lett.* **13** (1964) 508–509.

[23] G. Guralnik, C. Hagen, and T. Kibble, "Global Conservation Laws and Massless Particles," *Phys.Rev.Lett.* **13** (1964) 585–587.

[24] **Particle Data Group** Collaboration, J. Beringer *et al.*, "Review of Particle Physics (RPP)," *Phys.Rev.* **D86** (2012) 010001.

[25] J. Christenson, J. Cronin, V. Fitch, and R. Turlay, "Evidence for the 2 pi Decay of the k(2)0 Meson," *Phys.Rev.Lett.* **13** (1964) 138–140.

[26] M. Kobayashi and T. Maskawa, "CP Violation in the Renormalizable Theory of Weak Interaction," *Prog.Theor.Phys.* **49** (1973) 652–657.

[27] N. Cabibbo, "Unitary Symmetry and Leptonic Decays," *Phys.Rev.Lett.* **10** (1963) 531–533.

[28] **Planck Collaboration** Collaboration, P. Ade *et al.*, "Planck 2013 results. XVI. Cosmological parameters," arXiv:1303.5076 [astro-ph.CO].

[29] **Super-Kamiokande Collaboration** Collaboration, Y. Fukuda *et al.*, "Measurements of the solar neutrino flux from Super-Kamiokande's first 300 days," *Phys.Rev.Lett.* **81** (1998) 1158–1162, arXiv:hep-ex/9805021 [hep-ex].

[30] **KamLAND Collaboration** Collaboration, T. Araki *et al.*, "Measurement of neutrino oscillation with KamLAND: Evidence of spectral distortion," *Phys.Rev.Lett.* **94** (2005) 081801, arXiv:hep-ex/0406035 [hep-ex].

[31] **K2K Collaboration** Collaboration, M. Ahn *et al.*, "Measurement of Neutrino Oscillation by the K2K Experiment," *Phys.Rev.* **D74** (2006) 072003, arXiv:hep-ex/0606032 [hep-ex].

[32] **T2K Collaboration** Collaboration, K. Abe *et al.*, "First Muon-Neutrino Disappearance Study with an Off-Axis Beam," *Phys.Rev.* **D85** (2012) 031103, arXiv:1201.1386 [hep-ex].

[33] **T2K Collaboration** Collaboration, K. Abe *et al.*, "Measurement of Neutrino Oscillation Parameters from Muon Neutrino Disappearance with an Off-axis Beam," *Phys.Rev.Lett.* **111** (2013) 211803, arXiv:1308.0465 [hep-ex].

[34] B. Pontecorvo, "Mesonium and anti-mesonium," *Sov.Phys.JETP* **6** (1957) 429.

[35] Z. Maki, M. Nakagawa, and S. Sakata, "Remarks on the unified model of elementary particles," *Prog.Theor.Phys.* **28** (1962) 870–880.

[36] B. Pontecorvo, "Neutrino Experiments and the Problem of Conservation of Leptonic Charge," *Sov.Phys.JETP* **26** (1968) 984–988.

[37] **DAYA-BAY Collaboration** Collaboration, F. An *et al.*, "Observation of electron-antineutrino disappearance at Daya Bay," *Phys.Rev.Lett.* **108** (2012) 171803, arXiv:1203.1669 [hep-ex].

[38] **OPERA Collaboration** Collaboration, N. Agafonova *et al.*, "Evidence for $\nu_\mu \to \nu_\tau$ appearance in the CNGS neutrino beam with the OPERA experiment," arXiv:1401.2079 [hep-ex].

[39] **SNO Collaboration** Collaboration, Q. Ahmad *et al.*, "Direct evidence for neutrino flavor transformation from neutral current interactions in the Sudbury Neutrino Observatory," *Phys.Rev.Lett.* **89** (2002) 011301, arXiv:nucl-ex/0204008 [nucl-ex].

[40] **SNO Collaboration** Collaboration, Q. Ahmad *et al.*, "Measurement of the rate of $\nu_e + d \to p + p + e^-$ interactions produced by ^8B solar neutrinos at the Sudbury Neutrino Observatory," *Phys.Rev.Lett.* **87** (2001) 071301, arXiv:nucl-ex/0106015 [nucl-ex].

[41] **Super-Kamiokande Collaboration** Collaboration, S. Fukuda *et al.*, "Determination of solar neutrino oscillation parameters using 1496 days of Super-Kamiokande I data," *Phys.Lett.* **B539** (2002) 179–187, arXiv:hep-ex/0205075 [hep-ex].

[42] **MINOS Collaboration** Collaboration, D. Michael *et al.*, "Observation of muon neutrino disappearance with the MINOS detectors and the NuMI neutrino beam," *Phys.Rev.Lett.* **97** (2006) 191801, arXiv:hep-ex/0607088 [hep-ex].

[43] **Super-Kamiokande Collaboration** Collaboration, Y. Ashie *et al.*, "Evidence for an oscillatory signature in atmospheric neutrino oscillation," *Phys.Rev.Lett.* **93** (2004) 101801, arXiv:hep-ex/0404034 [hep-ex].

[44] **MINOS Collaboration** Collaboration, P. Adamson *et al.*, "Measurement of Neutrino Oscillations with the MINOS Detectors in the NuMI Beam," *Phys.Rev.Lett.* **101** (2008) 131802, arXiv:0806.2237 [hep-ex].

[45] A. Sakharov, "Violation of CP Invariance, c Asymmetry, and Baryon Asymmetry of the Universe," *Pisma Zh.Eksp.Teor.Fiz.* **5** (1967) 32–35.

[46] S. Weinberg, "The quantum theory of fields. Vol. 2: Modern applications,".

[47] S. R. Coleman and J. Mandula, "All Possible Symmetries of the S Matrix," *Phys.Rev.* **159** (1967) 1251–1256.

[48] N. Seiberg, "Naturalness versus supersymmetric nonrenormalization theorems," *Phys.Lett.* **B318** (1993) 469–475, arXiv:hep-ph/9309335 [hep-ph].

[49] **for the ATLAS, CMS collaborations** Collaboration, A. Mann, "SUSY Searches for Inclusive Squark and Gluino Production at the LHC," arXiv:1305.3545 [hep-ex].

[50] M. A. Shifman and A. Vainshtein, "Solution of the Anomaly Puzzle in SUSY Gauge Theories and the Wilson Operator Expansion," *Nucl.Phys.* **B277** (1986) 456.

[51] G. Giudice and R. Rattazzi, "Extracting supersymmetry breaking effects from wave function renormalization," *Nucl.Phys.* **B511** (1998) 25–44, arXiv:hep-ph/9706540 [hep-ph].

[52] J. Hisano and M. A. Shifman, "Exact results for soft supersymmetry breaking parameters in supersymmetric gauge theories," *Phys.Rev.* **D56** (1997) 5475–5482, arXiv:hep-ph/9705417 [hep-ph].

[53] R. Barbieri, S. Ferrara, and C. A. Savoy, "Gauge Models with Spontaneously Broken Local Supersymmetry," *Phys.Lett.* **B119** (1982) 343.

[54] A. H. Chamseddine, R. L. Arnowitt, and P. Nath, "Locally Supersymmetric Grand Unification," *Phys.Rev.Lett.* **49** (1982) 970.

[55] H. P. Nilles, "Dynamically Broken Supergravity and the Hierarchy Problem," *Phys.Lett.* **B115** (1982) 193.

[56] L. Alvarez-Gaume, J. Polchinski, and M. B. Wise, "Minimal Low-Energy Supergravity," *Nucl.Phys.* **B221** (1983) 495.

[57] E. Cremmer, P. Fayet, and L. Girardello, "Gravity Induced Supersymmetry Breaking and Low-Energy Mass Spectrum," *Phys.Lett.* **B122** (1983) 41.

[58] L. J. Hall, J. D. Lykken, and S. Weinberg, "Supergravity as the Messenger of Supersymmetry Breaking," *Phys.Rev.* **D27** (1983) 2359–2378.

[59] L. Randall and R. Sundrum, "Out of this world supersymmetry breaking," *Nucl.Phys.* **B557** (1999) 79–118, `arXiv:hep-th/9810155 [hep-th]`.

[60] G. F. Giudice, M. A. Luty, H. Murayama, and R. Rattazzi, "Gaugino mass without singlets," *JHEP* **9812** (1998) 027, `arXiv:hep-ph/9810442 [hep-ph]`.

[61] P. Fayet, "Supersymmetry and Weak, Electromagnetic and Strong Interactions," *Phys.Lett.* **B64** (1976) 159.

[62] P. Fayet, "Spontaneously Broken Supersymmetric Theories of Weak, Electromagnetic and Strong Interactions," *Phys.Lett.* **B69** (1977) 489.

[63] G. R. Farrar and P. Fayet, "Phenomenology of the Production, Decay, and Detection of New Hadronic States Associated with Supersymmetry," *Phys.Lett.* **B76** (1978) 575–579.

[64] P. Fayet, "Relations Between the Masses of the Superpartners of Leptons and Quarks, the Goldstino Couplings and the Neutral Currents," *Phys.Lett.* **B84** (1979) 416.

[65] S. P. Martin, "Extra vector-like matter and the lightest Higgs scalar boson mass in low-energy supersymmetry," *Phys.Rev.* **D81** (2010) 035004, `arXiv:0910.2732 [hep-ph]`.

[66] M. Ibe, S. Matsumoto, and T. T. Yanagida, "Pure Gravity Mediation with $m_{3/2}$ = 10-100TeV," *Phys.Rev.* **D85** (2012) 095011, `arXiv:1202.2253 [hep-ph]`.

[67] J. L. Feng, P. Kant, S. Profumo, and D. Sanford, "Three-Loop Corrections to the Higgs Boson Mass and Implications for Supersymmetry at the LHC," *Phys.Rev.Lett.* **111** (2013) 131802, `arXiv:1306.2318 [hep-ph]`.

[68] L. J. Hall and Y. Nomura, "A Finely-Predicted Higgs Boson Mass from A Finely-Tuned Weak Scale," *JHEP* **1003** (2010) 076, `arXiv:0910.2235 [hep-ph]`.

[69] G. Giudice and A. Masiero, "A Natural Solution to the mu Problem in Supergravity Theories," *Phys.Lett.* **B206** (1988) 480–484.

[70] N. Arkani-Hamed and S. Dimopoulos, "Supersymmetric unification without low energy supersymmetry and signatures for fine-tuning at the LHC," *JHEP* **0506** (2005) 073, `arXiv:hep-th/0405159 [hep-th]`.

[71] H. Georgi and S. Glashow, "Unity of All Elementary Particle Forces," *Phys.Rev.Lett.* **32** (1974) 438–441.

[72] N. Sakai, "Naturalness in Supersymmetric Guts," *Z.Phys.* **C11** (1981) 153.

[73] S. Dimopoulos and H. Georgi, "Softly Broken Supersymmetry and SU(5)," *Nucl.Phys.* **B193** (1981) 150.

[74] W. Siegel, "Supersymmetric Dimensional Regularization via Dimensional Reduction," *Phys.Lett.* **B84** (1979) 193.

[75] I. Jack and D. Jones, "Regularization of supersymmetric theories," `arXiv:hep-ph/9707278 [hep-ph]`.

[76] S. Gates, M. T. Grisaru, M. Rocek, and W. Siegel, "Superspace Or One Thousand and One Lessons in Supersymmetry," *Front.Phys.* **58** (1983) 1–548, `arXiv:hep-th/0108200 [hep-th]`.

[77] D. Stockinger, "Regularization by dimensional reduction: consistency, quantum action principle, and supersymmetry," *JHEP* **0503** (2005) 076, `arXiv:hep-ph/0503129 [hep-ph]`.

[78] H. Baer, J. Ferrandis, K. Melnikov, and X. Tata, "Relating bottom quark mass in DR-BAR and MS-BAR regularization schemes," *Phys.Rev.* **D66** (2002) 074007, `arXiv:hep-ph/0207126 [hep-ph]`.

[79] M. Shiozawa, "Preliminary results for the Super-Kamiokande Collaboration, presented at TAUP 2013, Asilomar, CA.,".

[80] K. Babu, E. Kearns, U. Al-Binni, S. Banerjee, D. Baxter, *et al.*, "Baryon Number Violation," `arXiv:1311.5285 [hep-ph]`.

[81] N. Sakai and T. Yanagida, "Proton Decay in a Class of Supersymmetric Grand Unified Models," *Nucl.Phys.* **B197** (1982) 533.

[82] S. Weinberg, "Supersymmetry at Ordinary Energies. 1. Masses and Conservation Laws," *Phys.Rev.* **D26** (1982) 287.

[83] C. Munoz, "Enhancement Factors for Supersymmetric Proton Decay in SU(5) and SO(10) With Superfield Techniques," *Phys.Lett.* **B177** (1986) 55–59.

[84] L. Abbott and M. B. Wise, "The Effective Hamiltonian for Nucleon Decay," *Phys.Rev.* **D22** (1980) 2208.

[85] Y. Aoki, E. Shintani, and A. Soni, "Proton decay matrix elements on the lattice," `arXiv:1304.7424 [hep-lat]`.

[86] S. Chadha and M. Daniel, "Chiral Lagrangian Calculation of Nucleon Decay Modes Induced by $d = 5$ Supersymmetric Operators," *Nucl.Phys.* **B229** (1983) 105.

[87] **JLQCD Collaboration** Collaboration, S. Aoki *et al.*, "Nucleon decay matrix elements from lattice QCD," *Phys.Rev.* **D62** (2000) 014506, `arXiv:hep-lat/9911026 [hep-lat]`.

[88] J. Hewett, H. Weerts, R. Brock, J. Butler, B. Casey, *et al.*, "Fundamental Physics at the Intensity Frontier," `arXiv:1205.2671 [hep-ex]`.

[89] L. Wolfenstein, "Parametrization of the Kobayashi-Maskawa Matrix," *Phys.Rev.Lett.* **51** (1983) 1945.

[90] J. Bagger and J. Wess, "Supersymmetry and supergravity,".

[91] S. L. Adler and W. A. Bardeen, "Absence of higher order corrections in the anomalous axial vector divergence equation," *Phys.Rev.* **182** (1969) 1517–1536.

[92] W. Pauli and F. Villars, "On the Invariant regularization in relativistic quantum theory," *Rev.Mod.Phys.* **21** (1949) 434–444.

[93] G. 't Hooft and M. Veltman, "Regularization and Renormalization of Gauge Fields," *Nucl.Phys.* **B44** (1972) 189–213.

[94] M. E. Machacek and M. T. Vaughn, "Two Loop Renormalization Group Equations in a General Quantum Field Theory. 1. Wave Function Renormalization," *Nucl.Phys.* **B222** (1983) 83.

[95] V. Novikov, M. A. Shifman, A. Vainshtein, and V. I. Zakharov, "Exact Gell-Mann-Low Function of Supersymmetric Yang-Mills Theories from Instanton Calculus," *Nucl.Phys.* **B229** (1983) 381.

[96] N. Arkani-Hamed and H. Murayama, "Holomorphy, rescaling anomalies and exact beta functions in supersymmetric gauge theories," *JHEP* **0006** (2000) 030, `arXiv:hep-th/9707133 [hep-th]`.

[97] A. Pomarol and R. Rattazzi, "Sparticle masses from the superconformal anomaly," *JHEP* **9905** (1999) 013, `arXiv:hep-ph/9903448 [hep-ph]`.

[98] N. Arkani-Hamed, G. F. Giudice, M. A. Luty, and R. Rattazzi, "Supersymmetry breaking loops from analytic continuation into superspace," *Phys.Rev.* **D58** (1998) 115005, arXiv:hep-ph/9803290 [hep-ph].

[99] S. Weinberg, "Effective Gauge Theories," *Phys.Lett.* **B91** (1980) 51.

[100] L. J. Hall, "Grand Unification of Effective Gauge Theories," *Nucl.Phys.* **B178** (1981) 75.

[101] T. Appelquist and J. Carazzone, "Infrared Singularities and Massive Fields," *Phys.Rev.* **D11** (1975) 2856.

[102] M.-x. Luo, H.-w. Wang, and Y. Xiao, "Two loop renormalization group equations in general gauge field theories," *Phys.Rev.* **D67** (2003) 065019, arXiv:hep-ph/0211440 [hep-ph].